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Feasibility Study of a Pulsed Thermonuclear Reactor

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Feasibility Study of a Pulsed Thermonuclear Reactor

by

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ABSTRACT

The minimal conditions of burning time, plasma temperature, and number density yielding net energy balance in a pulsed reactor are investigated. The analogue of a θ pinch is considered in which a long cylinder of $\beta = 1$ plasma is contained by a magnetic field, furnished by a single turn coil. The basic energy losses are joule heating of the coil by the magnetic field, heating of the coil by neutrons as they pass through it to the neutron blanket, and bremsstrahlung. The basic energy source is kinetic energy of the neutrons from d-t fusion reactions. Fuel burnup and α-particle heating of the plasma are also considered. Heat is removed from the beryllium-copper coil by means of turbulent flow of supercritical H_2O in channels. A coil temperature of $\sim 500^{\circ}C$ is consistent with the requirements of supercriticality of the H_2O. The energy required for pumping the heat-transfer coolant is taken into account in the energy balance. Two basic electrical models of the pulsed reactor are taken. In the first, all of the magnetic field B is furnished by the coil. In the second, the sustained-field θ pinch, a steady magnetic field is suddenly pulsed off by an opposite magnetic impulse of short duration. For B = 200 kG the minimum burning times τ_T in the two cases are about 15 and 1.5 msec, respectively. The $n\tau_T$ product in the first case is about 10 times that of the Lawson criterion. In the second case it is almost the same.

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CONTENTS

I. PULSED SYSTEMS

In a pulsed reactor the thermonuclear energy produced by the $D(t,n)He^4$ reaction would be used to overcome losses attendant upon plasma heating and containment and to produce additional useful output energy. In pulsed systems, as opposed to steady-state systems, the plasma is burned for a time τ_T and then purged. On the next pulse of magnetic field new, cold plasma is heated and burned again. In the present case we are guided by the θ-pinch analogy to consider a long toroidal or straight cylinder of $\beta = 1$ plasma, contained by a longitudinal magnetic field B which is furnished by a single-turn coil. (β is the ratio of plasma pressure to that of the external magnetic field.) We assume the plasma to have n/2 deuterons, n/2 tritons, and n electrons per cm^3 at a common temperature T.

The minimal conditions of plasma temperature, density, and burning time necessary to achieve a favorable balance between thermonuclear energy production and plasma losses are given by the Lawson criterion,[1] discussed in detail in Appendix I. In this criterion all losses external to the plasma itself are neglected, and one arrives at the following conditions for net energy gain per unit volume of plasma: There is a minimum plasma temperature near 25 keV at which the product $n\tau_T$ of number density and burning time is about $0.7 \times 10^{14}\ cm^{-3}$ sec. For a $\beta = 1$ plasma, pressure balance determines n in terms of the containing magnetic field B. Thereby τ_T is also determined. For a practical upper limit on B of 250 kG, $\tau_T \approx 2$ msec.

When one attempts to extend these considerations to determine plasma conditions sufficient for a net-power-producing reactor, the situation becomes less definite, since a particular model must be assumed in order to evaluate actual energy losses. One must not lose sight of the fact that the thermonuclear reactor problem is basically one of plasma physics, and that the materials technology which is necessary for its eventual solution will probably need to be developed to accommodate the exigencies of the plasma solution. Therefore, in considering feasibility of a reactor system, it may be premature to specify in terms of the present state of the art such things as energy sources for furnishing the magnetic field and heat exchange systems for conversion of thermonuclear heat to useful output. For example, at the large stored energies which are eventually necessary, magnetic energy storage, using cryogenic conductors or superconductors, is probably the most appropriate. Even the magnitude of the magnetic fields which can eventually be used in superconducting systems is presently unknown.

However, there are a number of basic elements which must comprise a thermonuclear reactor and which determine its losses when certain assumptions are made:

(1) The thermonuclear energy is emitted predominately as kinetic energy of 14-MeV neutrons. This must be converted to heat in a moderating blanket outside the plasma. For reasonable thermal efficiency the blanket should simultaneously act as a heat-exchange coolant at a temperature in excess of 200°C. The same blanket must also breed sufficient tritium to maintain a positive tritium budget against reprocessing losses.

(2) For a given $n\tau_T$ a fraction

$$f = \frac{1}{2} n\tau_T \overline{\sigma v} \tag{1}$$

of the d-t is burned up. Assuming the energy of the α particles to be retained and thermalized in the plasma, which also expands against constant magnetic field B, one can calculate the work ΔW done against the magnetic field. This represents electrical work done on the electrical energy source that furnishes the magnetic field and can partially make up the joule heating loss W_E, allowing the electrical system to be partly self sustaining. The internal energy of the plasma is increased an equal amount, allowing bremsstrahlung cooling to be made up by α-particle heating.

(3) The coil which furnishes the magnetic field is commonly conceived of as a hard superconductor when steady-state reactors are considered, and it is assumed to be placed outside the moderating blanket in order that heat deposited in it by the neutrons need not be removed by the refrigerator which maintains the temperature of the coil. In a pulsed reactor it may or may not be necessary to furnish the magnetic energy filling the volume of the blanket. In either case, in the models considered below, the pulsed coil is placed inside the blanket. The energy deposition by neutrons in the coil then becomes a major factor in the energy balance, and its treatment largely controls the materials technology. It is prudent to choose an inexpensive coil material which can be replaced when and if radiation damage becomes excessive.

Two basic models are chosen. Both use a hot, Be-Cu coil inside the neutron blanket. In the first model all of the magnetic field is furnished by this coil, so that none permeates the neutron blanket. The second model, discussed in Section X, is that of the sustained-field θ pinch,[2] in which a pulsed, hot, Be-Cu coil lies inside a steady field which can be furnished by a superconductor placed outside the neutron blanket. In this case, the pulsed electrical circuit is only slightly damped because of the very short duration τ of the magnetic impulse used to form the hot plasma. In the first model the circuit is heavily damped. The advantage of the hot Be-Cu coil over a cryogenic aluminum coil is that no refrigeration losses are incurred; and it is thus more efficient, even though the resistivity is two orders of magnitude higher than that of Al, cooled to liquid H_2 temperatures.

Owing to its proximity to the plasma, there is sufficient heat generated in the coil during the reactor pulse by electrical joule heating, bremsstrahlung, and nuclear reactions from the 14-MeV neutrons to drive its temperature well above its initial value, if one relied only upon the thermal conductivity and heat capacity of the coil material itself to accommodate the heat. It is possible, however, to remove this heat by flowing supercritical H_2O through coolant channels so that the temperature excursion stays within modest bounds. The pumping power required for the heat transfer must be taken into account in the overall energy balance.

The energy balance study will reduce itself primarily to determining approximately the minimum values the plasma burning time τ_T can take. The corresponding density values n are determined by the plasma temperature and the magnetic field.

II. PLASMA ENERGY RELATIONS

A. Plasma Energy and Magnetic Energy

Following θ-pinch experience,[3] we choose a system with $\beta = 8\pi p_p/B^2 \approx 1$ ($p_p = 2$ nkT is the plasma pressure). Thus the plasma energy content per cm length of reactor,

$$W_p = 1.508 \times 10^{-15} R_p^2 \, nkT, \qquad (J/cm) \qquad (2)$$

and the magnetic energy per cm length,

$$W_m = 1.25 \, B^2 R_c^2 \times 10^{-8}, \qquad (J/cm) \qquad (3)$$

are related by

$$nkT = 1.242 \times 10^7 \, B^2. \qquad (cm^{-3} \, keV) \qquad (4)$$

Here and throughout this paper kT is in keV, and B is in G (kG in Appendix II). R_p is the plasma radius and R_c the coil inner radius, both in cm. In terms of B and kT,

$$W_p = (3/2) \, (R_p/R_c)^2 \, W_m = 1.875 \times 10^{-8} R_p^2 B^2. \qquad (J/cm) \qquad (5)$$

B. Thermonuclear Power

The thermonuclear power per cm length is

$$P_T = (\pi/4) \, R_p^2 n^2 \, Q\overline{\sigma v}, \qquad (6)$$

where

$$Q = 18.9 \text{ MeV} = 3.03 \times 10^{-12} \text{ J.} \tag{7}$$

Here Q includes the 14.1 MeV kinetic energy of the d-t neutrons as well as the 4.8 MeV of kinetic energy derived from the Li^6 $(n,\alpha)t$ reaction which breeds tritium. The quantity $\overline{\sigma v}$ is the d-t fusion cross section, multiplied by the relative d-t velocity and averaged over a Maxwellian distribution at temperature T. A graph of $\overline{\sigma v}$ vs kT is given in Fig. 1. Substituting from Eq. (4) for n gives

$$P_T = 1.943 \times 10^{-2} \, B^4 R_p^2 \, Q\overline{\sigma v}/(kT)^2, \quad \text{(W/cm)} \tag{8}$$

where Q is also in keV. Values of $(kT)^2/Q\overline{\sigma v}$ for various kT values are given in Table I.

TABLE I

FUSION QUANTITIES USEFUL IN CALCULATING REACTOR ENERGY BALANCE
(The value Q = 18.9 MeV is assumed) (kT is in keV throughout)

kT	$(kT)^{\frac{1}{2}}/Q\overline{\sigma v}$	$(kT)^2/Q\overline{\sigma v}$
5	7.84×10^{12}	8.77×10^{13}
8	2.32×10^{12}	5.25×10^{13}
10	1.39×10^{12}	4.39×10^{13}
15	7.69×10^{11}	4.47×10^{13}
20	5.75×10^{11}	5.14×10^{13}
25	4.78×10^{11}	5.98×10^{13}
30	4.51×10^{11}	7.41×10^{13}
40	4.21×10^{11}	1.06×10^{14}
50	4.48×10^{11}	1.58×10^{14}
70	5.00×10^{11}	2.93×10^{14}

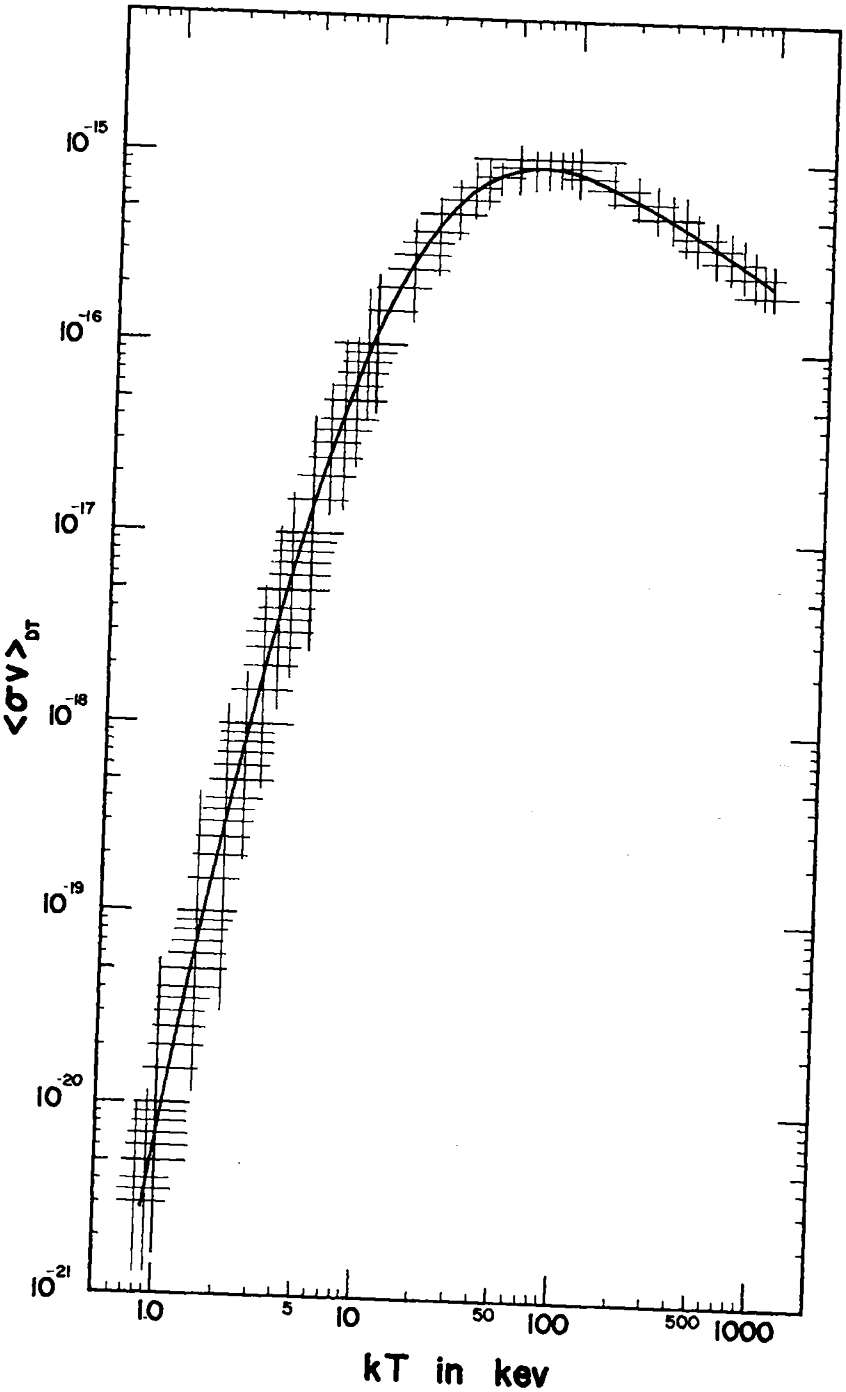

Fig. 1. The nuclear cross section for fusion reactions between deuterons and tritons, multiplied by their relative velocity and averaged over a Maxwellian distribution of velocities, as a function of plasma temperature.

