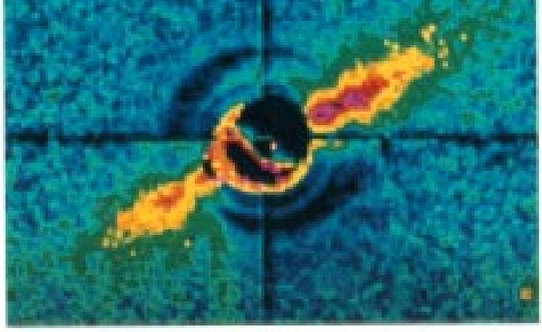
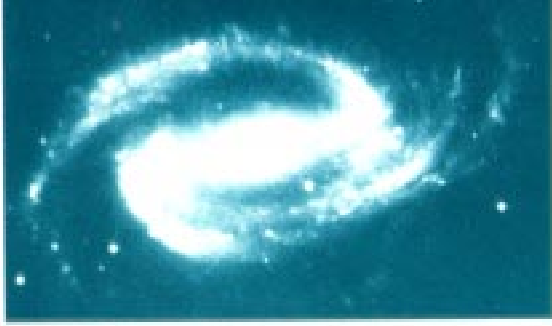






Angular Momentum
the cosmic pollutant

by Stirling A. Colgate and Albert G. Petschek

There seems to be too much angular momentum in the universe to allow the formation of stars or the accretion of matter onto variable x-ray sources. This fundamental problem begs for solution.

When we have too much of something and cannot find a way of getting rid of it, we often think of it as a pollutant. In the game of concentration and collapse of matter in the universe from clouds of gas to clusters of galaxies, galaxies, stars, planets, and black holes, angular momentum is the "pollutant" that prevents the game from being played to the absolute limit, namely, collapse into one awesome black hole for each cluster-sized cloud condensed from the early universe. Only at the scale of the whole universe does the energy in the Hubble expansion of the universe prevent collapse, independent of angular momentum constraints. On smaller scales there seems to be too much angular momentum to allow the collapse of clouds into dense objects.

◀ ***Examples of relatively dense astrophysical objects whose disk-like shapes indicate they have a net angular momentum. The central figure is Stephan's Quintet, a group of five interacting galaxies. Along the top from left to right are an edge-on view of the spiral galaxy NGC 4594, the star Beta Pictoris surrounded by a disk of dust, and the barred spiral galaxy NGC 1300. Along the bottom from left to right are the galaxy M81, an artist's conception of an x-ray binary, and the rings of Saturn. (Photo credits are given at the end of the article.)***

Yet our universe is populated by planets, stars, and black holes. How does nature get rid of the cosmic pollutant?

Models proposed for variable x-ray sources give a strong clue to this long-standing puzzle (see "X-Ray Variability in Astrophysics"). But before we restrict ourselves to x-ray variables, let's look at the problem more generally.

Angular Momentum, Weights, and Noncosmological Strings

The angular momentum of a weight of mass m whirled at velocity vat the end of a string of length R is mvR. Under ideal circumstances, that is, a rigid support for the string, no air friction, and no other external torques, the weight will continue to circle at the same velocity forever. In other words, its angular momentum $J = mvR$ is conserved. If the string is shortened, by pulling it through the support, then since the angular momentum must remain constant, the weight will speed up to a higher angular frequency $w = v/R$.

The inward force necessary to keep the weight moving in a circular path, $F_R = mv^2/R = mw^2R$, is supplied by the tension in the string. Since $F_R = J^2/mR^3$ in terms of the angular momentum, we see that the tension in the string increases very rapidly, as R^{-3}, as the string is shortened. In the cosmic game of collapse, the analogue of the tension in the string is the attractive gravitational force, which is proportional to R^{-2}. Since the required inward force goes as R^{-3}, while the available gravitational force goes as R^{-2}, there is bound to be a point beyond which gravity is unable to cause further collapse. This is the basis of a stable Keplerian orbit, like that of the earth around the sun or that of accreting matter around a compact star in an x-ray binary. Once in a stable orbit, the only way for matter to move farther inward is to lose angular momentum, but the puzzle is how? We also would like to estimate how much angular momentum has to be lost by whatever mechanism we devise.

Angular Momentum and the Universe

The universe as a whole does not seem to be rotating, as evidenced by the fact that the blackbody radiation believed to be a relic of the early universe is isotropic to better than one part in 10^4. Moreover, Tyson has found the orientations of a very large number of galaxies to be random. Since the net angular momentum appears to be zero over very large scales, the pollution is not as bad as it could be. Our problem is restricted to local patches of the universe where matter collapses to form relatively dense rotating objects such as those shown in the opening figure.

Galaxies Are Not a Problem. Let us consider the specific angular momentum $J_s = vR$ (angular momentum per unit mass) of a typical, modestly sized, spiral galaxy. The rotational velocity of matter at its outer edge, determined from the Doppler shifts of spectroscopic lines, is about 150 kilometers per second, and its radius is about 10 kiloparsecs (-3 X 10^{22} centimeters),* so J_s = 5 X 10^{29} centimeters squared per second. Suppose that, before condensing, the galactic matter occupied a space with a radius equal to one-half the average distance between galaxies, roughly 3 megaparsecs, or 300 times the galactic radius. If angular momentum was conserved in the collapse to the 10-kiloparsec galactic radius, the initial velocity of the matter must have been less by a factor of 300, or about 5 X 10^4 centimeters per second. This velocity, which is roughly the speed of sound in hydrogen at 150 kelvins, seems to be a reasonable value for the velocity of the turbulent eddies that must have existed when galaxies began to form. In fact, theoretical calculations suggest that density fluctuations in the early universe may produce velocities of this order. Theoreticians thus regard angular momentum in spiral galaxies not as a pollutant but as a much sought-after relic of an earlier history. This reasonable state of affairs is in sharp contrast to the problem angular momentum poses in the making of a star. Angular momentum may also be a problem in the formation of nearly spherical "elliptical" galaxies, which seem to have very little total angular momentum.

Collapse to Stars. The density of matter in our own galaxy before any of the matter collapsed into stars was roughly 0.1 to 1 hydrogen atom per cubic centimeter, or about 10^{-24} gram per cubic centimeter. To form a star, this dilute matter must have collapsed to a density of about 1 gram per cubic centimeter, an increase by a factor of 10^{24}. The radius would have decreased by a factor of 10^8, the cube root of the density ratio. The Keplerian velocity at the stellar radius is about 10^6 to 10^7 centimeters per second, so the initial velocity needed to conserve angular momentum must have been 10^8 times smaller, or 10^{-2} to 10^{-1} centimeter per second. This velocity is unreasonably small for the gas in a turbulent rotating galaxy. A more reasonable velocity for gas clouds, or even for galactic rotation, over the radius of the space from which the stellar matter ought to have been drawn, would be 10^5 to 10^6 centimeters per second. With this value for the velocity, coordinated motion of even a small fraction of the matter will introduce too much angular momentum, by a factor of 1O^6 to 10^8, to allow collapse. This immense amount of excess angular momentum must somehow have been dumped before collapse.

