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Neutrino Disintegration of the Deuteron  
at LAMPF Energies



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# Neutrino Disintegration of the Deuteron at LAMPF Energies

by

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NEUTRINO DISINTEGRATION OF THE DEUTERON

AT LAMPF ENERGIES

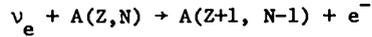
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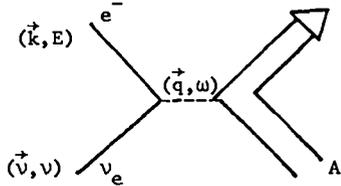
ABSTRACT

The differential cross section for the neutrino disintegration of the deuteron is calculated using effective range theory for neutrino energies from zero to 53 MeV.

The general formula for the neutrino-nucleus reaction



with the kinematics defined as



$$\vec{q} = \vec{v} - \vec{k}$$

$$\omega = E - \nu$$

is given by

$$d\sigma = 2\pi \frac{dn}{d\nu} \sum_{\text{lepton spins}} \frac{1}{2J_i + 1} \sum_{m_f} \sum_{m_i} |\langle f | \hat{H}_w | i \rangle|^2. \quad (1)$$

For relativistic electrons ( $E \gg m_e$ ) the multipole decomposition of the weak interaction Hamiltonian between nuclear states of definite total angular momentum and parity is (1,2)

$$\begin{aligned} \sum_{\text{lepton spins}} \frac{1}{2J_i + 1} \sum_{m_f} \sum_{m_i} |\langle f | \hat{H}_w | i \rangle|^2 &= 2 G^2 \frac{4\pi}{2J_i + 1} \cos^2 \frac{\theta}{2} \left\{ \sum_{J=0} |\langle J_f || M_J + \frac{\omega}{q} L_J || J_i \rangle|^2 \right. \\ &+ \left( \frac{1}{2} \frac{q_\lambda^2}{q^2} + \tan^2 \frac{\theta}{2} \right) \sum_{J=1} \left[ |\langle J_f || T_J^{e1} || J_i \rangle|^2 + |\langle J_f || T_J^{mag} || J_i \rangle|^2 \right] \\ &\left. - \tan \frac{\theta}{2} \left( \frac{q_\lambda^2}{q^2} + \tan^2 \frac{\theta}{2} \right)^{\frac{1}{2}} \sum_{J=1} 2 \operatorname{Re} \langle J_f || T_J^{mag} || J_i \rangle \langle J_f || T_J^{e1} || J_i \rangle^* \right\} \end{aligned}$$

(2)

where  $\cos \theta = \hat{k} \cdot \hat{\nu}$  and  $q_\lambda^2 = q^2 - \omega^2$ .

These multipole operators are the partial-wave decompositions of the operators

$$\tau_{\pm} (1, \vec{\sigma}, \vec{p}, \vec{\sigma} \cdot \vec{p}) e^{i \vec{q} \cdot \vec{r}}.$$

They are defined in references (1) and (2) and are made up of spherical Bessel functions in  $qr$ , vector spherical harmonics, and the nucleon spin and momentum operators.

The cross section for the nucleon reaction

$$\nu_e + N \rightarrow P + e^-, Q = .78 \text{ MeV}$$

is evaluated by taking the density of final states as

$$\frac{dn}{d\nu} = \frac{d^3k}{(2\pi)^3 d\nu} = \frac{k E d\Omega_e}{(2\pi)^3}$$

and evaluating the reduced matrix elements ( $J_i = \frac{1}{2}$ ,  $J_f = \frac{1}{2}$ ) of the major low momentum transfer terms, viz., the vector,

$$\begin{aligned} \langle \frac{1}{2} | M_0^+ \frac{\omega}{q} L_0 | \frac{1}{2} \rangle &= \langle \frac{1}{2} | j_0(qr) Y_0(\Omega_r) | \frac{1}{2} \rangle \\ &= \left( \frac{2}{4\pi} \right)^{\frac{1}{2}} \left( 1 - \frac{\omega^2}{q^2} \right) f_N(q) \end{aligned}$$

and the axial vector (with subscript 5 from  $\gamma_5$ )

$$\begin{aligned} \langle \frac{1}{2} | T_1^{e15} | \frac{1}{2} \rangle &= F_A \left( \frac{2}{3} \right)^{\frac{1}{2}} \langle \frac{1}{2} | j_0(qr) \vec{y}_{101} \cdot \vec{\sigma} | \frac{1}{2} \rangle \\ &= F_A \frac{2}{\sqrt{4\pi}} f_N(q) \end{aligned}$$