C. Bremsstrahlung

The Bremsstrahlung from the plasma is also an important factor, since it is absorbed by the coil or discharge-tube wall. Taking a Gaunt factor of 1.14 (appropriate to $kT_e \approx 15$ keV), the emitted power per unit length of plasma is

$$P_B = 1.72 \times 10^{-30} R_p^2 n^2 (kT)^{\frac{1}{2}}, \qquad \text{(W/cm)} \tag{9}$$

where kT is in keV. Comparing with Eq. (6), the ratio of the bremsstrahlung and thermonuclear powers is

$$P_B/P_T = 1.37 \times 10^{-14} (kT)^{\frac{1}{2}} /Q\overline{\sigma v}, \tag{10}$$

where kT and Q are both in keV. Values of $(kT)^{\frac{1}{2}} /Q\overline{\sigma v}$ are given in Table I.

III. FUEL BURNUP, ALPHA-PARTICLE HEATING, AND DIRECT CONVERSION OF FUSION ENERGY

The fraction of d and t ions which react during time τ_T is

$$f = \tfrac{1}{2} n\tau_T \overline{\sigma v}. \tag{1}$$

The number of α particles per cm length of plasma, produced and held in or close to the plasma at high magnetic fields, is

$$N_\alpha = \tfrac{1}{2} \pi R_p^2 nf, \tag{11}$$

where R_p and n refer to the plasma in which α-particle effects are negligible ("before" burnup).

In the case of a reactor the time τ_T is sufficiently large that equipartition occurs between the new α particles, which are born in the plasma with kinetic energy $E_\alpha = 3.52$ MeV, and the remaining d and t ions (cf. Section XII. A below). Hence, the plasma will be heated. It will also expand, pushing magnetic flux out of the coil and into the energy source, leading to direct recovery of some thermonuclear energy. For unit length of plasma, energy conservation is expressed by

$$W_f - 3\pi R_p^2 nkT = N_\alpha E_\alpha - \pi(B^2/8\pi) (R_f^2 - R_p^2), \tag{12}$$

where R_f, and T_f, and W_f are the plasma radius, temperature, and energy after the α particles have heated and expanded the plasma. The last term on the right is the work ΔW done against the magnetic field, whose variation is neglected.

For equipartition

$$W_f = 3\pi R_f^2 \left[n_f + (3/2)n_\alpha\right] kT_f, \tag{13}$$

where n_f and n_α are the d-t and α-particle densities in the plasma after α-particle heating. Pressure balance requires

$$(2n_f + 3n_\alpha)\, kT_f = B^2/8\pi. \tag{14}$$

In both Eqs. (13) and (14) the contributions of the electrons (two per α particle) are taken into account. Hence

$$W_f = (3/2)\, \pi\, R_f^2 (B^2/8\pi). \tag{15}$$

In addition, pressure balance before expansion gives

$$2\, nkT = B^2/8\pi. \tag{16}$$

Substituting Eqs. (16), (15), and (11) into Eq. (12) gives for the expansion of the plasma

$$R_f^2/R_p^2 = 1 + fE_\alpha/(10\ kT). \tag{17}$$

Conservation of particles requires

$$R_f^2(n_f + 2n_\alpha) = R_p^2 n. \tag{18}$$

This can also be written

$$N_f + 2N_\alpha = \pi R_p^2 n. \tag{19}$$

Combining Eqs. (18), (16), (14), and (11) gives for the temperature after α-particle heating

$$kT_f/kT = R_f^2/R_p^2 (1-f/4). \tag{20}$$

Finally, combining Eqs. (20), (18), (16), and (14) gives for the final density of d-t ions

$$n_f = (1-f)n\, R_p^2/R_f^2. \tag{21}$$

Evaluating the last term of Eq. (12) gives, for the work ΔW to drive magnetic energy back into the energy source,

$$\Delta W = W_m (R_p/R_c)^2 \;(fE_\alpha/10kT). \tag{22}$$

In order to show the effects of burnup, various quantities are plotted versus f in Fig. 2 for a plasma temperature (before burnup) of 15 keV. Since the quantity $n^2\,\overline{\sigma v}$ is a strongly decreasing function of f, it is clear that the plasma would not be ignited by the α-particle heating. Note, however, that $P_T \propto n^2\,\overline{\sigma v}\,R_f^2$ is a slowly decreasing function of f. In order to set an approximate limit on f it is instructive to take R_c = 10 cm, R_p = 5 cm. For the plasma not to expand to the wall, f can be as large as ~ 0.12. This would also not quench the reaction unduly. According to Eq. (1) this requires that $n\tau_T \lesssim 1.5 \times 10^{15}$ at kT = 15 keV. Note that Eq. (22) can also be written

$$\Delta W = (0.4\, E_\alpha/Q)P_T\tau_T = 0.0745\; P_T\tau_T. \tag{23}$$

The increase in plasma internal energy, owing to α-particle heating, is

$$\Delta U = W_f - W_p = \Delta W. \tag{24}$$

IV. RESISTIVE ENERGY LOSS IN THE MAGNET COIL

For a sine wave of magnetic field having half-period τ the magnetic field penetrates the conductor to an "electrical" skin depth

$$\delta_E = 0.503 \times 10^4\; \eta^{\frac{1}{2}}\; \tau^{\frac{1}{2}}, \qquad \text{(cm)} \tag{25}$$

where η is the resistivity in Ω-cm. A current density $j \approx 0.4\pi\delta_E\, B$ arises and hence an energy loss per unit length $W_E' \approx (\tau/2)\; 2\pi R_c \delta_E j^2 \eta$. In order to determine the constant in this dimensional analysis for W_E' exactly, a comparison was made with numerical calculations of the penetration of a pulse of magnetic field into a cylindrical conductor. Details of the computation are given in Appendix II.

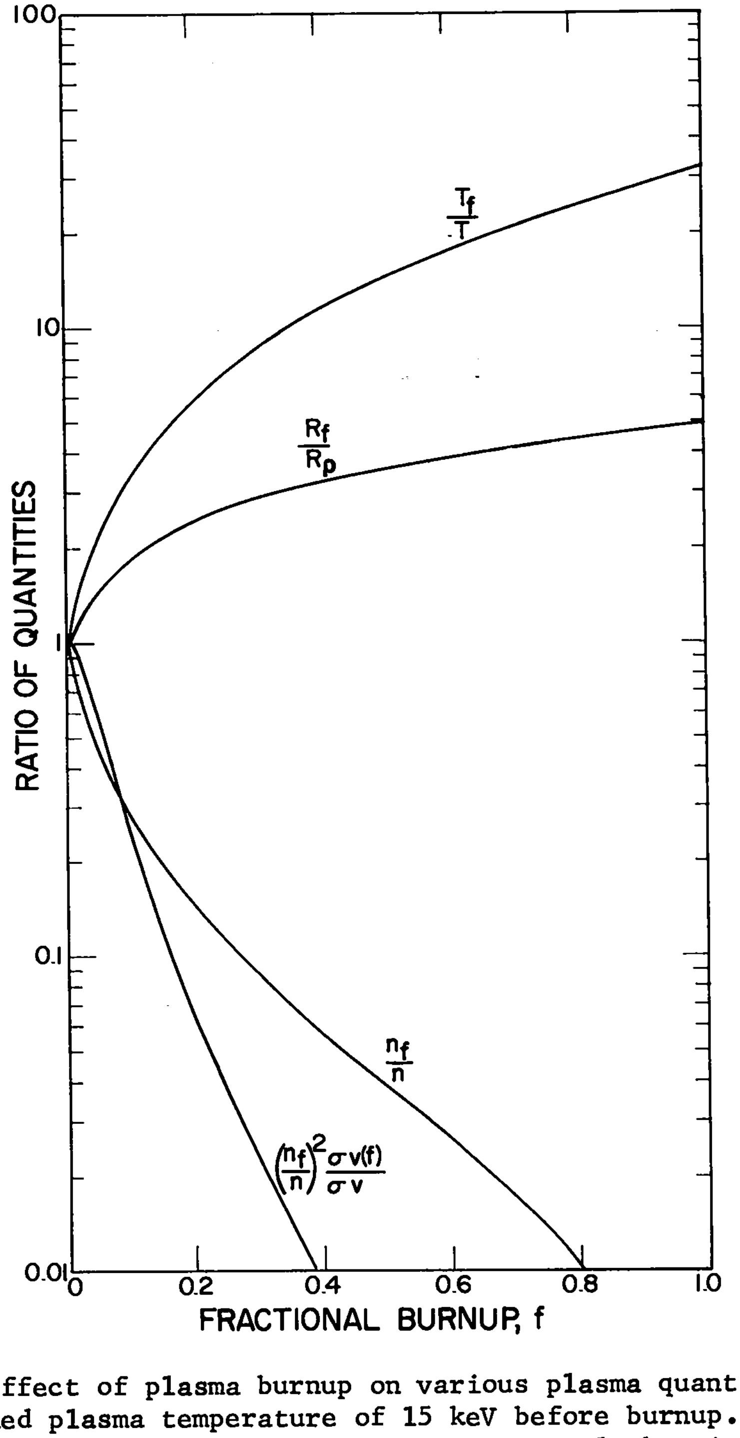

Fig. 2. The effect of plasma burnup on various plasma quantities for an assumed plasma temperature of 15 keV before burnup. T_f/T is the ratio of plasma temperature after α-particle heating to that before. R_f/R_p and n_f/n are the corresponding ratios of plasma radii and densities.

A complete skin penetration problem was programmed for the IBM-7094. The field profile inside the conductor was determined as a function of time and the total energy loss then determined as a function of τ. The result for the joule-heating energy loss is

$$W'_E = 2.25 \times 10^{-4} \ \eta^{\frac{1}{2}} \tau^{\frac{1}{2}} B^2 R_c. \qquad \text{(J/cm)} \qquad (26)$$

Some additional surface area at the same field and resistivity is assumed in order to account for current at the feed slot of the coil. An additional length of two times the coil circumference is taken. Multiplying Eq. (26) by three gives

$$W_E = 6.76 \times 10^{-4} \ \eta^{\frac{1}{2}} \tau^{\frac{1}{2}} B^2 R_c. \qquad \text{(J/cm)} \qquad (27)$$

This can also be written

$$W_E/W_m = 5.40 \times 10^4 \ \eta^{\frac{1}{2}} \tau^{\frac{1}{2}}/R_c. \qquad (28)$$

Here W_m is taken as $(\pi R_c^2)B^2/8\pi$, even in the case of large skin depth. The time τ is evaluated in analogy to the slightly damped case as twice the time lapse between events when the magnetic field has $1/\sqrt{2}$ its peak value.

V. LOSS OF ENERGY TO THE COIL MATERIAL BY NEUTRONS

A. Elastic Strength and Wall Thickness of the Coil

A major contribution to the coil heating is that due to the interaction of the 14-MeV neutrons with the copper nuclei as they pass through it. This heating is proportional to P_T and to the thickness ΔR_c of the coil. The coil thickness is in turn set by its strength against the pressure $B^2/8\pi$ of the magnetic field. In the sustained-field case this pressure must be supported on the outside as compressive pressure and leads to compressive stress $\tau_{\theta\theta}$ in the coil material. In the ordinary pulsed case the magnetic pressure is inside the coil and leads to tensile stress $\tau_{\theta\theta}$. If R_c be the inner coil radius and R_o its outer radius the tensile stress is given by

$$\tau_{\theta\theta} = \frac{B^2}{8\pi} \frac{R_o^2 R_c^2}{R_o^2 - R_c^2} \left(\frac{1}{R_o^2} + \frac{1}{r^2} \right), \tag{29}$$

with an analogous expression for the compressive stress. The maximum stress occurs at the inner (outer) radius, and a thickness

$$\Delta R_c = \frac{B^2}{8\pi\tau_y} \frac{R_c^2 + R_o^2}{R_c + R_o} \tag{30}$$

is required, where τ_y is the maximum permissible stress in the coil. In the tensile case the yield stress of 500°C Be-Cu is approximately 70,000 psi.[4] With a safety factor of two, τ_y reduces to 35,000 psi. For the compressive case τ_y could be taken considerably larger since coil destruction in this case is actually determined by instability of motion for metallic flow. In the tensile and compressive cases two measures are assumed in order to reduce the thickness as much as possible over Eq. (30): (1) The metal is prestressed so that its inner (outer) surface starts out in compression (tension) at 35,000 psi. (2) The stress is equalized, by prestressing or by radial ribs, so that outer and inner portions of the coil play an equal role in supporting the magnetic pressure. The net result is to allow one to set τ_y = 70,000 psi and to use the simple, thin-coil formula for coil thickness:

$$\Delta R_c = B^2 R_c / 8\pi\tau_y = 8.29 \times 10^{-12} \, B^2 R_c, \qquad \text{(cm)} \tag{31}$$

where B is in G, and R_c is in cm. Equation (31) gives a coil thickness of 3.3 cm at 200 kG.