For collapse to a neutron star, 10^{14} times more dense than a normal star, the problem would be worse by a factor equal to the sixth root of 10^{14}. (Since the Keplerian velocity is proportional to $R^{-1/2}$, $J_s = vR$ is proportional to $R^{1/2}$, or $p^{-1/6}$.) Thus we have another factor of about 100, or a total of 10^{10} times too much angular momentum.

We have discussed only the initial and final states involved in star formation, both of which are spherical. It must not be imagined, however, that the collapse is spherical throughout. Angular momentum conservation prevents collapse only in the directions perpendicular to the rotation axis; collapse parallel to the rotation axis is not inhibited and thus occurs first. This leads to formation of a disk whose radius is almost as large as the initial radius of the cloud. But the disk still has the large initial angular momentum that seems to prevent further collapse. Where and what are the galactic dumping grounds for this angular momentum?

"The unit of distance called a parsec is equal to about 3 X 10^{18} centimeters; its name is derived from parallax-second. A parsec is the distance at which the direction to an object, viewed from the earth at opposite phases of the earth's orbit around the sun, changes by 1 second of arc. The pointing accuracy of a typical old-fashioned telescope is about 1 second of arc.

Magnetic Fields

Only external torques can alter the angular momentum of a system. An obvious way to apply a global external torque to dilute ionized matter is through magnetic fields. Indeed this is an often-invoked panacea for the problem. The difficulty is that magnetic fields with the necessary strength and dimension are not observed in the universe. Furthermore, even if our estimates of magnetic field strengths (obtained from observations of the Faraday rotation of the polarization angle of radio waves caused by their passage through a magnetic field) are erroneous, we are faced with the following dilemma. If matter is to be strongly affected by magnetic fields, as is reasonable for partially or fully ionized matter, it is also reasonable that the matter is strongly tied to the field lines. Hence, as a not so extraneous conclusion, magnetic confinement fusion should be simple. The fact that it is not means that ionized matter escapes magnetic fields deceptively easily. Suppose, to the contrary, that matter and field are strongly coupled. Then purely two-dimensional radial collapse by our factor of 10^8 would mean that a region of uniform galactic field of 3 X 10^{-6} gauss would be compressed by a factor of 10^{16} in the newly formed star, and the field would increase to 3 X 10]0 gauss. This is too much field by many orders of magnitude. The pressure of such a field, $B^2/8\pi \approx 4 \times 10^{19}$ dynes per square centimeter, is larger than the pressure inside the newly formed star by a factor of 10^4 to 1O^6. Hence, magnetic field must escape easily from the collapsing matter even though it cannot escape too easily if it is to remove the extra angular momentum. Such a balance between field escape and field trapping seems most unlikely—although possible.

Thin Keplerian Accretion Disks

The hydrodynamics of thin accretion disks provides a more plausible mechanism for getting rid of angular momen-

AZIMUTHAL VELOCITIES OF ROTATING WHEEL AND KEPLERIAN DISK

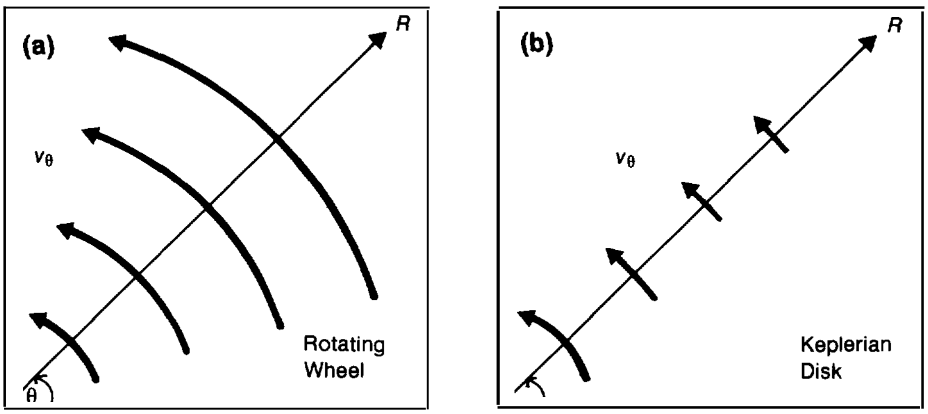


Fig. 1. Distribution of azimuthal velocities v_θ on (a) a rotating wheel and (b) a Keplerian disk. In a rotating wheel $v_\theta \propto R$ and $J_s \propto R^2$, so angular momentum is concentrated at the periphery. In a Keplerian disk $v_\theta \propto R^{-1/2}$ and $J_s \propto R^{1/2}$. The velocity shear in the disk tends to equalize the velocities and therefore transport angular momentum toward the periphery, that is, make the disk more like a wheel. In a gaseous disk ordinary molecular diffusion is too slow to explain the transport of angular momentum required for star formation. ◀

turn, or at least for allowing its transport outward as matter accretes toward a central point.

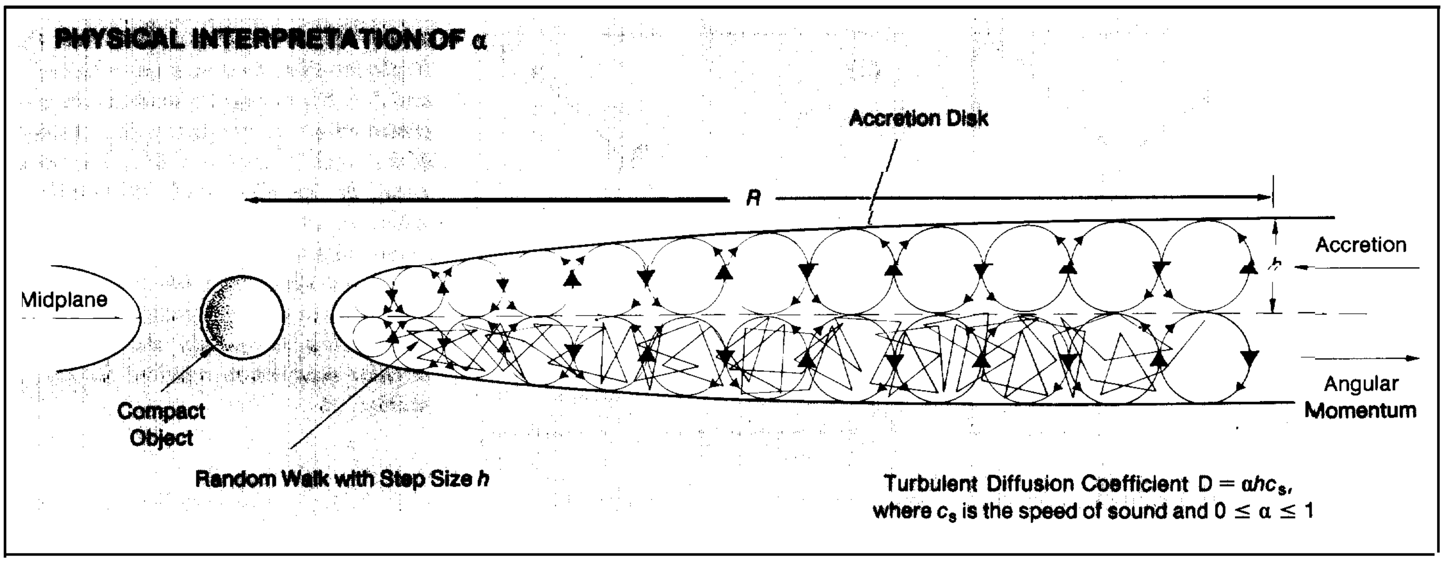
In variable x-ray sources and cataclysmic variables, accretion disks form around small, dense stars as matter from a companion star is pulled toward the compact object and trapped into orbit by the strong gravitational field. Such accretion disks resemble the rings around Saturn, but, whereas the rings around Saturn are evidently composed of solid chunks of matter that occasionally bump into one another, an accretion disk is composed of gaseous matter. The gaseous disks that eventually collapse to form isolated stars are thought to be quite similar.