and the vector

$$\begin{aligned} \langle \frac{1}{2} | T_1^{\text{mag}} | \frac{1}{2} \rangle &= \left( \frac{2}{3} \right)^{\frac{1}{2}} \frac{q\mu_v}{2M} \langle \frac{1}{2} | j_0(qr) \vec{y}_{101} \cdot \vec{\sigma} | \frac{1}{2} \rangle \\ &= \left( \frac{q\mu_v}{2M} \right) \frac{2}{\sqrt{4\pi}} f_N(q) \end{aligned}$$

where  $f_N(q)$  is the nucleon form factor and  $\mu_v = \mu_p - \mu_n$  is the nucleon vector magnetic moment.

Substituting in Eqs. (1) and (2) gives

$$\frac{d\sigma_{\nu N}}{d\Omega_e} = \frac{G^2 k E}{4\pi^2} \left\{ 2 \left( 1 - \frac{\omega^2}{q^2} \right)^2 \cos^2 \frac{\theta}{2} + 4 \left[ F_A^2 + \left( \frac{q\mu_v}{2M} \right)^2 \right] \left( \frac{1}{2} + \frac{1}{2} \sin^2 \frac{\theta}{2} \right) - 8 F_A \left( \frac{q\mu_v}{2M} \right) \sin \frac{\theta}{2} \right\}.$$

In the approximation  $q^2 = \nu^2 + k^2 - 2 \nu k \cos \theta \approx 2\nu^2 (1 - \cos \theta)$ , i.e.  $\omega = 0$ , the angular distribution can be integrated to give

$$\sigma_{\nu N} = \frac{G^2}{\pi} k E \left[ 1 + 3 F_A^2 + \frac{2Q}{3} \left( \frac{q\mu_v}{2M} \right)^2 - 16 F_A \left( \frac{q\mu_v}{2M} \right) \right].$$

These last two terms are usually neglected at low neutrino energies, however, at  $\nu = 53 \text{ MeV}$  the last term contributes + 2.6. This cross section, evaluated with the coupling constants,

$$G = 1 \times 10^{-5} / M^2, F_A = -1.24$$

is shown in Fig. (1) in the region of electron-neutrino energies available from the decay of stopped muons:

$$\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e.$$

The cross section for the breakup of the deuteron

$$\nu_e + D \rightarrow P + P + e^-, Q = -1.44 \text{ MeV}$$

has a density of final states

$$\frac{dn}{d\nu} = \frac{d^3k}{(2\pi)^3} \frac{d^3p}{(2\pi)^3} \frac{1}{d\nu} = \frac{k E d\Omega_e}{(2\pi)^3} \frac{M p}{2(2\pi)^3} dE d\Omega_p$$

where  $p$  is the relative momentum between the two final protons

$$\nu = E + \frac{p^2}{M} + Q.$$

The nuclear recoil energy has been neglected.