In the tensile case it should be noted that smaller coil thicknesses than those of Eq. (31) could be used by transferring some of the magnetic pressure through the coil to the outer neutron blanket and a heavy outer shell. Owing to the compressibility of the neutron blanket coolant, its hydraulic pressure would probably have to be increased simultaneously with the magnetic pressure. In the calculations below, however, the values given by Eq. (31) will be used, both in tension and compression. As noted above, this is particularly conservative in the compression case.

B. Nuclear Interactions of 14-MeV Neutrons with the Coil Material

A compilation of these interactions and the corresponding energy depositions in the copper is given in Table II. It can be summarized by an energy loss times cross section, $\sigma E = 4.1$ b-Mev. The nuclear energy deposited per cm length of coil per second is

$$P_N = \Delta R_c n_c (\sigma E) \left(\frac{\pi}{4} R_p^2 n^2 \overline{\sigma v}\right), \tag{32}$$

$$P_N = \Delta R_c n_c (\sigma E/Q)\ P_T, \tag{33}$$

where n_c is the number density of copper atoms ($8.42 \times 10^{22}\ cm^{-3}$). Substituting Eq. (31) and the value of σE gives

$$P_N/P_T = 1.52 \times 10^{-13} B^2 R_c. \tag{34}$$

Note that of the various interactions given in Table II, only the (n,p) process leads to loss of neutrons in passing through the coil. Hence, 14-MeV neutron absorption is negligible.

VI. CHARACTERISTIC TIMES AND THE DUTY FACTOR

Two times have been identified: The time τ_T characteristic of d-t burning and the time τ equal to the half period of the magnetic field when it is sinusoidal. Since the thermonuclear power P_T varies as B^4 [Eq. (8)], the relation between these two times in the case of a sinusoid is given by

$$\tau_T = \int_0^{\tau} \sin^4(\pi t/\tau)dt = (3/8)\tau. \tag{35}$$

It is necessary to define a duty factor ξ = (time between pulses)/τ_T. In the sinusoidal case a reasonable time between pulses would be 2τ; in which case $\xi = 16/3$. However, for the sustained-field θ pinch no such period is defined. In this case the time between pulses would be more nearly that required to purge the "used" plasma from, say, a 1-m length of reactor. Assuming that it could be done at a molecular speed of 10^5 cm/sec, about 10^{-3} sec would be required. This is less than 5.33 times the value of τ_T found below for energy balance.

In general we shall assume ξ values in excess of 16/3.

The time τ_c characteristic of containment is that during which $nkT \propto B^2$ is large. Hence

$$\tau_c = \int_0^{\tau} \sin^2(\pi t/\tau)dt = (1/2)\tau. \tag{36}$$

Therefore the containment and burning times are related by

$$\tau_c = (4/3)\tau_T. \tag{37}$$

TABLE II

CROSS SECTIONS OF Cu TO 14-MeV NEUTRONS AND ASSOCIATED ENERGIES DEPOSITED IN SOLID COPPER

(Cross Sections are averaged for the two stable isotopes)

Process	Cross Section (b)	Mean Energy Deposited (MeV)
Inelastic gamma rays	0.64	5.0
Elastic scattering	1.45	0.04
(n,np)	0.097	7.3
(n,p)	0.006	12.8
(n,2n)	0.76	0.10
Total	2.95	

Mean cross section times energy deposited = 4.12 b-MeV
= 6.60×10^{-13} b-J.

Average energy deposited = 1.4 MeV.

VII. HEATING OF THE COIL AND FEED SLOT

A. The Thermal Skin Depth

During an impulse of duration t heat will penetrate a distance given approximately by the thermal skin depth[5]

$$\delta_T = (2K_v/\pi C_v)^{\frac{1}{2}} t^{\frac{1}{2}} , \tag{38}$$

where K_v is the thermal conductivity, and C_v is the heat capacity per unit volume of the conductor. This is approximately the largest distance over which heat is accessible in time t. For time $\xi\tau_T$ between reactor pulses we take

$$\delta_T = (2K_v/\pi C_v)^{\frac{1}{2}} \xi^{\frac{1}{2}} \tau_T^{\frac{1}{2}}. \tag{39}$$

For copper at 500°C, $K_v = 1.6$ W/cm°C[6] and $C_v = 3.7$ J/cm^3°C,[7] so that

$$\delta_T = 0.521 \xi^{\frac{1}{2}} \tau_T^{\frac{1}{2}}. \tag{40}$$

B. Temperature Rise from a Single Reactor Pulse

Before considering transfer of the neutron, electrical, and bremsstrahlung heating, it is useful to estimate the temperature rise of the coil material from each effect during a single reactor pulse. In the following we take as a typical example: B = 200 kG, $\tau_T = 0.04$ sec, kT = 15 keV, $R_c = 10$ cm, $R_p = 5$ cm.

1. <u>Neutron Heating</u>. In this case the temperature rise is approximately

$$\delta T_N = P_N \tau_T / 2\pi R_c \Delta R_c C_v. \tag{41}$$

Substituting Eqs. (34), (31), and (8) gives $\delta T_N = 54$°C. Since the heat is deposited uniformly over the coil thickness ΔR_c it causes no appreciable temperature gradient.

2. <u>Electrical Heating</u>. This heat is deposited in a volume approximately equal to $6\pi R_c \delta_E$, including feed slot. The temperature rise is approximately

$$\delta T_E = W_E / 6\pi R_c \delta_E C_v. \tag{42}$$

Substituting Eqs. (27) and (25) gives $\delta T_E = 60^\circ C$. For the example considered δ_E is comparable to ΔR_c, so that again no appreciable temperature gradient arises.

3. Bremsstrahlung Heating. Even though the bremsstrahlung energy per pulse is small compared to that of neutron and electrical heating, it has a more serious effect, since the corresponding x rays penetrate only a small fraction of a mm into the coil or discharge-tube material. Therefore, during time τ_T this heat is deposited in a small volume $2\pi R_c \delta_T$, where δ_T is given by Eq. (38) with $t = \tau_T$. The mean temperature rise is approximately

$$\delta T_B = P_B \tau_t / 2\pi R_c \delta_T C_v . \qquad (43)$$

Substituting Eq. (9) gives $\delta T_B = 300^\circ C$. This is not large enough to melt the material, even if it is considered to be copper rather than a more refractive skin material such as tantalum. However, it generates such a high temperature gradient that it could be expected to strain the material of the inner skin past its yield point if it were rigidly attached to the body of the coil. (This problem arises also in steady-state reactors, where bremsstrahlung impinges on the inside of a metal vacuum vessel.) We visualize two possible measures against this effect. The first measure is to take special pains to remove heat from the inner skin. The second is to allow a metal skin of thickness $< \delta_T$, with a longitudinal slot, to be in circumferentially sliding contact with the inner coil surface so that it can change its circumferential length rather than crumpling.

VIII. NECESSARY HEAT-TRANSFER COEFFICIENTS

Both the neutron and electrical heat are deposited throughout the coil volume of thickness $\sim \Delta R_c$. Since ΔR_c is much larger than δ_T, this heat must be removed by channels, which we imagine to be parallel to the coil axis and to have diameter d. They are assumed to be spaced in a regular array by a distance $x\delta_T$, where x can be no larger than two. Since we have seen that a single reactor pulse causes only moderate temperature rise, we are free to decrease the rate of heat removal by allowing it to take place over the whole interval $\xi\tau_T$ between pulses. Increasing ξ mitigates the heat-transfer problem at the cost of increasing the reactor length for a given total power output. We consider the heat-transfer problem in three parts, corresponding to removal of neutron and electrical heat from the circular body of the coil, electrical heat from the feed slot, and bremsstrahlung heat from an inner skin. In each case we calculate the wetted perimeter S of the coolant channels, as well as the area A of coolant flow. In addition the heat-transfer coefficient h is computed for an assumed local temperature difference ΔT between coolant and coil material.

A. Neutron and Electrical Heat in the Coil

We assume a hole spacing $x\delta_T$, where δ_T is given by Eq. (39) or (40). In coil thickness ΔR_c the wetted perimeter (equal to heat-transfer area per unit length) is

$$S_\Sigma = 19.8\ R_c \Delta R_c d/x^2 \delta_T^2, \qquad \text{(cm)} \tag{44}$$

where all dimensions are in cm. Here Eq. (31) reduces to

$$\Delta R_c = 5.77 \times 10^{-7} B^2 R_c/\tau_y, \quad \text{(cm)} \tag{45}$$

where B is in G and τ_y, the yield strength of the coil material, is in psi. The flow area is

$$A_\Sigma = 5.30 \times 10^{-3}\ R_c \Delta R_c d^2/x^2 \delta_T^2, \quad (\text{ft}^2) \tag{46}$$

where linear dimensions on the right-hand side of Eq. (46) are all in cm. The hydraulic radius[8] $D_\Sigma = A_\Sigma/S_\Sigma$, which determines the Reynolds number of the flow, is just d/4.

The heat-transfer coefficient necessary to remove the neutron and electrical heat is

$$h_\Sigma = P_\Sigma/S_\Sigma \Delta T_\Sigma \xi, \qquad (\text{W/cm}^2\,^\circ\text{C}) \tag{47}$$

where

$$P_\Sigma = P_N + W_E/3\tau_T, \qquad (W/cm^2) \qquad (48)$$

and ΔT_Σ is the local temperature difference between the coil material and the coolant in ^oC. In British units

$$h_\Sigma = 1.762 \times 10^3 \; P_\Sigma/S_\Sigma \Delta T_\Sigma. \qquad (Btu/ft^2 hr^oF). \qquad (49)$$

In this expression for h_Σ, P_Σ is in W/cm and S_Σ is in cm. This grouping of units will be followed in the analogous cases of h_E and h_B below.

B. Electrical Heating at the Feed Slot

Here the effective thickness containing channels is δ_E over a length of $4\pi R_c$. Hence, using Eq. (25),

$$S_E = 3.24 \times 10^5 \; \eta^{\frac{1}{2}} \tau_T^{\frac{1}{2}} \; R_c d_E / (x_E \delta_T)^2, \qquad (cm) \qquad (50)$$

$$A_E = 87.3 \; \eta^{\frac{1}{2}} \tau_T^{\frac{1}{2}} \; R_c d_E^2 / (x_E \delta_T)^2, \qquad (ft^2) \qquad (51)$$

$$h_E = (1.76 \times 10^3)(2 \; W_E/3)/S_E \Delta T_E \xi \tau_T, \quad (Btu/ft^2 hr^oF) \qquad (52)$$

$$h_E = (3.16 \times 10^{-6}) B^2 x_E^2 \delta_T^3 / d_E \Delta T_E \xi \tau_T. \qquad (Btu/ft^2 hr^oF) \qquad (53)$$

C. Bremsstrahlung Heating at the Inner Coil Surface

Here a single row of holes, spaced by $x_E \delta_T$, is assumed on a circular cylinder close to the coil inner surface. Then

$$S_B = 19.8 \; R_c d_B / x_B \delta_T, \qquad (cm) \qquad (54)$$

$$A_B = 5.312 \times 10^{-3} R_c d_B^2 / x_B \delta_T, \qquad (ft^2) \qquad (55)$$

$$h_B = 89.3 \; P_B x_B \delta_T / R_c d_B \Delta T_B \xi. \qquad (Btu/ft^2 hr^oF) \qquad (56)$$

IX. HEAT TRANSFER BY SUPERCRITICAL, TURBULENT COOLANT FLOW

An effective means of achieving large heat transfer is to flow a fluid at fully developed turbulence through channels of diameter d, with the coolant at a moderate temperature difference from the material to be cooled. In the following we assume supercritical water so that single-phase flow is assured and no two-phase surface films at the wetted surface will develop. Efficient contact of the coolant is achieved by its highly turbulent motion. At the high temperature necessary to keep the water supercritical, the heat removed from the coil can be used at a high thermal efficiency to obtain useful work. This is an essential reason for choosing hot Be-Cu as the coil material, since the energy losses in the coil material become much less expensive than, for example, at cryogenic temperatures. The (water) fluid velocity u is characteristically high, of the order of 1000 ft/sec, and the pressure drop in the coolant channel is correspondingly high (~1000 psi). We assume a module length L_c for cooling, where various modules would be run in parallel or series parallel at a common pressure and temperature rise T_R of the coolant. In all cases considered here the ratio L_c/d turns out to be large, of the order of 100.

A. Properties of Supercritical Water

In order to assure supercriticality of the water coolant we take a mean operating point in the region $T \approx 700^\circ F$ and a mean pressure p of approximately 1000 psi. This allows considerable excursions of T and p in the single-phase region (cf. the Mollier chart on p. 10-A6 of Ref. 9). Mean values of the density,[10] viscosity,[11] thermal conductivity,[12] and specific heat for the water coolant are given in Table III. The specific heat C is an average over a load line on the Mollier chart through the points ($p = 1500$ psi, $T = 600^\circ F$) and (400 psi, $1100^\circ F$). These coefficients will be expressed in the British units of Table III throughout this paper.

TABLE III

MEAN TRANSPORT COEFFICIENTS OF SUPERCRITICAL WATER COOLANT FOR AN AVERAGE TEMPERATURE OF 700°F AND A PRESSURE OF 1000 PSI

Quantity	Symbol	Magnitude
Specific Heat	C	0.85 Btu/lb°F
Thermal Conductivity	K_c	0.27 Btu/hr ft°F
Viscosity	μ_c	0.055 lb/hr ft
Density	ρ_c	1.6 lb/ft^3

B. Dynamics of the Heat Transfer

The basic quantities in the heat transfer are the following: the fluid flow velocity u (which will be expressed in ft/sec), the hydraulic radius $D = d/4$(cm), the coolant channel length L_c(cm), the temperature rise T_R of the coolant in passing through the channel (°C), the local temperature difference ΔT between coolant and coil material (°C), and the pressure drop Δp across a coolant channel (psi).