Let us look at a likely state for matter that has partially collapsed and run into the angular momentum barrier. As explained earlier, it will go into a stable Keplerian orbit. Now matter of slightly different angular momenta will go into orbit at slightly larger or smaller radii and have slightly different velocities. If this matter is in gaseous form, it will "rub" with this differential velocity, and the friction will lead to a torque and hence a change in angular momentum. The direction of the rub tends to make the gaseous disk rotate more like a solid body or a wheel. In other words, matter at the periphery tends to speed up, increasing its angular momentum, while matter near the center slows down, decreasing its angular momentum (Fig. 1). The friction does just what we want—it transports angular momentum from the inside to the outside of the disk, allowing the inner matter to collapse and the outer matter—a small fraction—to be spun up and flung off, carrying with it all the excess angular momentum. So what's the rub?

The interaction between adjacent mass elements moving at different velocities can be characterized by the kinematic viscosity D (the ratio of dynamic viscosity to density). We use the unlikely symbol D for kinematic viscosity because this is really a diffusion coefficient. It describes how fast a viscous wave (velocity shear) relaxes due to molecular motion. To carry out an order-of-magnitude calculation, we use the kinematic viscosity of hydrogen at room temperature, which (in centimeters squared per second) happens to be almost equal to $10^{-3}/p$ when the density p is expressed in grams per cubic centimeter. As in any diffusion phenomenon, the relaxation time τ equals the square of the diffusion distance divided by the diffusion coefficient, or R^2/D. Combining these equations with an expression for the density, we find for τ, the time required for angular momentum to "diffuse" out of the disk, a value of 3×10^{17} seconds times the mass of the central object (in solar masses) and divided by the thickness of the disk (in parsecs). The thickness of the disk is much less than its radius, which must become much smaller than 1 parsec in the course of star formation. Hence the time required to form a star in this way would exceed the age of the universe, at most 6×10^{17} seconds, by several factors often. Yet stars abound. Clearly we need a better rub or more viscosity.

The Rub, or *a*

A model that assumes a large viscosity was invented by Shakura and Sunyaev to explain the apparently rapid accretion of matter from a disk onto a compact star in x-ray and cataclysmic variables. This model invokes turbulence as the source of the viscosity but does not describe how the turbulence is driven. The strength of the turbulence is parametrized by a coefficient *a*, which can be varied between O and 1. Calculations based on this hypothetical turbulent viscosity have been very successful in duplicating the apparent accretion rates in x-ray variables. The value of cc turns out to be quite large, implying that the accretion disk is highly turbulent. Such calculations, and even more detailed calculations of accretion in cataclysmic variables, strongly suggest the validity of the model. Thus the elusive friction in Keplerian disks may have been identified. If so, we know how nature gets rid of the excess angular momentum that would otherwise prevent the formation of stars and hence us.



A

Fig. 2. Cross section of a thin Keplerian accretion disk around a compact object. Large eddies with radii equal to the half-thickness of the disk transport angular momentum in the radial direction R. The diffusion caused by these eddies can be approximated by a random walk with step size h and velocity of the order of the sound speed c_s. Shakura and Sunyaev modeled this transport by a turbulent kinematic viscosity hc_s.

A Physical Interpretation of *a*. Turbulence is the enhanced transport of matter due to relatively large-scale, random motions of a fluid. If there are velocity gradients in the matter, then the effect of turbulence is to transport momentum across a mean velocity shear; it acts like viscosity or friction. The maximum rate of transport by turbulence is determined by the maximum size of the eddies; that is, the diffusion caused by turbulence can be approximated by a random walk with a step size equal to the diameter of the largest eddy. Since the largest eddy that can "fit" in the disk and transport matter in the radial direction is a round eddy whose diameter is *h*, the half-thickness of the disk (Fig. 2), and since the maximum velocity of such an eddy is the local sound speed c_s, the maximum possible random-walk diffusion coefficient, or turbulent kinematic viscosity *D*, is hc_s. Thus Shakura and Sunyaev parameterized the turbulent kinematic viscosity by αhc_s, where $0 \le \alpha \le 1$. To match observations α must be between 0.03 and 1.

What is the Origin of the Turbulence?

We are accustomed to the ubiquity of turbulence in fluids with velocity shears and large Reynolds numbers. Since these conditions are met in most accretion disks, it seems reasonable to expect turbulence to supply the necessary fluid friction. But, as Lord Rayleigh pointed out more than a century ago, the constraint of angular momentum conservation is strong enough to stabilize the shear flow of a Keplerian disk against shear-produced, or Helmholtz, instabilities. Hence these instabilities alone cannot drive the turbulence. Another possibility is that the turbulence is driven by heat convection. Turbulence always produces friction, but now we must ask, conversely, whether the heat produced by the friction from velocity shear is enough to drive the turbulence. In the next section we will explore this possibility as an example of how difficult it is to produce the large values of a required to transport momentum outward in Keplerian accretion disks.

Convection-Driven Turbulence. In an accretion disk, friction from velocity shears should give rise to inhomogeneous heating concentrated near the midplane of the disk. This differential heating can create instabilities that lead to turbulent motion. To transport angular momentum outward, the turbulent motion must be isotropic (or nearly so) rather than just in the "easy" azimuthal direction. It is easy to create eddies whose axes are in the radial direction and whose velocities are azimuthal, but we need an instability strong enough to overcome the stabilizing effect of angular momentum and drive radial motions. If these instabilities exist, the turbulent motion provides an effective viscosity (an "eddy" viscosity) far larger than the molecular viscosity and can transport angular momentum at the rate required for the evolution of the disk.