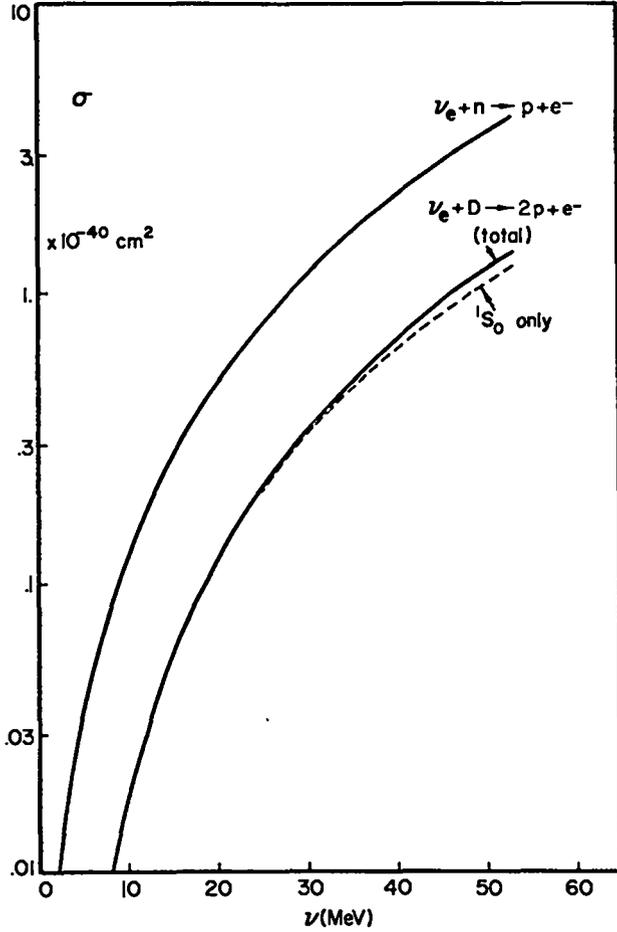


Fig. 1. Electron-neutrino reaction cross section for the free neutron and the deuteron. The dashed line is for the singlet S-wave final state while the solid line includes the contribution of higher partial waves.

The main transition at low momentum transfer is the  ${}^3S_1 \rightarrow {}^1S_0$  induced by the operators  $T_1^{el_5}$  and  $T_1^{mag}$ . (The operator  $M_0$  or  $L_0$  would induce  ${}^3S_1 \rightarrow {}^3S_1$  which is forbidden for identical particles in the final state.) By angular momentum recoupling one can show

$$\begin{aligned} & \langle {}^1S_0 || T_1^{el_5 \text{ or } mag} || {}^3S_1 \rangle \\ &= \frac{1}{\sqrt{2}} \langle \frac{1}{2} || T_1^{el_5 \text{ or } mag} || \frac{1}{2} \rangle I_0(p, q) \end{aligned}$$

$$I_0(p, q) = \int_0^\infty d^3r \psi_p(r) j_0\left(\frac{qr}{2}\right) \psi_0(r)$$

where  $\psi_0$  and  $\psi_p$  are the initial and final S-wave radial wave functions of the bound deuteron and the continuum protons. The double differential cross section obtained by integrating over the (isotropic) proton angular distribution is

$$\begin{aligned} \frac{d^2\sigma_{\nu D}}{d\Omega_e dE} &= \frac{4}{3} \frac{G^2}{(2\pi)^2} kE \frac{M_p}{(2\pi)^2} \\ &\times \left\{ \left( \frac{1}{2} + \frac{1}{2} \sin^2 \frac{\theta}{2} \right) \left[ F_A^2 + \left( \frac{qH_V}{2M} \right)^2 \right] \right. \\ &\left. - 2 \sin \frac{\theta}{2} F_A \left( \frac{qH_V}{2M} \right) \right\} I_0^2(p, q) . \end{aligned}$$

Effective-range theory provides simple, reliable wave functions for the  ${}^3S_1$  bound state and the  ${}^1S_0$  continuum state

$$\psi_0(r) = \sqrt{\frac{2\alpha}{1 - \alpha r_t}} \frac{e^{-\alpha r}}{r} \frac{1}{\sqrt{4\pi}}$$

$$\psi_p(r) = \frac{\sin(pr + \delta_s)}{pr}$$

$$p \cot \delta_s = -\frac{1}{a_s} + \frac{1}{2} r_s p^2$$

where  $\alpha = .232 \text{ fm}^{-1}$ ,  $r_t$  (triplet effective range) = 1.75 fm,  $r_s = 2.79$  fm, and  $a_s$  (singlet scattering length for protons) = -7.82. These functions give

$$\begin{aligned} I_0^{er}(p, q) &= \sqrt{4\pi} \sqrt{\frac{2\alpha}{1 - \alpha r_t}} \frac{1}{p q/2} \\ &\times \left\{ \frac{\cos \delta_s}{4} \ln \frac{\alpha^2 + (p + q/2)^2}{\alpha^2 + (p - q/2)^2} + \frac{\sin \delta_s}{2} \right. \\ &\left. \left[ \tan^{-1} \frac{2\alpha (q/2)}{\alpha^2 + p^2 - (q/2)^2 + \epsilon\pi} \right] \right\} \end{aligned}$$

$$\epsilon = 0 \text{ for } \alpha^2 + p^2 - (q/2)^2 \geq 0$$

$$\epsilon = 1 \text{ for } \alpha^2 + p^2 - (q/2)^2 < 0$$

The total cross section, obtained by numerically integrating over the electron energy and angular distribution, is shown in Fig. (1).