The basic dimensionless relation which defines the heat-transfer coefficient h in terms of D, u, and the coolant transport coefficients is the Dittus-Boelter equation,[13] which applies to single phase flow and has been found to apply for such varied substances as gases, water, and liquid hydrogen:

$$(D/K_c)h = 0.023\, R^{0.8}(Pr)^{0.3}, \qquad (57)$$

where

$$R = Du\rho_c/\mu_c \qquad (58)$$

is the (dimensionless) Reynolds number of the flow, using self-consistent units.

In the units of this paper

$$R = 29.53\ u\rho_c d/\mu_c \ . \qquad (59)$$

The quantity Pr is the Prandtl number of the coolant

$$Pr = \mu_c C/K_c . \qquad (60)$$

For typical parameters in the present case ($d \approx 0.8$ mm, $u \approx 1.5 \times 10^3$ ft/sec) $R \approx 10^5$ and $Pr \approx 0.2$.

It is useful to define the coolant mass flow rate

$$G = \rho_c uA, \text{ (lb/sec)} \qquad (61)$$

where A is the coolant flow area in ft^2. The pressure drop Δ_p necessary to drive the coolant through the channels is given by the Fanning equation,[14]

$$\Delta p = (\rho_c u^2/2g)(4fL_c/d), \qquad (62)$$

where g is the acceleration of gravity, and f is a friction factor,[14] given over the present range of Reynolds numbers by

$$f = 6.6 \times 10^{-2}/R^{0.233} . \qquad (63)$$

In the present system of units

$$\Delta p = 4.32 \times 10^{-4}\ \rho_c u^2 (fL_c/d) . \quad \text{(psi)} \qquad (64)$$

The temperature rise of the fluid after flow through the channels is

$$T_R = L_c P/\xi GC, \qquad (65)$$

where P/ξ is the power being removed per unit length in all channels. (Here the duty factor is explicitly included to avoid confusion in applying the earlier power equations.) In the present units

$$T_R = 5.27 \times 10^{-4}\ L_c P/\xi GC . \quad (^{\circ}C) \qquad (66)$$

Finally the pumping power per unit length of reactor necessary to drive the fluid is given by

$$P_p = 1.953 \times 10^2\ G\Delta P/L_c \rho_c . \quad \text{(W/cm)} \qquad (67)$$

The foregoing equations, when used with a specific expression for the heat-transfer coefficient h (in Btu/ft^2 hr^{o}F), determine all the quantities in the heat-transfer problem. Reasonable choices of independent parameters must lead to reasonable values of derived parameters. When this is settled upon, the pumping power P_p is determined as the "cost" of the heat transfer to be taken into account in the overall energy balance of the reactor.

C. Neutron and Electrical Heating in the Coil

Substituting Eqs. (49), (44), and (38) into Eq. (59) gives the following relation between channel diameter and fluid velocity (in ft/sec)

$$ud = 0.911 \times 10^8 \frac{\mu_c^{5/8}}{K_c^{7/8}\ \rho_c C^{3/8}} \left(\frac{K_v \tau_y}{C_v}\right)^{5/4} \left(\frac{x\tau_T^{1/2}}{R_c B}\right)^{5/2} \left(\frac{P_\Sigma}{\Delta T_\Sigma}\right)^{5/4} . \qquad (68)$$

Here K_v and C_v involve the units J, cm, sec, and ^{o}K, and τ_y is in psi. The flow rate (in lb/sec) is

$$G = 3.98 \times 10^7 \frac{\mu_c^{5/4}}{K_c^{7/4}\ \rho_c C^{3/4}} \left(\frac{K_v \tau_y}{C_v}\right)^{3/2} \left(\frac{x\tau_T^{1/2}}{R_c B}\right)^{3} \frac{P_\Sigma^{5/2}}{\xi\ u\Delta T_\Sigma^{5/2}} . \qquad (69)$$

The channel length (in cm) per cooling module is

$$L_c = 2.11 \times 10^{11} \frac{\mu_c^{5/8}}{K_c^{7/8}\ \rho_c^2 C^{3/8}} \left(\frac{K_v \tau_y}{C_v}\right)^{5/4} \left(\frac{x\tau_T^{1/2}}{R_c B}\right)^{5/2} \frac{P_\Sigma^{5/4}\ \Delta p}{fu^3\,\Delta T_\Sigma^{5/4}} . \qquad (70)$$

The temperature rise (in ^{o}C) of the coolant is

$$T_R = 2.79 \left(\frac{C_v}{K_v \tau_y}\right)^{1/4} \frac{K_c^{7/8}}{\rho_c C^{5/8}\,\mu_c^{5/8}} \left(\frac{R_c B}{x\tau_T^{\ 1/2}}\right)^{1/2} \frac{\Delta p \Delta T_\Sigma^{5/4}}{fu^2 P_\Sigma^{1/4}} . \qquad (71)$$

The pumping power per unit length (in W/cm) is

$$P_p = 3.68 \times 10^{-2} \frac{\mu_c^{5/8}}{K_c^{7/8}\ C^{3/8}} \left(\frac{K_v T_y}{C_v}\right)^{1/4} \left(\frac{x T_T^{1/2}}{R_c B}\right)^{1/2} \frac{P_\Sigma^{5/4} f u^2}{\xi\ \Delta T_\Sigma^{5/4}} . \quad (72)$$

D. Electrical Heating at the Feed Slot

In analogy to the above calculations, using Eqs. (50) through (53):

$$ud = 0.706 \times 10^{-9} \frac{\mu_c^{5/8}}{K_c^{7/8}\ \rho_c C^{3/8}} \left(\frac{K_v}{C_v}\right)^{5/4} \left(\frac{B^2 x_E^2}{\Delta T_E}\right)^{5/4} , \quad (73)$$

$$G = 6.85 \times 10^{-17} \frac{\mu_c^{5/4}}{K_c^{7/4}\ \rho_c C^{3/4}} \left(\frac{K_v}{C_v}\right)^{3/2} \frac{B^5 R_c T^{1/2} x_E^3}{u_E \xi T_T^{1/2} \Delta T_E^{5/2}} , \quad (74)$$

$$L_c = 1.65 \times 10^{-6} \frac{\mu_c^{5/8}}{K_c^{7/8}\ \rho_c^2 C^{3/8}} \left(\frac{K_v}{C_v}\right)^{5/4} \left(\frac{B^2 x_E^2}{\Delta T_E}\right)^{5/4} \frac{\Delta p_E}{u_E^3 f_E} , \quad (75)$$

$$T_R = 7.38 \times 10^{3} \frac{K_c^{7/8}}{\mu_c^{5/8}\ \rho_c C^{5/8}} \left(\frac{C_v}{K_v}\right)^{1/4} \frac{\Delta p_E \Delta T_E^{5/4}}{u_E^2 f_E (B^2 x_E^2)^{1/4}} , \quad (76)$$

$$P_p = 8.10 \times 10^{-9} \frac{\mu_c^{5/8}}{K_c^{7/8}\ C^{3/8}} \left(\frac{K_v}{C_v}\right)^{1/4} \frac{R_c \eta^{1/2}\ f_E u_E^2 x_E^{1/2}}{\xi \tau_T^{1/2}} \left(\frac{B^2}{\Delta T_E}\right)^{5/4} . \qquad (77)$$

E. Bremsstrahlung Heating at the Inner Coil Surface

Using Eqs. (54) through (56),

$$ud = 1.93 \frac{\mu_c^{5/8}}{K_c^{7/8}\ \rho_c C^{3/8}} \left(\frac{K_v}{C_v}\right)^{5/8} \left(\frac{x\tau_T^{1/2} P_B}{R_c \Delta T_B \xi^{1/2}}\right)^{5/4} , \qquad (78)$$

$$G = 2.48 \times 10^{-2} \frac{\mu_c^{5/4}}{K_c^{7/4}\ \rho_c C^{3/8}} \left(\frac{K_v}{C_v}\right)^{3/4} \left(\frac{x\tau_T^{1/2}}{R_c}\right)^{3/2} \frac{P_B^{5/2}}{u \Delta T_B^{5/2} \xi^{7/4}} , \qquad (79)$$

$$L_c = 4.51 \times 10^{3} \frac{\mu_c^{5/8}}{K_c^{7/8}\ \rho_c^2 C^{3/8}} \left(\frac{K_v}{C_v}\right)^{5/8} \left(\frac{x\tau_T^{1/2} P_B}{R_c \Delta T_B \xi^{1/2}}\right)^{5/4} \frac{\Delta p}{u^3 f} , \qquad (80)$$

$$T_R = 0.957 \times 10^{2} \frac{K_c^{7/8}}{\mu_c^{5/8}\ \rho_c C^{5/8}} \left(\frac{C_v}{K_v}\right)^{1/8} \left(\frac{R_c}{x\tau_T^{1/2}}\right)^{1/4} \frac{\xi^{9/8}\ \Delta p \Delta T_B^{5/4}}{u^2 f P_B^{1/4}} , \qquad (81)$$

$$P_p = 1.07 \times 10^{-3} \frac{\mu_c^{5/8}}{K_c^{7/8}\ C^{3/8}} \left(\frac{K_v}{C_v}\right)^{1/8} \left(\frac{x\tau_T^{1/2}}{R_c}\right)^{1/4} \frac{u^2 f P_B^{5/4}}{\xi^{9/8} \Delta T_B^{5/4}} . \qquad (82)$$

F. Sample Calculations of the Heat Transfer

In order to determine the feasibility of the heat transfer, sample calculations were made for typical parameters occurring near reactor energy balance. The results are given in Table IV. For each of the three cases the four parameters u, Δp, ΔT, and x were chosen until reasonable derived parameters L_c, T_R, P_p, and d were obtained. The values $T_R \approx 270^\circ C$ and $\Delta p = 1.1 \times 10^3$ psi are consistent with each other on the Mollier chart. A boundary condition on u is that it not be supersonic.

The channel length L_c is forced to be about 25 cm at $\tau_T = 0.02$ sec, this being a reasonable module length which is probably consistent with a longitudinal division of the coil for field-shaping purposes. The channel diameter is about 1 mm, which is a reasonable choice in view of past experience[15] with similar heat-transfer problems. It should also be feasible to produce such holes over the length L_c.

It will be noted that in the body of the coil the ratio of hole diameter to hole spacing is about 3.5. Correspondingly, about 10% of the coil volume is occupied by water (at 0.026 normal density). Assuming an average energy transfer to an H_2O molecule of 15 MeV at the total cross section of 2.25 b, the neutron kinetic energy loss of the water is only about 10^{-2} that to the copper (cf. Table II). The coil is weakened elastically by about 30% over the assumptions of Section IV. However, this is not a serious problem, since an increase of ΔR_c by 30% does not seriously affect the energy balance. It can also be compensated by a pulsed increase of hydraulic pressure of the neutron blanket. Finally, one notes that the ratio $d/x\delta_T$ is only about two in the bremsstrahlung layer. This is sufficient to allow support of the magnetic pressure.

It is useful to note the heat fluxes to which the heat transfers correspond in the examples of Table IV. These fluxes are given by the quantities $h\Delta T$ of Eqs. (49), (53), and (56), for the Σ, E, B cases; at $\tau_T = 0.020$ sec, they are, respectively, 1.0, 0.71, and 0.31 kW/cm^2.

TABLE IV

HEAT-TRANSFER PARAMETERS FOR THE CASE $kT = 15$ keV, $B = 200$ kG, $R_c = 10$ cm

$R_p = 5$ cm, $\xi = 16/3$, $\tau_y = 7 \times 10^4$ psi, $K_v = 1.6$ W/cm°K, $C_v = 3.7$ J/cm^3°C

(The mean transport coefficients of the supercritical

H_2O are those given in Table III)

Neutron and Electrical Heat in the Coil

τ_T (sec)	L_c (cm)	T_R (°C)	P_p (W/cm)	d (cm)	$x\delta_T$ (cm)
0.015	21.3	252	2.76×10^5	0.0961	0.274
0.020	28.1	254	2.46×10^5	0.1204	0.317
0.030	42.0	256	2.10×10^5	0.1669	0.388
0.050	71.1	260	1.75×10^5	0.2557	0.501

$u = 1.275 \times 10^3$ ft/sec, $\Delta p = 1.10 \times 10^3$ psi, $\Delta T = 36.41$°C, $x = 1.85$

Electrical Heating at the Feed Slot

τ_T (sec)	L_c (cm)	T_R (°C)	P_p (W/cm)	d (cm)	$x\delta_T$ (cm)
0.015	25.0	269	3.92×10^5	0.100	0.280
0.020	25.0	269	3.40×10^5	0.100	0.324
0.030	25.0	269	2.77×10^5	0.100	0.397
0.050	25.0	269	2.15×10^5	0.100	0.512

$u = 1.196 \times 10^3$ ft/sec, $\Delta p = 1.10 \times 10^3$ psi, $\Delta T = 34.42$°C, $x = 1.89$

Bremsstrahlung Heating of the Inner Coil Surface

τ_T (sec)	L_c (cm)	T_R (°C)	P_p (W/cm)	d (cm)	$x\delta_T$ (cm)	ΔT (°C)
0.015	21.3	244	6.21×10^4	0.138	0.267	10.77
0.020	28.1	237	6.40×10^4	0.173	0.308	10.38
0.030	42.0	226	6.71×10^4	0.239	0.378	9.79
0.050	71.1	210	7.21×10^4	0.366	0.489	8.99

$u = 1.639 \times 10^3$ ft/sec, $\Delta p = 1.10 \times 10^3$ psi, $x = 1.80$

G. The Heat-Transfer Cycle

The heat removed from the copper coil by the coolant is assumed to be given up in a heat exchanger to essentially the same prime mover that receives heat from the neutron blanket. However, there are two other sources of heat in this cycle. The pump which raises the H_2O pressure by Δp after it leaves the heat exchanger is assumed to be 80% efficient, delivering 20% of the pump power as heat to the fluid. Correspondingly an input power of $1.2\ P_p$ is required to run the pumps. As an approximation, it is assumed that the turbulent fluid flows through the channels at approximately constant velocity u. This requires that the hydraulic energy dissipated in friction [cf. Eq. (64)] also heats the fluid.