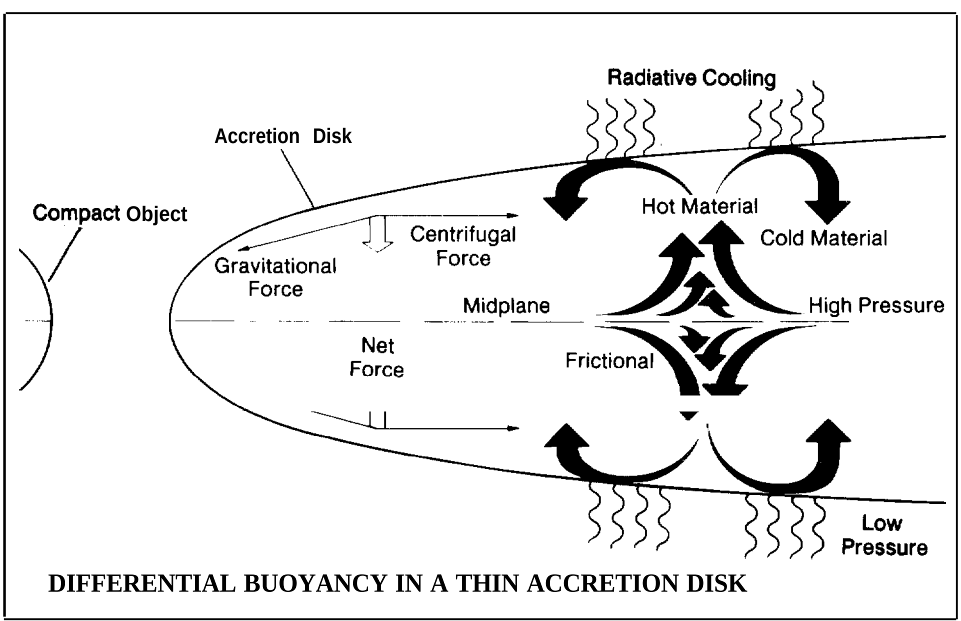
To see whether differential heating can drive the required instability, we must look at the structure of the accretion disk implied by the Shakura and Sunyaev model (see Fig. 2). As indicated above, a value of a near unity implies that the diameter of the eddies that interchange matter in the radial direction must be close to the half-thickness h of the disk, and their velocity must equal the sound speed c_s. Thus the interchange scale in the radial direction, if the eddies are round, will equal h. The internal energy in the eddy, which is determined by the sound speed c_s, will determine how much energy is available to drive the interchange. As Pringle has pointed out, the structure of Keplerian disks is such that $c_s/v = h/R$, where v is the azimuthal velocity and R is the radial distance.

The disk is densest in the midplane because the pressure is greatest there. The pressure is required to hold the material near the surface out against the component of gravity perpendicular to the disk (Fig. 3). Consequently frictional heating from velocity shears and therefore the temperature will be greatest at the midplane. The surface of the disk will be cooled by radiation. This configuration is Rayleigh-Taylor unstable in the direction perpendicular to the plane of the disk. Motions perpendicular to the disk and at the same radius do not transport angular momentum. On the other hand, motions in the radial direction do transport angular momentum and produce enhanced friction and a. The force from differential buoyancy must be large enough to create an eddy that interchanges matter over a distance $\Delta R \approx h$.

Now let's consider a single eddy and calculate how much work is needed to interchange two mass elements a distance AR while conserving angular momentum. We will assume an initial laminar stable state so that in the interchange of two adjacent elements of equal mass, the angular momentum of each is conserved separately (vR = constant). Then for each mass element the change in azimuthal velocity, Δv, will be such that $\Delta v/v = -\Delta R/R$. The change in specific kinetic energy of the two elements is $1/2[(v_1 + \Delta v_1)^2 - v_1^2] + 1/2[(v_1^2 + \Delta v_2)^2 - v_2^2]$. The angular momentum constraint, together with a little algebra, shows that the net change is equal to $(\Delta v)^2$. The net change in gravitational energy will be zero because we have assumed the interchange of two equal masses. Hence we need an energy equal to $(\Delta v)^2$ to effect the interchange. This energy must be provided by differential buoyancy.

The energy available from raising hot fluid and lowering cold fluid is the internal energy $E = c_s^2/(\gamma(\gamma-1))$. For a typical gas with $\gamma = 5/3$, $E \approx c_s^2$. Using the fact that $c_s/v = h/R = \Delta v/v$, we derive a maximum possible buoyancy energy of $E \approx (\Delta v)^2$. Since the work that must be done in the interchange is the same as the energy available in buoyancy, there is barely enough buoyancy to force an overturn or a circular eddy in the radial direction, especially at an eddy velocity near c_s. For the eddy to develop we need a nearly perfect heat engine that converts the heat of friction to potential and kinetic energy.

The Ideal Heat Engine. We can imagine an ideal heat engine driving the eddies. The heat is produced in our mass element of size $h = \Delta R$ at the midplane of the disk by turbulent friction due to the shear of the orbital velocity. The hot material expands adiabatically as it rises to the surface of the disk. There the remaining internal energy must be lost by radiation during the residence time of the mass element at the surface, that is, in the time $\Delta R/(\alpha c_s)$. Thus the diffusion coefficient for radiation, $(\beta c/3)(\Delta R/\tau)$, where β is the ratio of radiation energy density to total energy density $(aT^4/(aT^4 + nkT))$, c is the velocity of light, and τ is the optical depth of the disk, must be the same as the coefficient for turbulent mass transport. This implies that τ must be of order $\beta c/(3\alpha c_s)$. With these restrictions our mass element would cool, and it could then descend adiabatically with much smaller internal pressure, so less work must be done on it. When it reaches the midplane of the disk, radially displaced by ΔR, frictional heating



◀ ***Fig. 3. Accretion disks are denser at the midplane, where the gravitational potential is lower, than at points above and below it. The dense material will be heated by velocity shears and rise to the surface of the disk where it will cool by emitting radiation. The question to ask is whether the differential buoyancy is large enough to drive an eddy in the radial direction.***

