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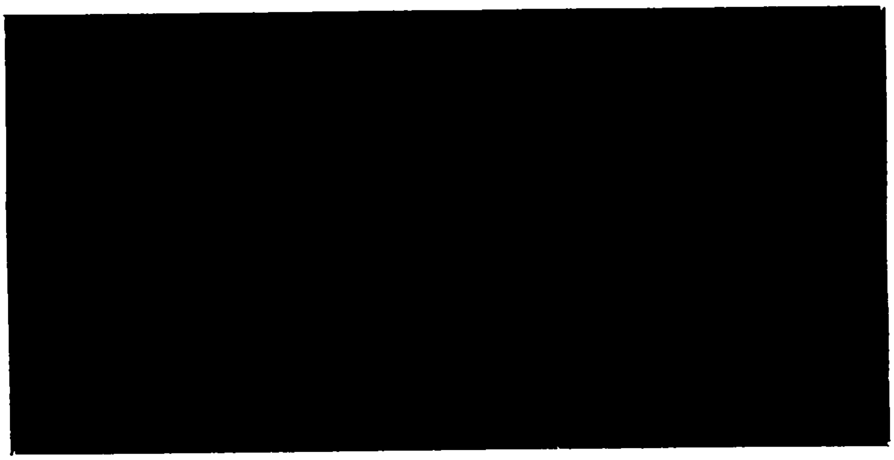

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AN OPTICAL FALLOUT ANALOGUE

by

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INSTRUMENTATION

ABSTRACT

The principle of the Optical Analogue is that a plate may be exposed in such a way that the resulting photographic density distribution represents the surface density of fallout debris from a nuclear detonation. The essential components are an optical filtering system which controls the intensity of light according to the assumed initial distribution of activity over height and particle size and according to the assumed decay rate, a size control system which depends on the lateral dimensions of the cloud of debris and adjusts the size of the light beam accordingly, and a position control system which moves the beam to the correct position on the plate as determined by the wind structure and the time of fall of the particles.

The machine described in this report is intended as a fallout forecasting instrument for use in test operations. The development of the basic design equations and design criteria follows from the intended application. The treatment is, however, general enough perhaps to be of use to persons interested in variants of the Analogue designed for other applications.

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Chapter 1

INTRODUCTION

For several years now various groups have attempted to solve the fallout problem using computing machines. The trend towards machine computing came with the realization that simple formulations of the fallout problem would never provide answers in the detail necessary for analysis of fallout patterns or for satisfactory forecasting during full-scale test operations. For a time simple hand calculations were all that our knowledge warranted. More complicated hand calculation schemes were developed as the quality of the input data improved. Certain of these schemes are still in use and are powerful but time-consuming tools. In certain situations these methods are still as good as machine calculations or better because of their versatility.

The machines which have been available for fallout work are, of course, general purpose digital computers. Very great advances have been made through the use of these machines. There are, however, certain limitations in these machines which make them cumbersome when applied to the fallout problem. The chief limitation is insufficient memory.

It appears now that a special-purpose analogue computer would be better suited to the fallout problem. The Optical Analogue described in the following chapters is such a computer.

The particular forms which the Optical Analogue may take are to some extent determined by the uses to which the machine will be put. Our ideas necessarily reflect such considerations. We believe, however, that the same general principles may be built into machines intended for quite different applications and that in those cases the engineering details would differ appreciably from the details incorporated in our version.

There is no attempt in this report to consider the many possible variants of the Optical Analogue, and to a large extent we have refrained from presenting the engineering features of our own. For one thing, the machine is not yet built and may differ a good deal from our present conception of it. For this reason this report does not contain the actual design calculations for our machine. It does, however, contain a fairly complete treatment of the general considerations. After the machine is built, we shall prepare a technical operating manual containing design calculations, engineering drawings, and operating and maintenance instructions.

Let us proceed now to the formulation of the fallout problem.

Chapter 2

FORMULATION OF THE FALLOUT PROBLEM

From the fallout forecasting point of view the problem begins with a stabilized cloud and a set of wind conditions. Both must be predicted from whatever information is available in order to establish the initial conditions of the problem. The most important single feature of the weather is almost certainly the wind structure, the velocity as a function of altitude. Other features take on more or less importance depending on the problem. Of these, the lapse rate has often a significant effect. In particular the presence or absence of a well-defined tropopause, and its height, may be very important indeed.