In the energy balance of Section X all energies having to do with the plasma and electrical system will be entered for the burning time τ_T. However, the pumps would run continuously, and their energy must be computed for time $\xi\ \tau_T$. Thus, $1.2\xi P_p \tau_T$ is entered as a loss. However, a fraction ϵ (= thermal efficiency) of this energy is available as work through the thermal cycle.

X. REACTOR ENERGY BALANCE

A. Components of the System for Energy Balance

The reactor actually comprises three separate systems insofar as energy balance is concerned: (a) the plasma itself; (b) the electrical system in which the external energy source is coupled through the magnetic field to the plasma; and (c) the thermal system which receives heat from the neutron blanket, as well as the heat-transfer coolant, and produces useful work at thermal efficiency ϵ. These systems may be intercoupled in various ways. For example, some of the output work of the thermal system must be coupled into the electrical system, since ΔW of Eq. (23) is insufficient completely to make up the electrical energy loss W_E.

One is free to consider various kinds of coupling between the systems. If we allow "used" plasma to give up its internal energy to the thermal system (however, preferably not to the coil) then all systems are coupled, and we obtain the most general energy balance,

$$\epsilon(P_T\tau_T - P_N\tau_T) + \Delta W + \epsilon(W_p + \Delta U - P_B\tau_T)$$
$$+ \epsilon(P_N\tau_T + P_B\tau_T + W_E + 1.2\,\xi P_p\tau_T) = 1.2\xi P_p\tau_T + W_p + W_E. \quad (83)$$

Here the primary source of energy is that of the d-t neutrons in the neutron blanket. This energy is diminished by $P_N\tau_T$ in passing through the coil and is converted to useful work at the thermal efficiency ϵ. The work ΔW is directly available to the electrical energy source. The internal energy of the plasma after α-particle heating is $W_p + \Delta U$, less the bremsstrahlung loss $P_B\tau_T$. In addition all energy deposited in the coil and feed section can be used at efficiency ϵ, as discussed in the preceding section; this accounts for the last term on the left of Eq. (83), where the heat deposited in the heat-transfer coolant is included. The loss terms on the right represent necessary input to the reactor system.

B. Energy Balance of the Plasma System Alone

Instead of solving Eq. (83) directly for τ_T it is simpler first to consider a separate energy balance for the plasma itself. The first balance to consider is that in which α-particle heating makes up the bremsstrahlung loss from the plasma. The condition for this is that

$$\Delta U \geq P_B\tau_T. \quad (84)$$

Substituting Eqs. (24), (23), and (10) gives

$$(kT)^{\frac{1}{2}} / Q\overline{\sigma v} \leq 5.44 \times 10^{12}. \quad (85)$$

Referring to Table I, this is seen to occur for $kT \geq 6.3$ keV. Since overall reactor energy balance occurs at larger kT values than this, there is extra internal energy available to make up other losses. Therefore we next calculate the balance of plasma quantities, assuming that the final plasma internal energy $W_p + \Delta U - P_B\tau_T$ is given to the thermal system, to be used at efficiency ϵ, along with $P_B\tau_T$, to make up for the original plasma energy W_p. This energy balance, which makes the plasma internally self sufficient, reads

$$\epsilon\,[P_B\,\tau_T + (W_p + \Delta U - P_B\tau_T)] = W_p. \tag{86}$$

Using Eqs. (24) and (5), with $\epsilon = 0.4$, this reduces to

$$\tau_T = \frac{1.943 \times 10^{-5}}{B^2}\,\frac{(kT)^2}{Q\overline{\sigma v}}\,. \tag{87}$$

Graphs of τ_T and $n\tau_T$ for this plasma balance alone are given in Fig. 3. Note that $n\tau_T$ is independent of B and is approximately 10 times the Lawson-limit values shown later in Fig. 7. The energy balance is independent of plasma radius and essentially makes use of α-particle heating as an energy source. Since the burning times τ_T of Fig. 3 are roughly equal to those found for the thermal and electrical energy balance discussed in the next paragraphs below, it is instructive and approximately correct to split off the plasma energy balance in this way and to treat the remainder separately.

C. Energy Balance of the Electrical and Thermal System

Subtracting Eq. (86) from (83) gives for the energy balance of the remainder of the system

$$P_T\tau_T + \Delta W = (1-\epsilon)\;(W_E + 1.2\;\xi P_p\tau_T). \tag{88}$$

As an indication of the values of τ_T sufficient for energy balance of the electrical-thermal system alone, we determine the root of Eq. (88) under the conditions $kT = 15$ keV, $B = 200$ kG, $R_c = 10$ cm, $R_p = 5$ cm, $\xi = 16/3$, $\epsilon = 0.4$. Substituting Eqs. (8), (24), and (27), and using Table I gives

$$8.25 \times 10^6\;\tau_T - 7.45 \times 10^5\;\tau_T^{\frac{1}{2}} = 3.8\;P_p\tau_T. \tag{89}$$

Here P_p represents the sum of the contributions from Eqs. (72), (77), and (82), tabulated for the present parameters in Table IV. Graphical solution of Eq. (89) yields the value $\tau_T = 0.017$ sec.

The two values of τ_T obtained from Eq. (89) and from Fig. 3 at

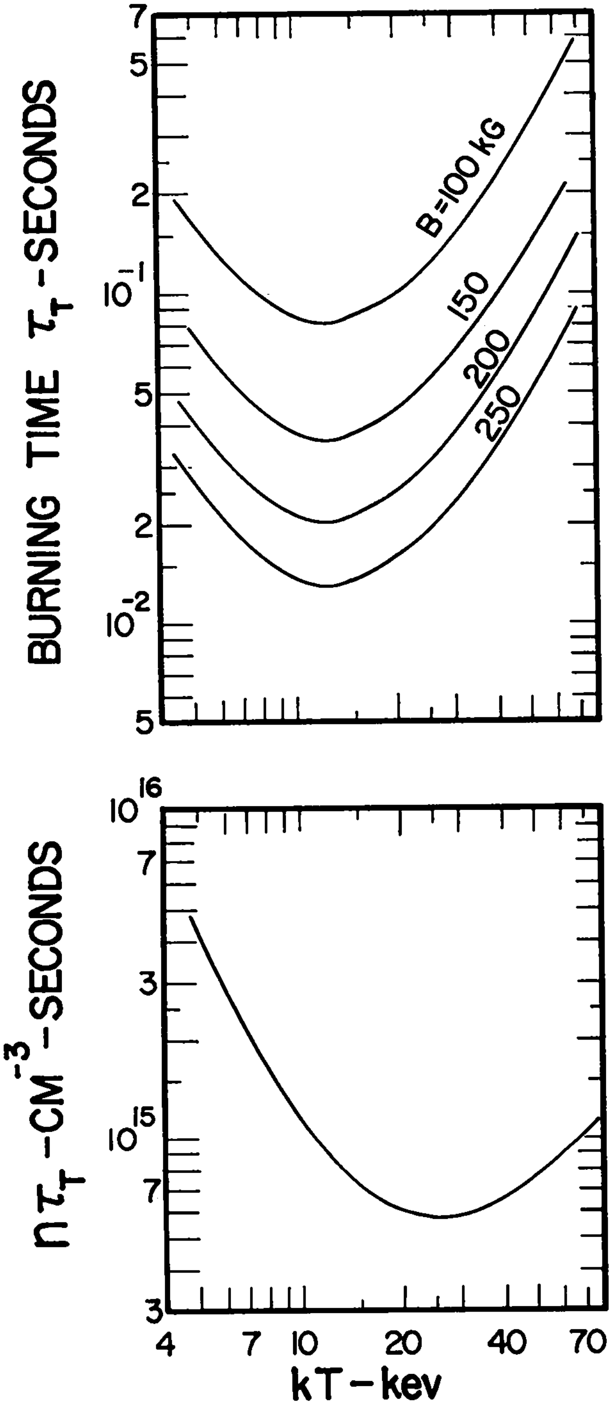

Fig. 3. Energy balance relations for the plasma system alone, corresponding to Eq. (87). The burning time τ_T and the product $n\tau_T$ are plotted versus kT, the plasma temperature, for various magnetic field values.

kT = 15 keV, 17 and 20 msec, are roughly consistent with each other, indicating a solution in the neighborhood of 18 msec for the parameters under consideration. The solution of the overall energy balance Eq. (83) to obtain the corresponding τ_T values is the subject of the next section.

D. Overall Energy Balance

The pump power P_p is a complicated function of τ_T, as can be seen from Eqs. (72), (77), and (82). Therefore, the mathematical procedure for solving Eq. (83) for τ_T is first to solve the equation (which is then quadratic in $\tau_T^{1/2}$) without the P_p term. These values of τ_T are then used to evaluate P_p, which is inserted as a provisional constant into Eq. (83) to obtain the next iterated value of τ_T, and so on. This procedure was carried out* on an IBM-7094 computer until the process converged to values of τ_T and all of the corresponding heat-transfer quantities. Graphs of τ_T and $n\tau_T$ versus kT for B = 150 and 200 kG are given in Fig. 4.

XI. ENERGY BALANCE FOR A SUSTAINED-FIELD THETA PINCH

A. Configuration and Power Losses

A pulsed system has been proposed for producing θ pinches with steady compression fields which could be furnished by superconducting magnets outside the coil and neutron blanket.[2] A waveform of the field is shown schematically in Fig. 5. An impulse of magnetic field of duration τ and magnetic field $\approx$ -B is applied to the coil volume which is normally filled with magnetic field B. The θ-pinch plasma is formed on the back swing of the negative impulse and held by the steady field B.

In analyzing this system, the time τ can be taken quite small. A practical value would appear to be about 10^{-5} sec. For the coil conductor we again assume 500°C Be-Cu for which $\eta = 7.9 \times 10^{-6}$ Ω-cm.

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*We are indebted to Mrs. Josephine Powers for performing this computation.

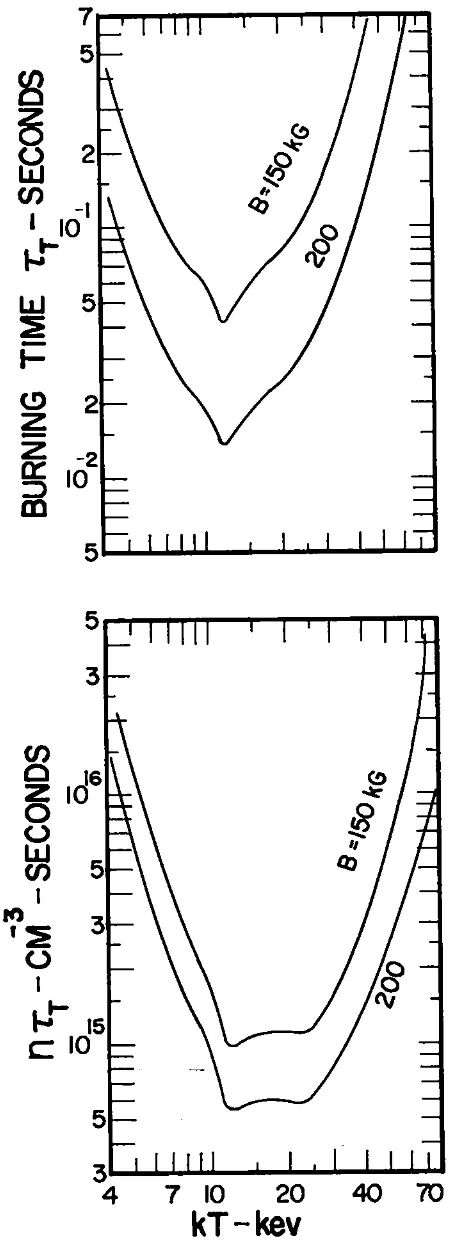


Fig. 4. Energy balance relations for the overall pulsed reactor system, corresponding to Eq. (83), with heat transfer included. The heat-transfer parameters for the 200-kG case are the same as those of Table IV (except for a gradual increase of u_B to 2×10^3 ft/sec between kT = 12 keV and kT = 5 keV). The heat-transfer parameters for the 150-kG case are the same except that u_E = 989 ft/sec and ΔT_E = 23.0°C. The thermal efficiency ϵ is taken as 0.4.