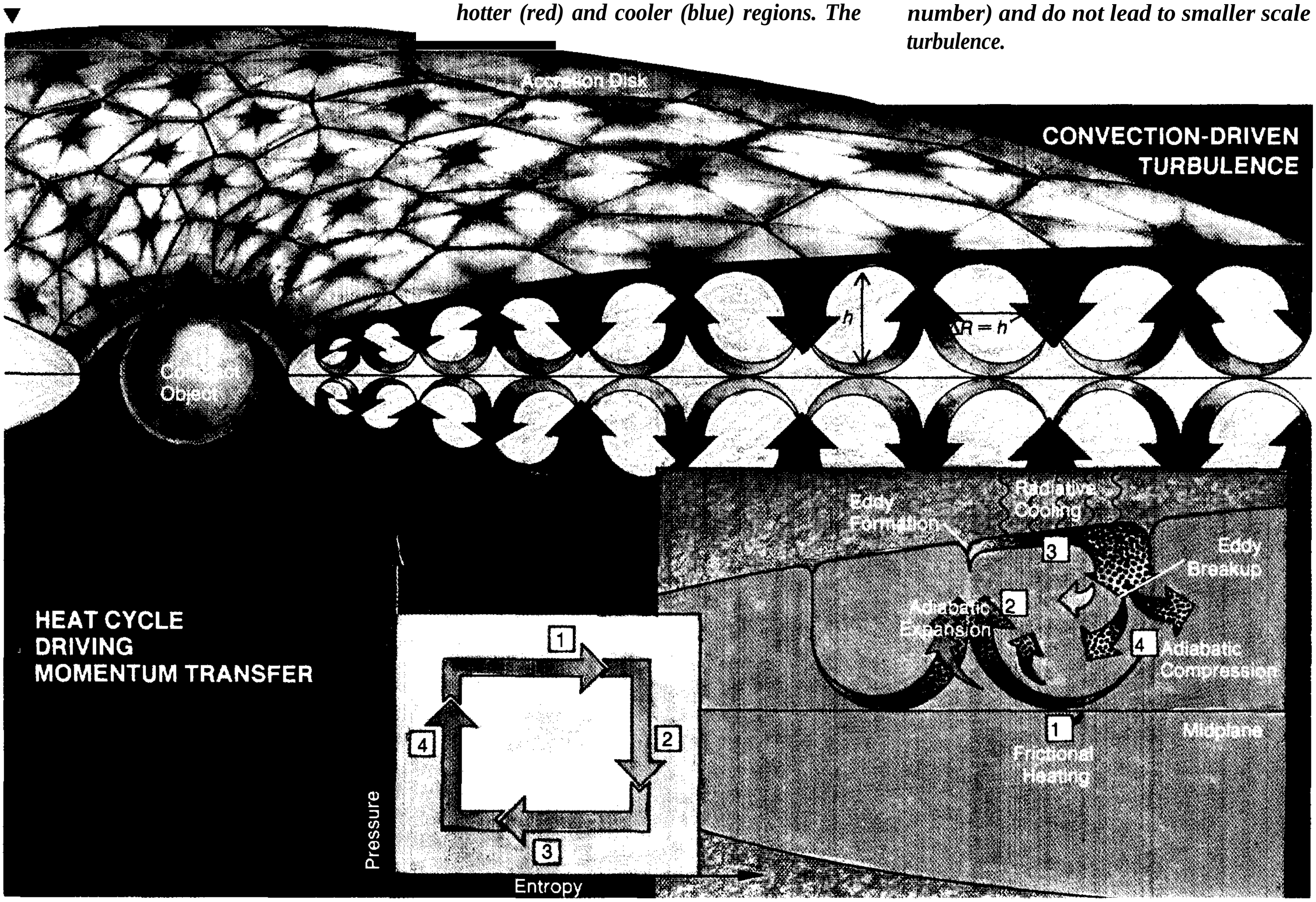
can start the cycle over again. This cycle as well as the alternating regions of hotter and cooler material that would result at the surface are illustrated in Fig. 4.

The required condition on the optical depth, together with the opacity of the material, determines the mass per unit area of the disk. Then the mass flow rate can be calculated from a and the sound speed. It is not known whether this mechanism leads to a self-adjusting disk in the sense that if the mass-injection rate changes, then a and the other parameters vary to maintain a consistent disk structure. It is also not known whether all astrophysical disks can be explained in the parameter space just outlined. Furthermore, nature does not like to make ideal heat engines, especially not in a turbulent environment because heat engines must be so perfect and turbulence is so random. Thus, the above ideal cycle, although conceptually feasible, seems difficult to justify as an explanation for the origin of a. Convection-driven turbulence does not seem strong enough to overcome the angular momentum barrier.

Thick Disks Beg the Question

The importance of angular momentum

Fig. 4. Convection-driven turbulence in a thin Keplerian accretion disk creates large eddies that break the angular momentum constraint by enhancing radial transfer of angular momentum. The energy available from differential buoyancy is barely enough to drive the eddies. Their formation would require a nearly perfect engine, that is, one in which nearly all the heat was converted to work as matter flows around the eddy. The hexagonal pattern of Benard-like cells shown might reproduced by heating at the midplane. The heat cycle that drives the overturn of the eddies produces alternating hotter (red) and cooler (blue) regions. The figure suggests the required breakup of the eddies into smaller scale turbulence after about half a cycle. (A persistent eddy would not produce any net transport.) This breakup is a difference between the eddies in the disk and those in a standard Benard cell, which are very slow (low Reynolds number) and do not lead to smaller scale turbulence.



constraints is illustrated in studies of thick accretion disks by Wojciech Zurek and Winy Benz of Los Alamos. They have performed numerical simulations of the evolution of thick disks with specific angular momentum independent of radius. Because the specific angular momentum is constant, the interchange of two equal mass elements requires no energy. The disks exhibit violent Helmholtz instabilities. As one would expect, less constraint leads to greater turbulence. The instabilities cause angular momentum to redistribute itself very quickly to $J_s \propto R^q$, where q is about 0.27 (see "Redistribution of Angular Momentum in Thick Disks"). Thus the disk becomes more Keplerian, but since these instabilities are damped the disk never becomes truly Keplerian $(q = 0.5)$.

Such models invite the question of how a disk can be formed with small and nearly uniform angular momentum. In the case of quasars and active galactic nuclei powered by the accretion of matter onto massive black holes, these disks might be formed by the gravitational breakup of stars scattered by interactions with other stars in the strong gravitational field close to the massive black hole. More specifically, stars in a dense galactic nucleus scatter at random. Occasionally one of these scattering events causes a star to approach the black hole with an impact parameter so small (several Schwartzschild radii) that the star deforms tidally and a fraction of the star is captured. Other stars of the cluster then have a slightly greater angular momentum because of its conservation. Jack Hills of Los Alamos calculated that a thick disk of low angular momentum is a reasonable outcome of such accretion.

Thus there may not be an angular momentum problem in feeding a black hole, but the original problem of making a star from tenuous gas remains. Either we must posit an initial gaseous state of tiny and thus statistically unlikely angular momentum, or we are left with, the imperative to find a transport mechanism for the cosmic pollutant. ■

AUTHORS

Stirling A. Colgate received his B.S. and Ph.D. degrees in physics from Cornell University in 1948 and 1952, respectively. He was a staff physicist at Lawrence Livermore Laboratory for twelve years and then president of New Mexico Institute of Mining and Technology for ten years. He remains an Adjunct Professor at that institution. In 1976 he joined the Theoretical Division at Los Alamos and in 1980 became leader of the Theoretical Astrophysics Group. He is also a Senior Fellow at the Laboratory, a member of the National Academy of Sciences, and Technical Director of the Gifthorse Program. His research interests include nuclear physics, astrophysics, atmospheric physics, and geotectonic engineering.

Albert G. Petschek is currently a Fellow at Los Alamos as well as being Professor of Physics at New Mexico Institute of Mining and Technology. He received his B.S. from MIT in 1947, his M.S. from the University of Michigan in 1948, and his Ph.D. from the University of Rochester in 1952. His research interests are in astrophysics, cloud physics, and fracture mechanics. The spring of 1978 found him in Israel as a Visiting Professor at Tel Aviv Univerity. He is a member of the American Physical Society, the American Association for the Advancement of Science, and the American Astronomical Society.