Unfortunately, the most important weather feature is frequently the most difficult to predict in the detail necessary for acceptable precision in the fallout forecast. It is primarily for this reason that fairly simple models with many uncertainties and a fair lack of reality may be used with as much success in many circumstances as the more complex models. It is also the reason why, in forecasting, the details of formation of the cloud and the effects of wind structure during this time are usually neglected.

Various cloud models are used by different people to calculate fall-out patterns. Before going into the details of the model in current use at the Los Alamos Scientific Laboratory (LASL), it will perhaps be useful to mention the features which all of the leading models have in common. These models invariably assume radiological dimensions, a particle size distribution, a vertical distribution of activity, and a radial distribution of activity. The implications are that one may assign a fraction of the total activity in the cloud to a specific particle size group which at stabilization occupies a specific space within the cloud and that one can perform the multiple integration to obtain the total activity. Further assumptions are that in the appropriate units (kilotons, megacuries at some reference time, etc.) this total activity is conserved and that the integration can therefore be performed at some later time over surface variables and with surface density of activity as the integrand. For forecasting purposes it is not necessary to perform the integration but only to assume that the integral exists. The thing of interest is the surface density of activity as a function of the surface coordinates. The chief observational data are of this type, and these, coupled with rather meager data on particle sizes, time of arrival, etc., have been used in an attempt to improve the basic features of the models -- dimensions, initial particle size distribution, and the activity functions. So far, it has proved impossible to designate any existing model as invariably superior to all of the others. Furthermore, it has proved impossible to establish separately any one of the variations on the principal features.

So far as dimensions are concerned, all models presume that the dimensions generally increase with energy release. In particular there is a strong tendency to presume that the maximum radiological height must correspond rather closely to the observed physical height. Most models assume activity at the top of the physical cloud, but there is wide disagreement on the fraction to be assigned to this level. There is perhaps wider disagreement on the radiological radius as a function of height. Some models claim success with radii corresponding closely to observed physical radii while others, compensating in other ways, seem to succeed best with appreciably smaller lateral dimensions. The very simple models frequently do well with no lateral extent whatsoever when used to forecast distant fallout.

One would guess that choices of particle size distribution might show fair agreement in the various models, but the complex nature of the formation and descent of active particles has left this feature about as uncertain as the others. Some models employ a distribution which is constructed entirely from observational data. Others attempt to modify the distribution by theoretical deductions on the mechanism of formation -- a very important point when the burst takes place over surfaces of different chemical and physical composition and when the burst height is changed. Still other distributions are selected chiefly for their analytic form in order to speed up or simplify calculations. Whatever their form and however they are selected, all particle size distributions are tested for their ability, when combined with the other model features and the observed weather, to reproduce observed patterns.

The activity functions are in the same uncertain state. Most models tend to agree, however, on the most expedient form of the radial distribution. To our knowledge they all assume circular symmetry and, with one exception, constant volume density of activity throughout the layer. An attempt was made at one time to place the activity in a fuzzily defined torus but so far as we are aware this model did not effect any noticeable improvement. The model currently in use at LASL employs a Gaussian radial distribution of the volume density which is analytically convenient and is perhaps a little more realistic than the widely-used step function, but does not appear to give particularly better patterns.

The disagreement on height functions is narrow only in the sense that there are two main schools of thought rather than one for each model. Even here, there are many cases in which one school will grant the other's point, but by and large there are those who believe that the observations are best explained by putting most of the activity in the puff of the cloud and those who place it in the stem. Unfortunately, the problem cannot be isolated from the problem of formation of active particles and their distribution in the stabilized cloud. It is certainly true that the real problem is to determine how to associate activity with particles and how to distribute these particles in the cloud. The activity functions become meaningful only in association with the particles. In most models, nevertheless, they are associated with the particles largely by implication, and one speaks of them generally as independent functions. The chief reason for this apparent disassociation lies in the general failure to estimate the actual numbers of active particles of various sizes in different parts of the cloud. Though estimates have been made of the total mass of particulate matter which is swept up into the cloud, the fallout models invariably deal with relative numbers of particles.