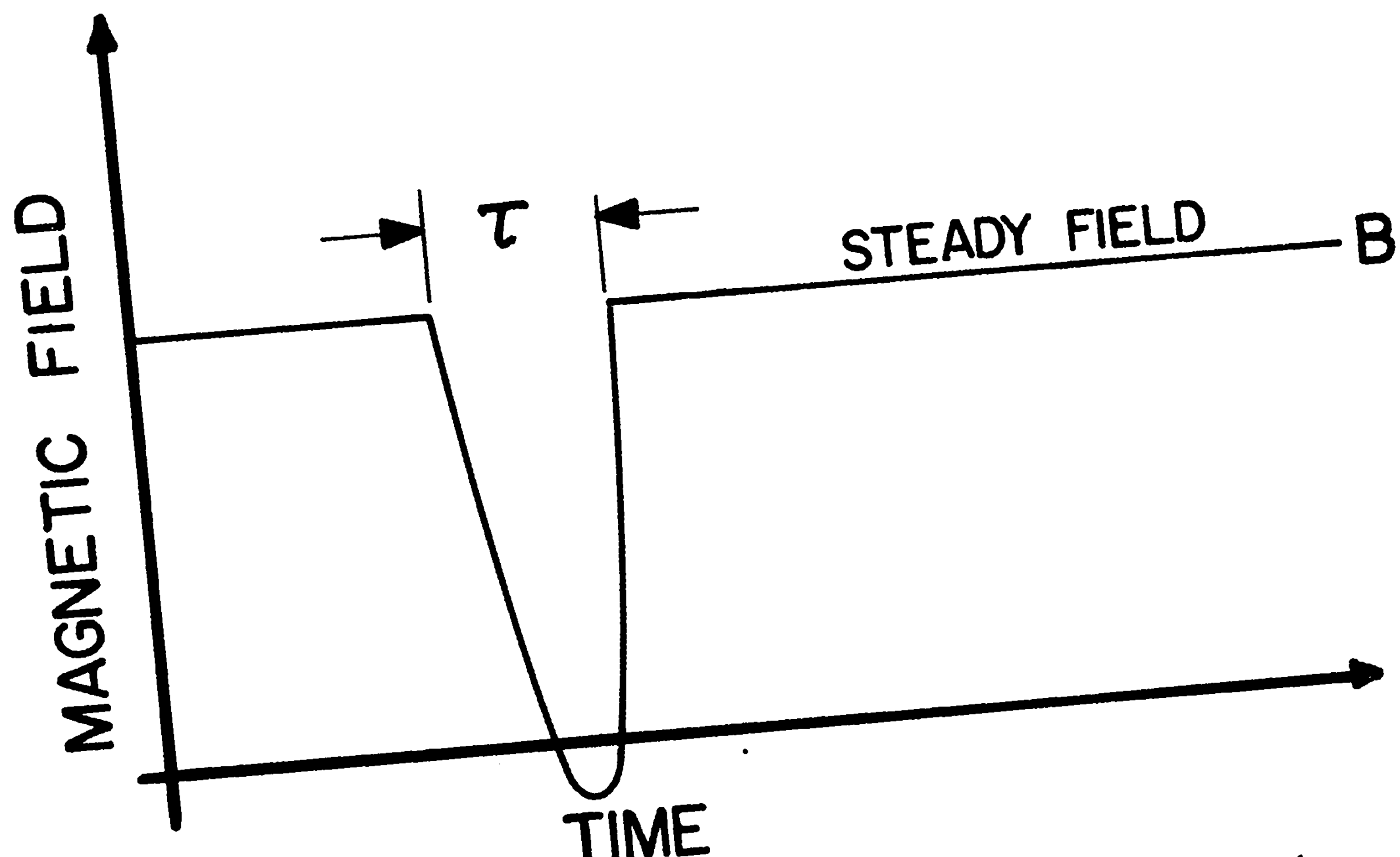


Fig. 5. Time history of the magnetic field in the sustained-field θ pinch.

The coil wall must now support the outer pressure $B^2/8\pi$ during time τ. Similar considerations to those discussed before for the tensile case lead to identical expressions for ΔR_c and P_N [cf. Eqs. (31) and (34)].

The power sources and losses during the burning time τ_T are the same as those in the previously discussed pulse case, with the exception that W_E is now negligible. This can be seen from Eq. (28), since τ is now very small. Similarly we neglect W_E in the heat-transfer problem.

B. The Energy Balance

Since W_E can be neglected, Eq. (83) reduces to

$$\epsilon(P_T\tau_T + \Delta U) + \Delta W = (1 - \epsilon)\ (W_p + 1.2\xi\ P_p\tau_T)\ , \tag{90}$$

where P_p now includes no contribution from W_E. Solutions of τ_T and $n\tau_T$ for the overall reactor energy balance Eq. (90) are given in Fig. 6.

It is seen that energy balance occurs here for $\tau_T \approx 1.5$ msec. This burning time is approximately the same as that of the Lawson criterion. Thus, a sustained-field pulsed reactor presents a more favorable energy balance than does the ordinary pulsed reactor discussed earlier.

XII. COLLISIONAL EFFECTS

A. Thermalization of the 3.5-MeV Alpha Particles

The α particles produced by d-t reactions in the cylindrical plasma column have comparatively small gyro radii outside it (2.7 cm at 200 kG), and they may be considered to spend most of their time inside the plasma. There they lose energy to d and t ions of the plasma, and they will come to a common temperature if the time required for this thermalization is shorter than the burning time τ_T of the plasma.

The first process of α-particle energy loss to be considered is that of collisions with the plasma ions (who mean mass m_i is taken to be 2.5 atomic units). If w be the initial α-particle kinetic energy and v its velocity, then the mean time τ_α for losing a major fraction of its

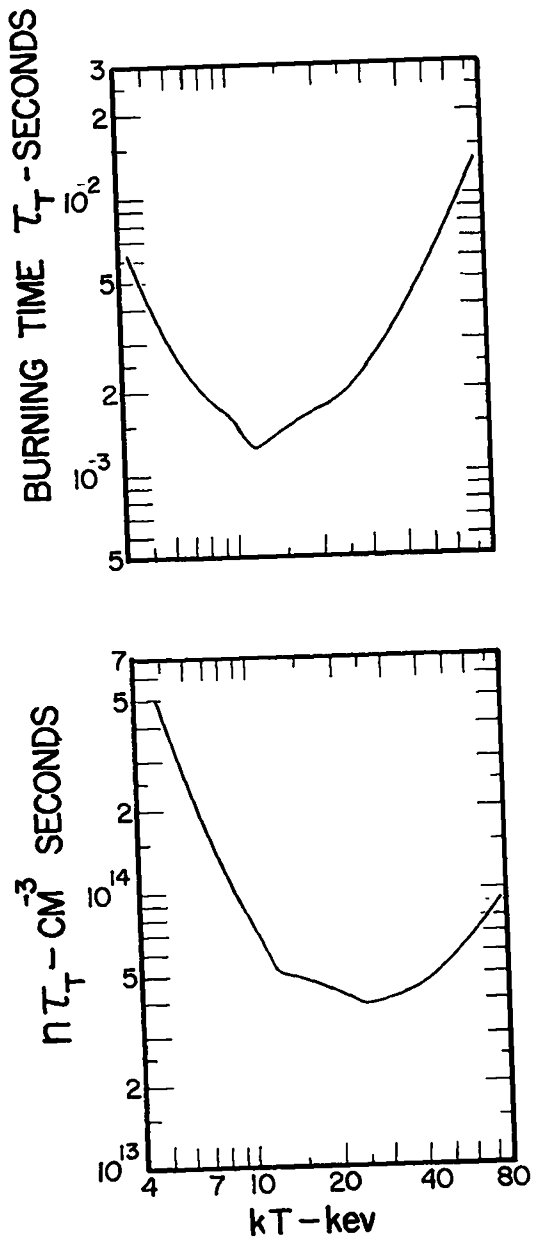


Fig. 6. Energy balance relations for sustained-field θ pinch, corresponding to Eq. (90) with heat transfer included, for the following parameters: $B = 200$ kG, $R_c = 10$ cm, $R_p = 3.5$ cm, $\xi = 15$, $u_\Sigma = 4.239 \times 10^2$ ft/sec, $u_E = 1.105 \times 10^3$ ft/sec, $u_B = 1.143 \times 10^3$ ft/sec, $x_\Sigma = x_E = 1.8$, $x_B = 1.9$, $\Delta p_\Sigma = \Delta p_E = \Delta p_B = 1.1 \times 10^3$ psi, $\Delta T_\Sigma = 3.0$ °K, $\Delta T_E = 2.93$ °K. Other parameters are the same as for Fig. 4. The quantity u_B increases to 1.5×10^3 ft/sec between kT = 10 and 5 keV.

energy by Coulomb collisions with the ions is approximately[16]

$$\tau_\alpha = (m_i/m_\alpha)w^2/4\pi n_i z_i^2 z_\alpha^2 e^4 v_\alpha \ell n(2/\theta_m). \quad (91)$$

Here m_i, n_i, and z_i are the mass, number density, and nuclear charge of the plasma ions; and θ_m ($= 3.9 \times 10^{-10}$ rad) is the minimum scattering angle to be counted for Coulomb collisions.[16] Evaluation of Eq. (91) for $n_i = 3 \times 10^{16}$ cm^{-3} and kT = 15 keV yields $\tau_\alpha = 0.008$ sec. As noted in Ref. 16, a roughly equal contribution to the slowing down of the α particles occurs because of direct nuclear collisions. Therefore the thermalization time of the α particles is shorter than the minimum burning times of the reactor, which lie in the range from 15 to 30 msec.

B. Classical Diffusion of the Plasma Across the Magnetic Field

As a criterion for diffusion, we take the time required for the plasma radius to double:[17]

$$\tau_2 = (10^{-9} R_p^2/\eta_p)\ (B^2/2nkT), \quad (92)$$

where η_p is the plasma resistivity in Ω-cm (1.9×10^{-8} Ω-cm at kT = 15 keV). In terms of the plasma β

$$\tau_2 = 4.71 \times 10^{-8} R_p^2/\beta\eta_p. \quad (93)$$

In the case $R_p = 5$ cm, β = 1 the diffusion time is 64 sec. For all parameters considered here, the classical diffusion times are from two to three orders of magnitude greater than the burning times.

APPENDIX I

THE LAWSON CRITERION

In deriving this criterion[1] all losses external to the plasma itself are neglected. The energy expenditures considered are the following irreducible ones:

(1) The energy required to produce each cm^3 of plasma having n/2 deuterons, n/2 tritons, and n electrons per cm^3 at a common temperature T:

$$w_p = 3nkT. \tag{I-1}$$

(2) The energy radiated away as bremsstrahlung during the time τ_T that plasma burns:

$$w_B = 4.8 \times 10^{-31} \; n^2 \tau_T (kT)^{\frac{1}{2}}. \tag{I-2}$$

Here w_B is in joules per cm^3, and kT is in keV. (The Gaunt factor of bremsstrahlung theory is here taken as unity.)

The thermonuclear energy produced by each cm^3 of plasma is given by

$$w_T = \tfrac{1}{4}n^2 \; Q\overline{\sigma v} \; \tau_T, \tag{I-3}$$

where Q is taken here to be the energy released per d-t fusion reaction (17.6 MeV), and $\overline{\sigma v}(T)$ is the fusion cross section averaged over all relative velocities as shown in Fig. 1.

In the Lawson criterion w_T, as well as w_p and w_B, is considered as a source of thermal energy, and it is assumed that this energy is converted to useful output at a thermal efficiency ϵ. Then the minimal condition for useful energy production is given by

$$\epsilon(w_T + w_p + w_B) \geq w_p + w_B. \tag{I-4}$$

Substituting for these quantities gives

$$\tfrac{1}{4}Q\overline{\sigma v}(n\tau_T) \geq \frac{1-\epsilon}{\epsilon}\left[\,3\ kT + 4.8 \times 10^{-31}\,(kT)^{\frac{1}{2}}\,n\tau_T\right]. \tag{I-5}$$

For $kT \gtrsim 5$ keV, the bremsstrahlung term can be neglected, giving for the Lawson criterion

$$n\tau_T \gtrsim 12\,\frac{1-\epsilon}{\epsilon}\,kT/Q\overline{\sigma v}. \tag{I-6}$$

A graph of the Lawson criterion (including bremsstrahlung) for $\epsilon = \frac{1}{3}$ and $Q = 17.6$ MeV is given in Fig. 7. In this plot of $n\tau_T$ versus kT the region above the curve represents net energy gain per unit volume of plasma. At the minimum near $kT = 25$ keV the value of $n\tau_T$ is about 7×10^{13} cm^{-3} sec. This accounts for the often-quoted figure $n\tau_T \approx 10^{14}$ as the goal of containment experiments.

For $\beta = 1$ the pressure balance equation is

$$2nkT = B^2/8\pi. \tag{I-7}$$

Hence at the minimum of the Lawson curve ($kT = 25$ keV)

$$\tau_T = 1.41 \times 10^8/B^2, \quad \text{(sec)} \tag{I-8}$$

where B is in G. The maximum value of B set by the strength and melting of conductors is approximately 500 kG. Therefore the minimum value of containment time for satisfaction of the Lawson criterion is approximately 0.5 msec. A more practical upper limit on B is 250 kG; then $\tau_T \approx 2$ msec.