Photo Credits

Stephan's Quintet. A combination of best available photographs in different colors taken with the 200-inch reflector at Palomar Observatory.© Halton C. Arp; reproduced with permission.
NGC 4594. Palomar Observatory photograph; reproduced with permission.
Beta Pictoris. A color-coded map created from optical images of Beta Pictoris and the similar star Alpha Pictoris, both taken with a coronagraph and a charge-coupled device at the Las Campanas Observatory in Chile. The disk surrounding Beta Pictoris was revealed by plotting the ratio of the intensity of scattered light around Beta Pictoris to that around Alpha Pictoris. Reproduced with permission of Bradford A. Smith, University of Arizona, and Richard J. Terrile, Jet Propulsion Laboratory.
NGC 1300. Palomar Observatory photograph; reproduced with permission.
M81. A color-enhanced image in which radiation from gas in the spiral arms of the galaxy appears blue and radiation from older stars in the disk appears orange. The image was composed from single-color photographs taken with Palomar Observatory's 48-inch Schmidt telescope.© Halton C. Arp; reproduced with permission.
Rings of Saturn. An optical image in real color created from data collected in October 1980 by NASA's Voyager I satellite. The rings have been enhanced with additional color. Reproduced with permission of the NASA Jet Propulsion Laboratory.

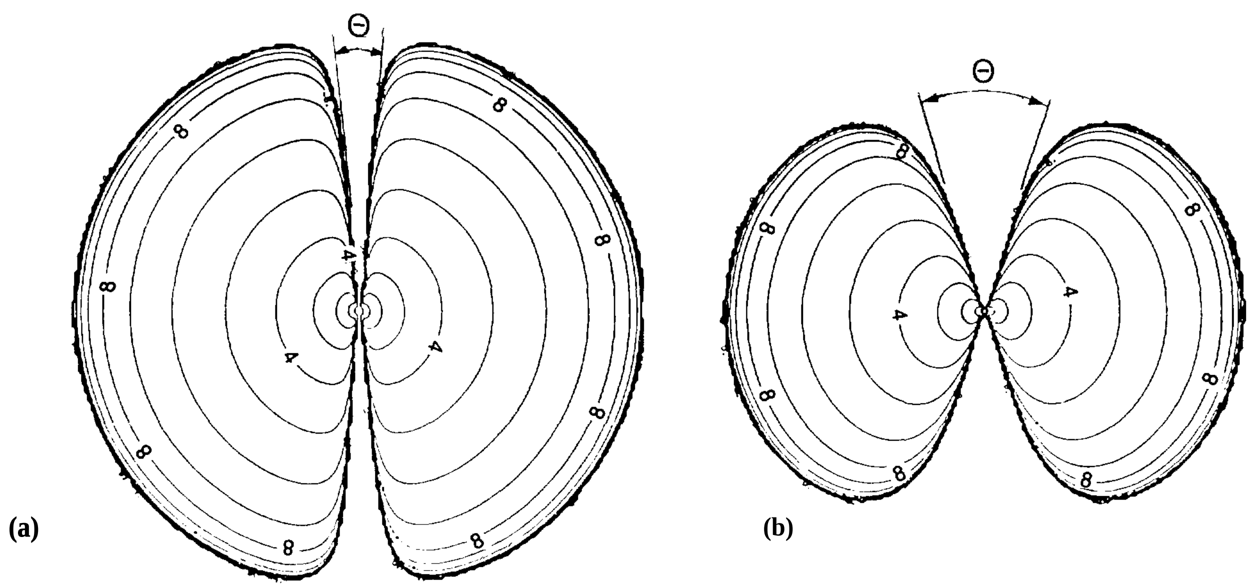
Further Reading

Pringle, J. E. 1981. Accretion discs in astrophysics. *Annual Review of Astronomy and Astrophysics* 19:137-162.

Shakura, N. I. and Sunyaev, R. A. 1973. Black holes in binary systems. Observational appearance. *Astronomy and Astrophysics* 24:337-355.

Redistribution of Angular Momentum in Thick Accretion Disks

by Wojciech H. Zurek and Winy Benz



CROSS SECTIONS OF THICK ACCRETION DISKS

A

Fig. 2. Isodensity contours of two tori with approximately the same inner and outer radii, but with different distributions of specific angular momentum: (a) a torus with J_s = constant and (b) a torus with J_s - $r^{0.27}$ Note the change in the funnel opening angle from -10° in (a) to -30° in (b). The density decreases by twelve orders of magnitude from the innermost to the outermost contours.

The importance of the distribution of angular momentum is illustrated in our recent numerical simulations of thick accretion disks. These torus-like disks, in contrast to the flat, pancake-like Keplerian disks, have a height above the equatorial plane comparable to their extent in the radial direction. Until recently the constant specific angular momentum (J_s = constant) variety of such non-Keplerian accretion disks around massive black holes was considered the best model for the central "powerhouse" in quasars and active galactic nuclei. However, doubts about the validity of that hypothesis were raised in 1984 by analyses of the stability of the disks by Papaloizou and Pringle. Our numerical simulations confirm those first suspicions. More important, we were able to demonstrate that growing instabilities in a constant-J, torus rapidly redistribute angular momentum, causing the torus to become thinner and more Keplerian. Hence thick accretion disks with constant J_s cannot be regarded as models for astrophysical objects.

The equilibrium configuration of thick accretion disks, for constant J_s and large pressure forces (sound speeds comparable to rotational velocities), looks like a fat torus, or doughnut (Fig. 1a). The sides of the torus form a funnel, with a small opening angle 6, about the rotation axis. As noticed by Lynden-Bell, this is a perfect shape for a deLaval nozzle,* that is, for accelerating the enormous supersonic jets observed to be emanating from so many active galactic nuclei. Moreover, the steep walls of the funnel allow the luminosity of the torus to exceed the Eddington limit, a property that would be indeed useful in modeling variable quasars with huge power outputs.

The first question to ask is how such non-Keplerian accretion disks can exist, since matter at small radial distances r has "too much" angular momentum (more than the Keplerian value) and matter at large r has "too little" angular momentum. The pressure in the disk is responsible for maintaining this distribution of angular momentum: the matter at large r is being prevented from moving inward by the pressure, and the matter at small r is being kept from moving outward by the pressure-mediated weight of the outer parts of the disk.

Proponents of J_s = constant accretion disks assume that the effective turbulent viscosity of such a disk is very small, so that it is all but impossible to transport angular momentum outward as matter accretes toward the massive body at the center of the disk. However, if the central massive body is a black hole (as it is almost certain to be for quasars and active galactic nuclei, including the nucleus at the center of our own Milky Way), then it is possible to get rid of the angular momentum of accreting matter by pushing it into the black hole. The necessary push can be provided by applying pressure from far away and "force-feeding" the black hole with gas.