The electron-neutrino spectrum from mu-plus decay at rest is given (with  $\nu$  in MeV) by

$$\frac{dN}{d\nu} = \frac{12}{(53)^4} \nu^2 (53 - \nu) .$$

The product  $\frac{dN}{d\nu} \times \sigma_{\nu D}$  is plotted in Fig. (2) showing  $\nu = 43$  MeV as the most probable interaction energy. When  $\sigma_{\nu N}$  and  $\sigma_{\nu D}$  are averaged over the neutrino spectrum we obtain

$$\langle \sigma_{\nu N} \rangle = 1.52 \times 10^{-40} \text{ cm}^2, \quad \langle \sigma_{\nu D} \rangle_{s\text{-wave}} = .44 \times 10^{-40} \text{ cm}^2 .$$

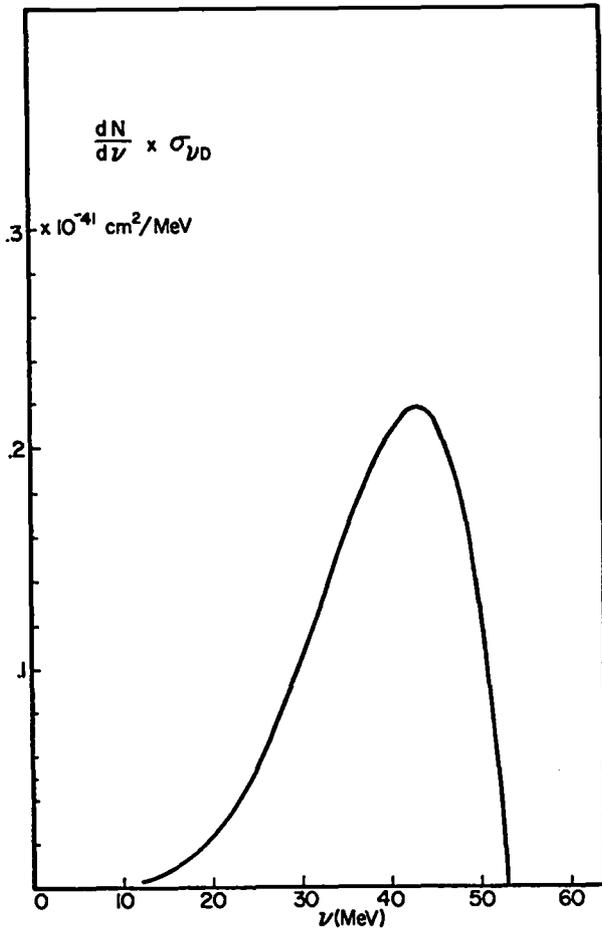


Fig. 2. The product of the deuteron (S-wave) cross section and the electron-neutrino spectrum from stopped mu-plus decays.

The electron energy spectrum at  $\nu = 43$  MeV is shown in Fig. (3). The most probable electron energy (41.2 MeV) is close to the maximum available because of the phase space weighting and the preference of the two outgoing nucleons for low relative energies. The weighted angular distribution of electrons is shown in Fig. (4) indicating a strong fore-aft asymmetry when the weak magnetism term is taken into account.

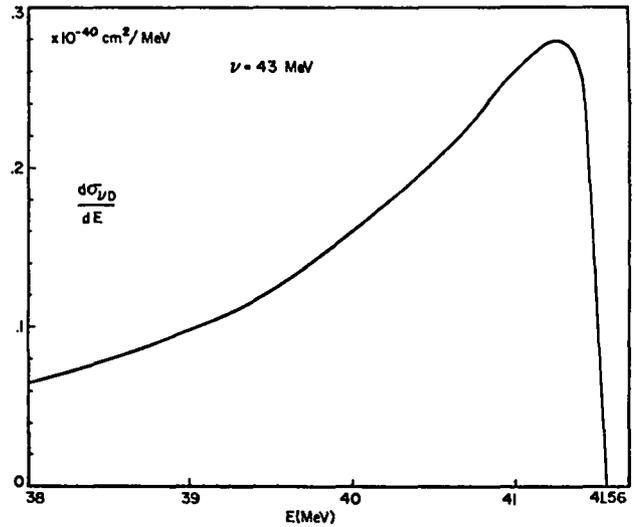


Fig. 3. Energy spectrum of electrons produced by 43-MeV neutrinos.