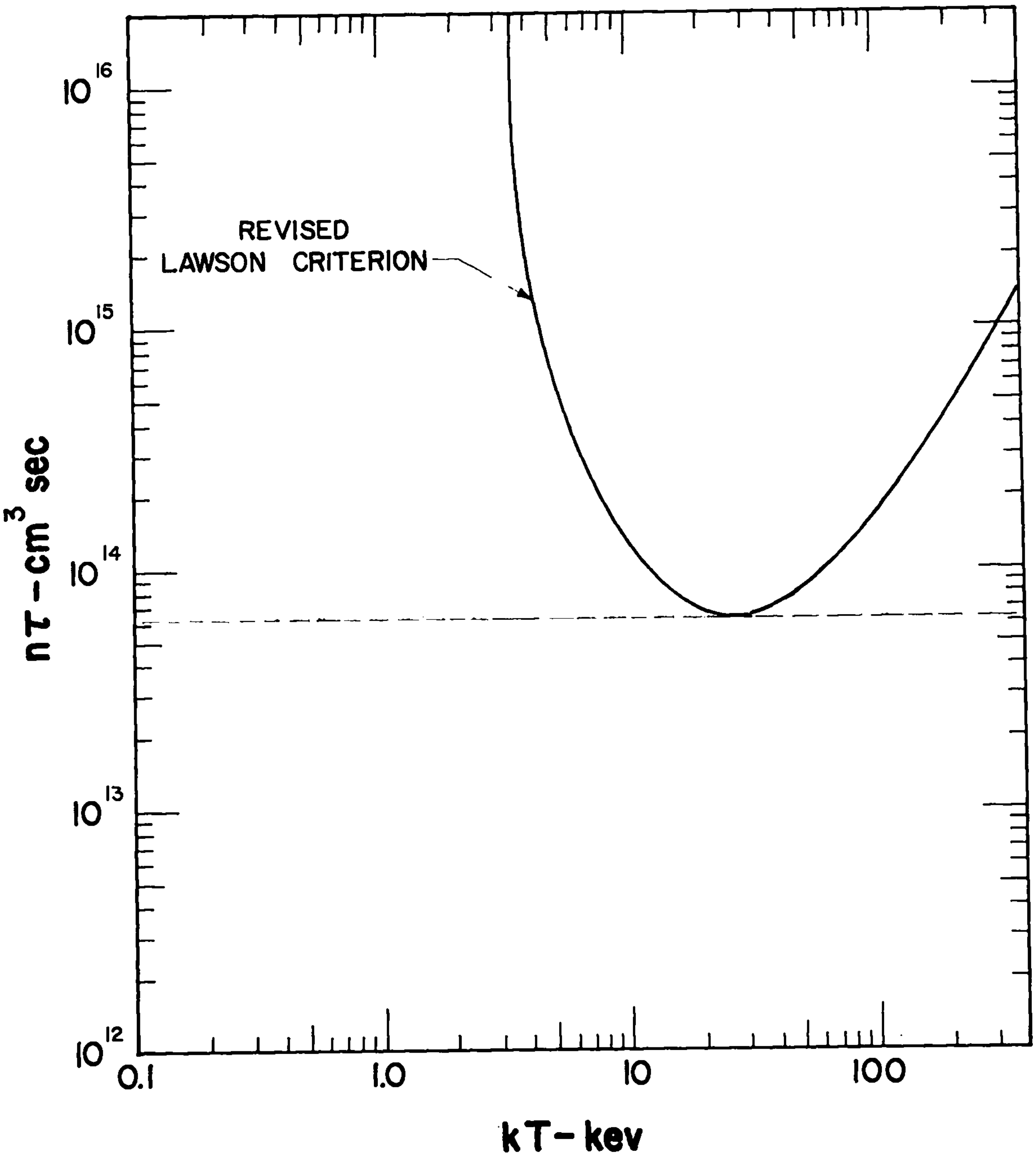


Fig. 7. The Lawson criterion for the minimum product of density and burning time to produce net energy balance against plasma energy losses versus plasma temperature.

APPENDIX II

ELECTRICAL ENERGY DISSIPATION IN THE THETA-PINCH COIL AND FEED SLOT

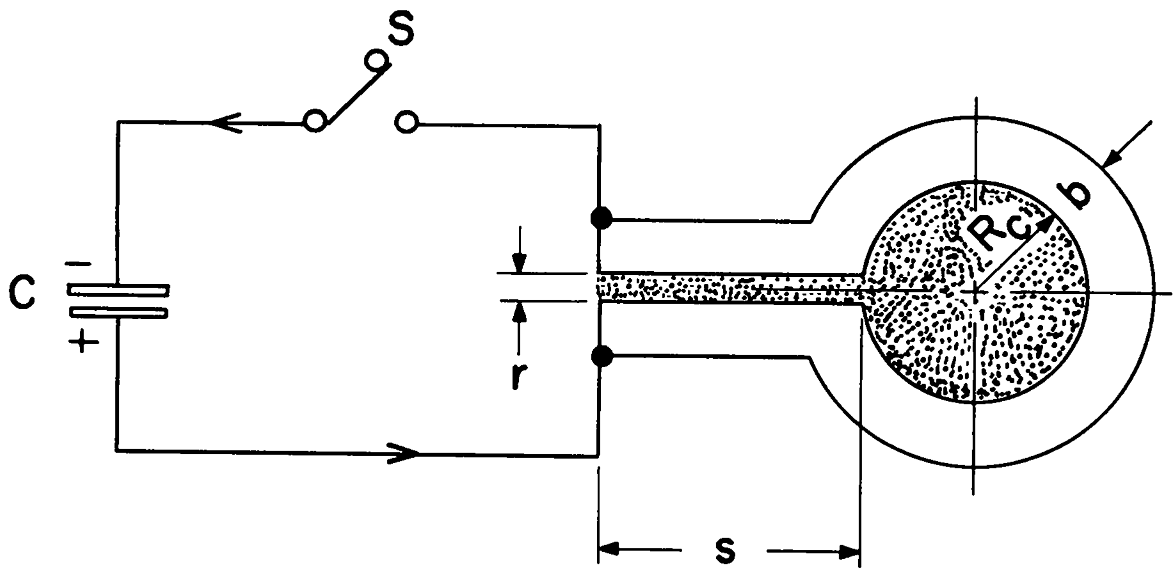
A. Model for Computation

The object is to compute the energy dissipation in a thick coil and feed slot as a function of the magnitude B and duration τ of an approximately sinusoidal impulse of magnetic field. In order to obtain the scaling of this problem the model of Fig. 8(a) is taken, in which a charged capacitor furnishes the electrical energy. The penetration of the magnetic field into the coil and feed-slot walls is derived as a function of time and the joule heating computed.

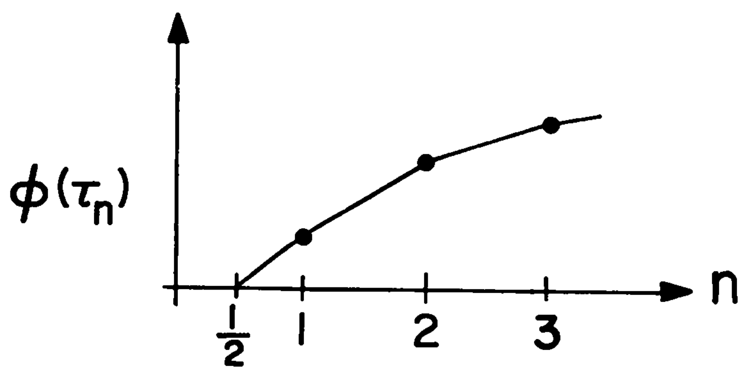
Even though a capacitive energy source is assumed here for convenience, this does not imply that such would be used in a thermonuclear reactor. In fact some form of magnetic energy storage presently appears more practical. The relations derived here depend primarily on the nature of the magnetic impulse itself, which would not depend greatly upon the nature of the energy source.

B. Computation of the Field Penetration and Joule Heating

In the physical system illustrated schematically in Fig. 8(a), the effect of the pinched plasma itself is not included, but the magnetic field is allowed to fill up the dotted region inside the conductor. When the switch S is closed the condenser C begins to discharge. Owing to the current I which flows in the circuit, a magnetic field develops in the dotted region. Some magnetic field penetrates outward from the dotted region into the conductor. The total back emf E_t due to the rate



(a)



(b)

Fig. 8. (a) Schematic diagram of the pulsed-magnetic field system used as a model in the field-penetration computations. (b) Scheme for numerical integration of the boundary magnetic field versus time.

of rise of the magnetic field is conveniently divided into two parts,

$$E_T = E + E_I. \tag{II-1}$$

The contribution E_I comes from the dotted region of empty space, and E comes from the rate of increase of field as it penetrates the conductor. We can immediately write E_I,

$$E_I = -\int(\partial B/\partial t)\cdot dA = -(rs + \pi R_c^2)(\partial B/\partial t), \tag{II-2}$$

since the field uniformly fills the empty space. The field inside the empty space is given by

$$B = \hat{z}\; 4\pi I/\ell_m, \tag{II-3}$$

where the z-direction is out of the page in Fig. 8(a), and ℓ_m is the length of the pinch coil. Therefore, Eq. (II-2) becomes

$$\begin{aligned} E_I &= -\left[4\pi(rs + \pi R_c^2)/(100\ \ell_m)\right](dI/dt) \\ &= -\left[\pi(rs + \pi R_c^2)/(25\ \ell_m)\right](dI/dt), \end{aligned} \tag{II-4}$$

where now E_I is given in kV; ℓ_m in m; r, s, and R_c in cm; I in MA; and t in μsec. Therefore,

$$E_I = -L\ dI/dt, \tag{II-5}$$

where

$$L = \pi(rs + \pi R_c^2)/(25\ \ell_m) \tag{II-6}$$

is the open space inductance in nh. The circuit equation can therefore be written as

$$10^3 Q/C - L dI/dt + E = 0, \tag{II-7}$$

where Q is the charge in C, C is the capacitance in μF, and E is in kV.

The contribution E in kV is

$$E = -(1/100)\int(\partial B/\partial t)\cdot dA, \tag{II-8}$$

where B is in kG, dA in cm, and t in μsec. As a first approximation, we assume that the field does not penetrate very far into the metal. The behavior along the inside perimeter of the metal will then be fairly independent of the shape of the conductor. We then integrate along the perimeter, obtaining

$$E = -\left[(s + \pi R_c)/50\right] \int_o^b (\partial B/\partial t)\,dx, \tag{II-9}$$

where now x indicates distance into the conductor. We assume that $b(= \Delta R_c)$ is large enough so that we can treat essentially the penetration of a field into an infinite plane conductor. Therefore, we only need to integrate over a range of x for which the field gives an appreciable contribution to the integral (II-9). We thus integrate only up to b' which will be determined below in terms of a penetration depth.

It will turn out that the plane solution approximation will be violated when the field penetrates large distances into the conductor. In such a case we really need to consider two-dimensional effects due to the shape of the conductor and limitations due to its finiteness. The mathematical complications are prohibitive in doing this properly, so we simply accept the qualitative validity of penetration into a plane infinite medium no matter how deep the penetration. Integrating up to b', we replace Eq. (II-9) by

$$E = -\left[(s + \pi R_c)/50\right] \int_o^{b'} (\partial B/\partial t)\,dx. \tag{II-10}$$

Assuming a constant resistivity η, the equation for the field penetration is

$$\partial B/\partial t = (\eta/4\pi)(\partial^2 B/\partial x^2)\,, \tag{II-11}$$

where the units of η must be mΩ-cm in order to agree with the units of B, x, and t.

Our basic problem is to solve Eq. (II-11) in a semi-infinite medium with a time-dependent boundary condition. This is mathematically equivalent to a heat flow problem solved by Carslaw and Jaeger.[18] We simply replace their κ and $v(x,t)$ by our $\eta/4\pi$ and $B(x,t)$, respectively. The solution is written as

$$B(x,t) = (2/\sqrt{\pi}) \int_{\mu_o}^{\infty} \phi(t - x^2/(\eta/\pi)\mu^2)e^{-\mu^2} d\mu, \quad \mu_o \equiv x\sqrt{\eta t/\pi} \; . \qquad \text{(II-12)}$$

where $\phi(\tau)$ is the boundary field as a function of time. We define the penetration depth δ as follows:

$$\delta \equiv \sqrt{(\eta t/\pi)} \, . \qquad \text{(II-13)}$$

Also, we introduce the scaled distance y by

$$x = \delta y. \qquad \text{(II-14)}$$

Then, we write Eq. (II-12) in the following form:

$$B(y,t) = (2/\sqrt{\pi})\int_{y}^{\infty} \phi\left[t(1 - y^2/\mu^2)\right]e^{-\mu^2} d\mu. \qquad \text{(II-15)}$$

As mentioned above, we integrate only out to a finite distance which we denote by Y. Then, Eq. (II-15) is replaced by

$$B(y,t) = (2/\sqrt{\pi})\int_{y}^{Y} \phi\left[t(1 - y^2/\mu^2)\right]e^{-\mu^2} d\mu. \qquad \text{(II-16)}$$

For example a good value is $Y = 3$. The exponential $e^{-\mu^2}$ will provide an adequate cutoff for the numerical integral. In terms of Y, the b' of Eq. (II-10) is

$$b' = \delta Y. \qquad \text{(II-17)}$$

The numerical procedure goes as follows: We mark off the time scale in terms of the index n and represent the boundary field $\phi(\tau_n)$ in terms of this scale as in Fig. 8(b).