Using an idealized model, Papaloizou and Pringle challenged the foundations of the thick accretion disk theory by showing that constant- J_s tori are violently unstable against nonaxisymmetric shear-driven perturbations. The instabilities revealed by their linear analysis are Helmholtz instabilities and in some ways are analogous to "fire-hose" instabilities. However, the gas does not stream out in random directions, as would water from a hose left unattended. Instead the gas deflected from its equilibrium orbit by the instability is bound by the gravitational potential and so produces density inhomogeneities, pressure gradients, and sound waves, which, in turn, produce more deflections, which lead to more sound waves.

In a second paper Papaloizou and Pringle extended the stability analysis to include very thin tori (like slender bicycle tires) with J_s varying as r^q. For $q < 2 - \sqrt{3} \cong 0.2679$, the tori were found to be unstable. For greater values of q a large class of unstable modes is stabilized. Their linear analysis did not, however, reveal the ultimate fate of the original configuration.

The action of a deLaval nozzle is described by M. L. Norman and K.-H. A. Winkler in "Supersonic Jets," Los Alamos Science Number 12, 1985.

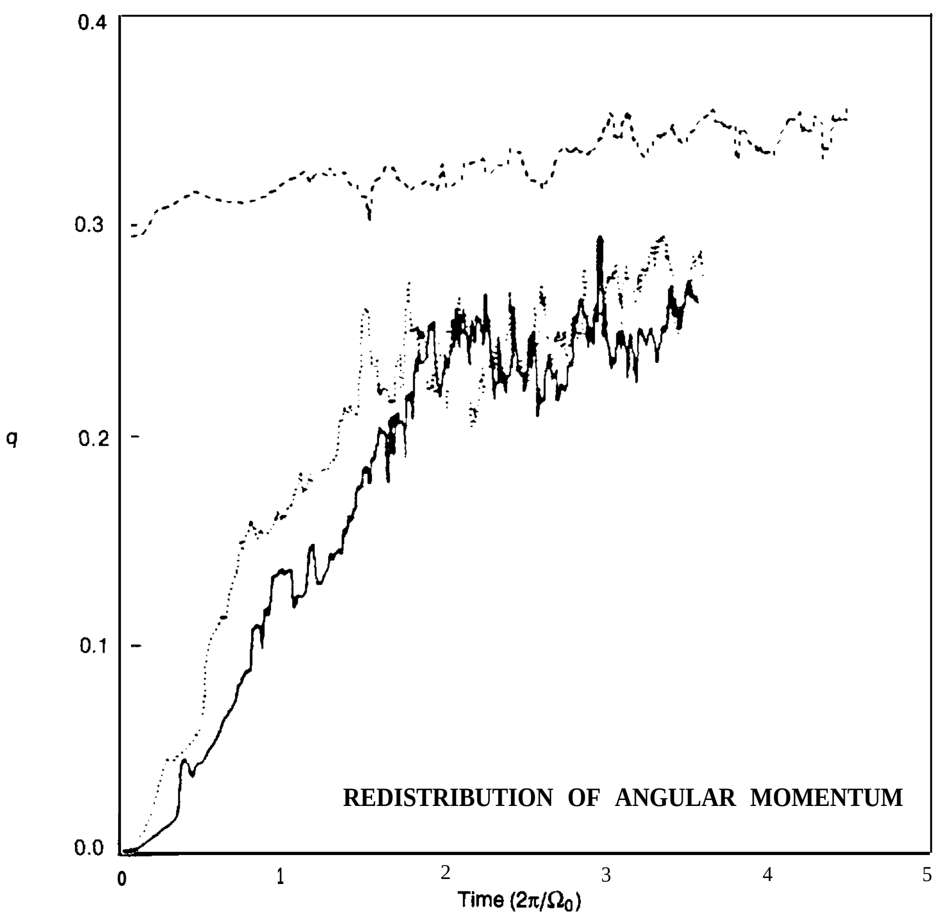
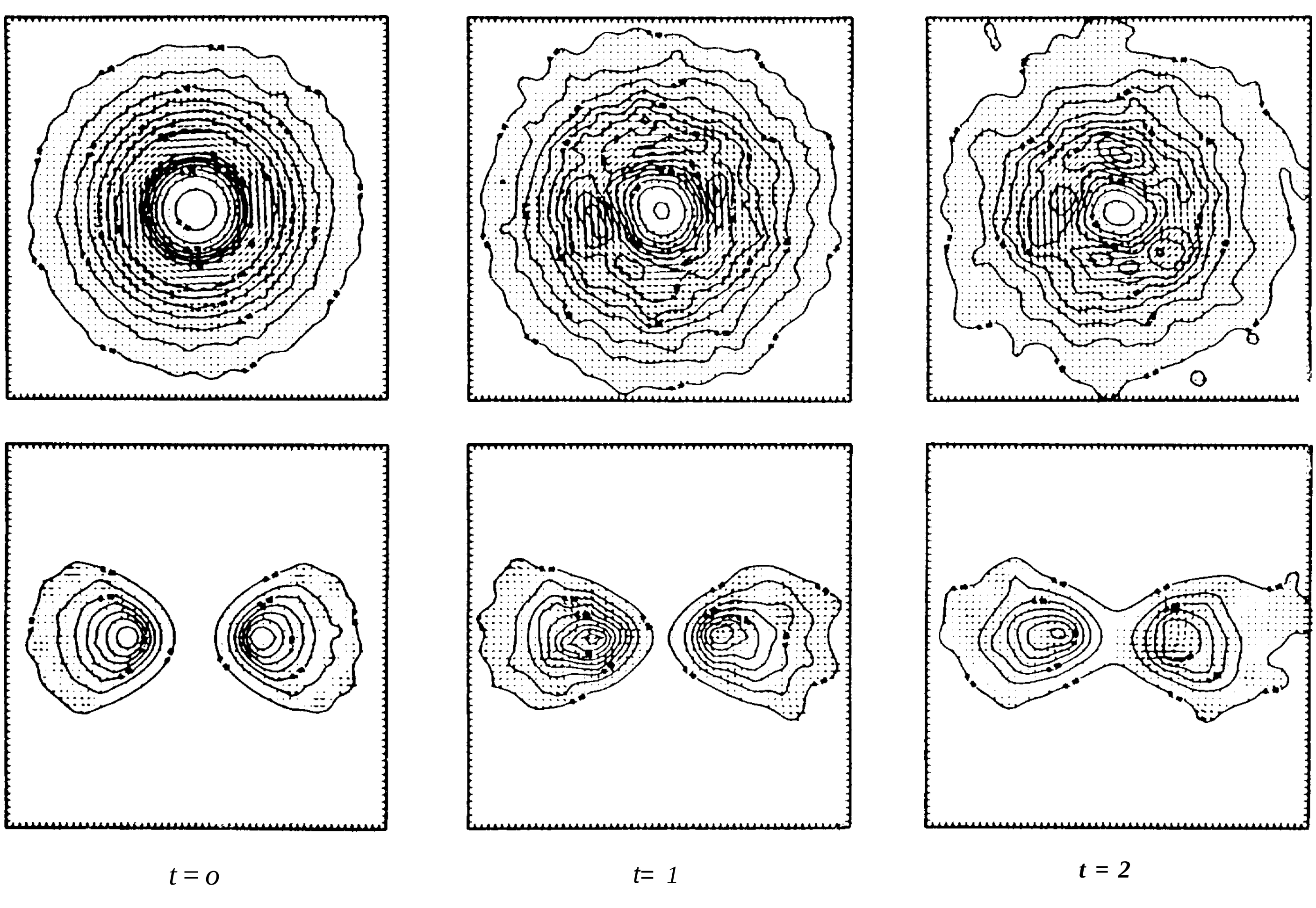