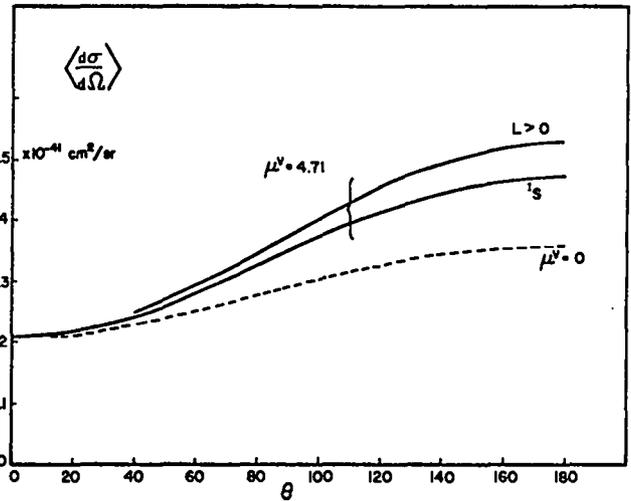


Fig. 4. Angular distribution of all electrons averaged over the neutrino spectrum. The curves show the effects of the magnetic moment term and higher partial waves.

The contribution of higher nucleon partial waves can be obtained by adding the  $L = \text{even}$  singlet partial waves using

$$\psi_p = \frac{1}{2} \left( e^{i \vec{p} \cdot \vec{r}} + e^{-i \vec{p} \cdot \vec{r}} \right) - j_0(pr)$$

and the  $L = \text{odd}$  triplet partial waves using

$$\psi_p = \frac{1}{2} \left( e^{i \vec{p} \cdot \vec{r}} - e^{-i \vec{p} \cdot \vec{r}} \right)$$

The cross section is augmented by these higher partial waves at the higher neutrino energies and back angles as shown in Figs. (1) and (4). The spectrum averaged cross section for the deuteron increases to

$$\langle \sigma_{VD} \rangle_{\text{total}} = .48 \times 10^{-40} \text{ cm}^2$$

The present calculation differs from the extension of the Kelly and Überall calculation<sup>3</sup> by Chen<sup>4</sup> in that the momentum dependence of the radial integral has been taken into account and the weak magnetism term and higher partial waves added.

The spectrum-averaged cross sections for the free neutron and deuterium can be compared to the neutrino-electron elastic scattering and to recent shell model calculations for carbon<sup>1</sup> and oxygen<sup>5</sup>.

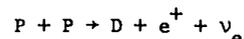
$$\langle \sigma_{Ve} \rangle = .0053 \times 10^{-40} \text{ cm}^2$$

$$\langle \sigma_{VC} \rangle = .146 \times 10^{-40} \text{ cm}^2$$

$$\langle \sigma_{VO} \rangle = .052 \times 10^{-40} \text{ cm}^2$$

These results show that the deuteron is an important target for neutrino interactions because of its large reaction cross section and because this cross section can be reliably computed. The proposed LAMPF experiment of Hughes, Nemethy, Duclos, Burman, and Cochran will utilize a  $6\text{m}^3$  deuterated-water Cerenkov counter separated by  $6\text{m}$  of shielding from the LAMPF beam stop. With  $1/3 \text{ mA}$  of  $750 \text{ MeV}$  protons incident on the beam stop a  $\nu_e + D \rightarrow 2P + e^-$  event rate of 40 per day can be expected for the deuteron spectrum averaged cross section.

The deuteron cross section is closely related to the weak reaction



which governs the rate of proton fusion in the sun and other stars. This reaction, a  $^1S_0 \rightarrow ^3S_1$  transition, has never been observed in the laboratory. Therefore, a measurement of the neutrino disintegration of the deuteron will help our understanding of the astrophysical process.

#### References

1. J. S. O'Connell, T. W. Donnelly and J. D. Walecka, "Semi-leptonic Weak Interactions with  $C^{12}$ ," Phys. Rev. C, 6, 719 (1972).
2. J. D. Walecka, "Semi-leptonic Weak Interactions in Nuclei" for the book Muon Physics, edited by V. W. Hughes and C. S. Wu, Stanford preprint ITP-391.
3. F. J. Kelly and H. Überall, Phys. Rev. Letters 16, 145 (1966).
4. H. H. Chen, University of California Irvine, Internal Report UCI-10P19-58.
5. T. W. Donnelly, private communication.