The $\phi(\tau_n)$ will be a numerical function which we build up as we progress in time. The argument τ_n is given by

$$\tau_n = (n - \tfrac{1}{2})\Delta t, \qquad \text{(II-18)}$$

and the switch S is to be closed at $n = \frac{1}{2}$.

The difference equation for the external circuit is

$$10^3 Q^{n+\frac{1}{2}}/C - L\,(I^{n+1} - I^{n})/\Delta t + E^{n+\frac{1}{2}} = 0. \qquad \text{(II-19)}$$

Generally, we will know $Q^{n+\frac{1}{2}}$ and I^n, but not I^{n+1} and $E^{n+\frac{1}{2}}$. So we guess a value of $E^{n+\frac{1}{2}}$ (for our first step at $n = 0$, we guess $E^{\frac{1}{2}} = 0$) and solve for I^{n+1},

$$I^{n+1} = I^n + (\Delta t/L)(10^3 Q^{n+\frac{1}{2}}/C + E^{n+\frac{1}{2}}). \qquad \text{(II-20)}$$

From I^{n+1}, we get $\phi(t^{n+1})$ from Eq.(II-3):

$$\phi(t^{n+1}) = 4\pi\ I^{n+1}/\ell_m. \qquad \text{(II-21)}$$

This is a guessed value, whereas all those earlier on the curve of Fig. 8(b) are presumed to be known. Therefore, we have enough information to carry out the integral (II-16) numerically. The factor ϕ in the integral is obtained by computing the argument and then interpolating in the table of values which has been accumulated so far. This then gives the field $B(y,t)$ over any chosen y mesh at t^{n+1}. The corrected back emf $E^{n+\frac{1}{2}}$ is obtained from the integral (II-10). Caution must be exercised in evaluating $\partial B/\partial t$. This is the physical quantity to be evaluated for constant x. However, we have computed $B(y,t)$. But from Eqs. (II-14) and II-13),

$$y = \frac{x}{\delta} = \left(x/\sqrt{(\eta/\pi)}\right)\tau^{-\frac{1}{2}}. \qquad \text{(II-22)}$$

Then,

$$B(y,t) = B[y(x,t),t]. \qquad \text{(II-23)}$$

Hence

$$\frac{\partial B}{\partial t}\Big|_x = \frac{\partial B}{\partial y}\frac{\partial y}{\partial t}\Big|_x + \frac{\partial B}{\partial t}\Big|_y . \qquad \text{(II-24)}$$

Now,

$$\partial y/\partial t\ \Big|_x = -(1/2\tau)(x/\delta) = -y/2t. \qquad \text{(II-25)}$$

Thus, Eq. (II-24) becomes

$$\partial B/\partial t \,\Big|_x = \partial B/\partial t \,\Big|_y - (y/2t)(\partial B/\partial y) \,\Big|_t . \qquad \text{(II-26)}$$

Using Eq. (II-26), we can evaluate $\partial B/\partial t|_x$ conveniently in our (y,t) mesh. Then, writing Eq. (II-10) in the form

$$E = -\left[(s + \pi R_c)\delta/50\right] \int_0^Y (\partial B/\partial x) \,\Big|_x \, dy, \qquad \text{(II-27)}$$

we can evaluate the corrected $E^{n+\frac{1}{2}}$, since we know $B^{n+1}(y)$ from Eq. (II-16), and we can use convenient numerical analogs to Eqs. (II-26) and (II-27).

We can reiterate $E^{n+\frac{1}{2}}$ by means of Newton's method until we obtain the desired accuracy. This then also gives a correct value for I^{n+1}. We then advance the charge by the formula

$$Q^{n+3/2} = Q^{n+\frac{1}{2}} - I^{n+1}/\Delta t, \qquad \text{(II-28)}$$

and we are ready to proceed to the next time step.

C. Results

A typical computed wave form of magnetic field inside the coil, for one of the more damped cases considered, is shown in Fig. 9. The lines on the diagram show a method for deriving the period τ of the impulse which applies both to heavily and slightly damped impulses, namely:

$$\tau = 2\left[t_2(B_{max}/\sqrt{2}) - t_1(B_{max}/\sqrt{2})\right]. \qquad \text{(II-29)}$$

Figure 10 shows profiles of the magnetic field inside the hot copper conductor ($\eta = 7.9 \times 10^{-6}$ Ω-cm) at various times during the magnetic impulse of Fig. 9.

For a variety of capacitances, with initial voltages V_o adjusted to give constant B_{max}, the field penetration problem was run on an IBM-7094 computer. In each case the ratio of total energy dissipation W_E in the conductor to magnetic energy W_M in the circular coil space was computed. By convention we take W_E to be the total energy lost by the capacitor from time zero until the vacuum magnetic field again passes

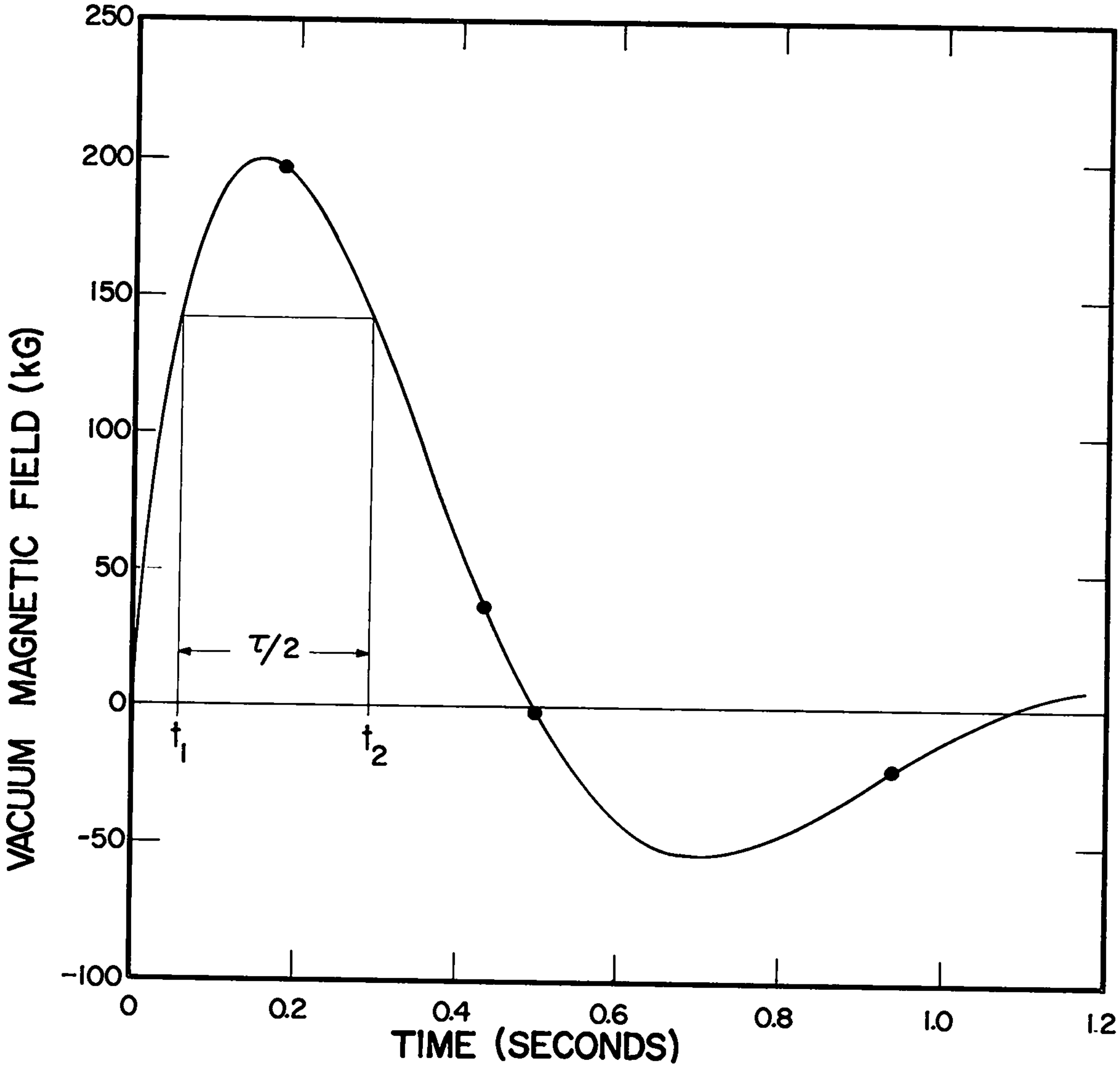


Fig. 9. Illustrative waveform of magnetic field in the vacuum of the single turn coil of Fig. 8(a) for the following parameters: $C = 10^4$ fd, $V_o = 35$ V, $R = 10$ cm, $\ell_m = 10$ cm, $r = 0.2$ cm, $S = 62.8$ cm, $\eta = 7.9 \times 10^{-6}$ Ω-cm. The vertical lines illustrate the method of defining a half-period τ, even in heavily damped cases.

through zero. At this instant there is some magnetic energy trapped in the conductor [as in Fig. 10(c)], and this energy is assumed to be dissipated [as would occur if the switch of Fig. 8(a) were opened]. This small amount of magnetic energy is therefore included in W_E. The result of the computations, for B = 150 and 200 kG and various τ values between 2.1×10^{-4} and 2.1 sec, is given by Eq. (28) with its constant numerical coefficient. The constant of this equation is used to normalize those of Eqs. (26) and (27) also.

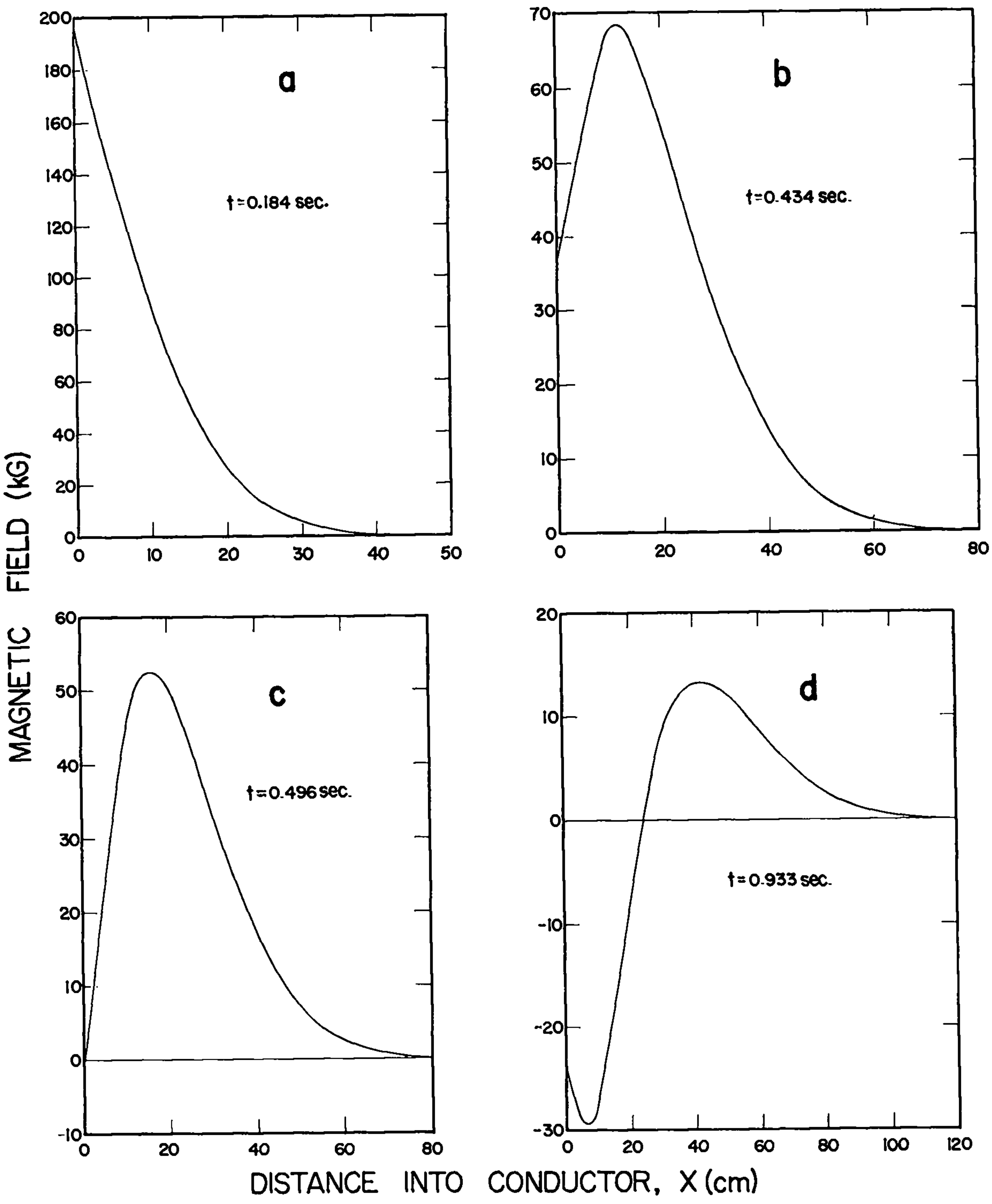


Fig. 10. Profiles of magnetic field inside the conductor, corresponding to the points marked on the waveform of Fig. 9.

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