Fig. 2, The exponent q as a function of time, where q is calculated from a power-law fit ($J_s \sim r^q$) to the specific angular momentum distribution obtained from the numerical simulation. The results are shown for three simulations. Note that, for the two disks with initially constant specific angular momentum, q increases rapidly from O to about 0.27 within about two rotation periods. After the critical q = q_c-0.27 is reached, the redistribution of angular momentum slows down to the rate observed in a disk with initial J_s - $r^{0.3}$, It is not yet known whether this slow rate of angular momentum transport is caused in part by nonaxisymmetric instabilities or is totally explained by a numerical viscosity that is an unavoidable artifact of such calculations. We are planning to study this problem further.

We have extended such stability analyses to the nonlinear regime by adding a small random density perturbation (of the order of 1 percent) to an initial equilibrium configuration with constant J_s (see Fig. 3, t = 0). These numerical experiments not only confirm that such disks are unstable but also show that a fat accretion torus is forced to undergo a "crash diet": instabilities redistribute the angular momentum very quickly (on the time scale of about a rotation period J_s) from J_s = constant to J_s - r^{q_c}, where q_c turns out invariably to be 0.27 (Fig. 2). Note that this value for the

Fig. 3. A computer-generated time sequence showing the three-dimensional evolution of the central region of a thick accretion disk with initially constant specific angular momentum. The upper panels show isodensity contours in the equatorial plane of the central region; the lower panels show isodensity contours in a plane parallel to the rotation axis. The density decreases by one to two orders of magnitude over the region shown. Time is expressed in rotation periods of the density maximum. The velocity field is indicated by means of arrows whose lengths are normalized to the maximum value of the velocity in each frame. Following the introduction of a small nonaxisymmetric perturbation, the growth of instabilities causes a rapid redistribution of angular momentum that, in turn, flattens the disk and fills in the central "hole." This simulation was made with a three-dimensional hydrodynamics code that uses the so-called smoothed particle hydrodynamics method (Lucy 1977). This free Lagrangian approach to solving the usual equations of hydrodynamics replaces the continuum by a finite set of spatially extended particles. Thus no mesh is required, and the usual problems as-

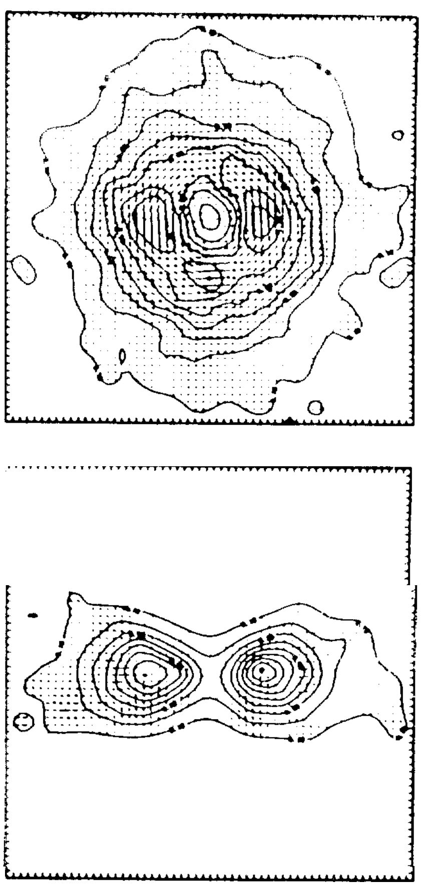
TIME EVOLUTION OF A THICK DISK



t = o *t= 1* ***t = 2***

exponent is about the same as that obtained by Papaloizou and Pringle for the stabilization of "bicycle tire" tori. Figure 3 shows the time evolution of the disk.

sociated with its rezoning are bypassed. The simulation is made by computing the trajectories of 1000 extended particles that interact through pressure forces in a central gravitational potential. Since the particles are allowed to move without any constraints in all three spatial directions and since a mesh is not needed, this method is particularly suited for the simulation of highly distorted flows.



t = 3

As the angular momentum is redistributed, the fat torus becomes much thinner and much more Keplerian in appearance. Moreover, the narrow funnel invoked to explain the formation and collimation of relativistic jets becomes much wider (Fig. 1 b) and therefore less effective in producing collimated jets and super-Eddington luminosities.

Regarding the question of angular momentum transport discussed in the main text, our calculations show that, at least for $q < q_c$, shear-driven instabilities provide a powerful source of the "rub," that is, of *a,* the turbulent viscosity. The next obvious question—not addressed properly by the calculations performed to date—is whether the shear-driven instabilities will provide a mechanism for a when $q > q_c$. Can these instabilities generate wave-like excitations and "interesting" *a* values in disks that are "barely" stable ($J_s = r^{q_c}$) or almost Keplerian ($J_s = r^{o5}$)? We are now exploring this question with one of the new three-dimensional hydrodynamics codes developed at Los Alamos. ■

Further Reading

Abramowicz, Marek A., Calvani, Massimo, and Nobili, Luciano. 1980. Thick accretion disks with super-Eddington luminosities. *The Astrophysical Journal* 242:772-788.

Fishbone, Leslie G. and Moncrief, Vincent. 1976. Relativistic fluid disks in orbit around Kerr black holes, *The Astrophysical Journal* 207:962-976.

Lucy, L. B. 1977. A numerical approach to the testing of the fission hypothesis, *The Astronomical Journal* 82:10131024.

Lynden-Bell, D. 1977. Gravity power. *Physica Scripta* 17:185-191.

Papaloizou, J. C. B. and Pringle, J. E, 1984. The dynamical stability of differentially rotating discs with constant specific angular momentum. *Monthly Notices of the Royal Astronomical Society* 208:721-750.

Papaloizou, J. C. B. and Pringle, J. E. 1985, The dynamical stability of deferentially rotating discs—IL *Monthly Notices of the Royal Astronomical Society* 213:799-820.

Zurek, W. H. and Benz, W. Redistribution of angular momentum by nonaxisymmetric instabilities in a thick accretion disk, Accepted for publication in the 1 September 1986 issue of *The Astrophysical Journal.*

AUTHORS

Wojciech Zurek earned a Ph.D. in theoretical physics from the University of Texas, Austin, in 1979 and is now a J. Robert Oppenheimer Fellow in the Laboratory's Theoretical Astrophysics Group. Winy Benz is a postdoctoral fellow in the same group, He earned his Ph.D. in astronomy and astrophysics from the University of Geneva in Switzerland in 1984.