Let us turn now to the mathematical formulation of the fallout problem. To begin with, we shall consider a quantity Y, called the effective radiological yield. Since this may be an unfamiliar quantity it will be useful to define it more fully. It is best described by stating a few of the things it is not. It is not in general the total energy release. It is not the fission yield. It is not the sum of fission yield and activation yield. It is, instead, that amount of activity which in one way or another is precipitated from the cloud in a short enough time and with sufficient surface density to have immediate physiological importance. In most practical cases to date, the most important component of the effective radiological yield has been the group of γ-emitting fission products with half-lives in the range from a few hours to a few years. In consequence the most hazardous fallout has resulted from devices with large fission yield fired at or near the surface of the ground. All high air bursts have a small effective radiological yield, and pure fusion yields will have at most only a neutron activation component.

Like the total energy release W, the effective radiological yield Y must be predicted. Y depends upon the fission yield, neutron activation, character of the surface, burst height, and the materials near the device (concrete shields, tower or barge materials, etc.). In general the quantity Y is very difficult to predict within a factor of two or three but in the cases of most importance, large fission yields fired on the ground, it is usually taken as some large fraction of the fission yield.

According to the assumptions of the fallout models one may write the equation

$$d^4Y = C(h,r,\theta,s)\ dh\ r\ dr\ d\theta\ ds \tag{2.1}$$

where $C(h,r,\theta,s)$ is a distribution function which describes the concentration of effective radiological yield per unit volume and per unit of particle size range throughout the stabilized cloud. If Y is in kilotons, then the function C gives the number of kilotons in a differential volume element at h, r, and θ which will be deposited at some later time and at some point on the earth's surface by particles in the size range between s and $s + ds$.

So far it is not important to know what physical characteristics the variable s represents. For the present it will be convenient to describe the various particles by their rates of fall f, rather than by their dimensions or masses. In general the fall rate of a given particle will vary with altitude because of differences in air density, viscosity, and vertical components of the wind. The time at which such a particle arrives at the surface is obtained from the equation

$$t_s(h) = \int_h^0 \frac{dh}{f_s(h)} \tag{2.2}$$

where the signs are chosen to make the time positive. The surface pattern of the particles may be calculated from the wind structure from the equation

$$\underset{\sim}{R}_s(h) = \underset{\sim}{r} + \underset{\sim}{e}_\rho \int_h^0 V_\rho\,(h,\rho,\phi,t)\,\frac{dh}{f_s(h)} + \underset{\sim}{e}_\phi \int_h^0 V_\phi(h,\rho,\phi,t)\,\frac{dh}{f_s(h)} \tag{2.3}$$

where $\underset{\sim}{R}$ is the final position of the particle, $\underset{\sim}{r}$ is the initial radial displacement of the particle at height h, and V_ρ and V_ϕ are the point components of the horizontal wind at each position of the trajectory. The vertical component has been absorbed in the fall rate $f_s(h)$, and $\underset{\sim}{e}_\rho$ and $\underset{\sim}{e}_\phi$ are unit vectors.

Equation 2.1 can be integrated since it is merely a definition of the distribution function C. Equations 2.2 and 2.3 can be integrated in principle, but in practice the integrands are known only approximately. Before discussing the precision with which Equations 2.2 and 2.3 may be solved under various conditions, it will be worthwhile to jump forward for a moment to a discussion of the precision that is needed.

Equation 2.2 gives the time at which a certain number of kilotons equivalent of radioactive materials will be deposited on the earth's surface. At this time these materials will have an activity measurable in some suitable radiation units such as megacuries, and they will be spread over some differential area. By the appropriate conversion factor, one may assign to this area a dose rate in roentgens per hour. If one knows the decay rate of the active materials, one may calculate also the total dose by integrating the dose rate from the time of arrival of the materials to infinity. In particular, if fission products predominate, the decay is approximately proportional to $t^{-1.2}$ for most of the time of interest (a few hours or days, anyway) and one must make an error of a factor 32 in the time of arrival in order to make an error of a factor 2 in the integrated dose. The total dose is quite insensitive to the time of arrival because of the fifth root dependence. On the other hand, if the main component of the fallout consists of neutron-activated materials, two statements may be made. If the time of arrival is short compared with the average half-life, the error in integrated dose resulting from an error in time will be small. If the time of arrival is long compared with the half-life, the error in integrated dose will be large but the dose itself will generally be small. In sum, Equation 2.2 may be solved to a factor 2 without in the important cases having a significant effect on the total dose.

The situation with Equation 2.3 is somewhat different. Equation 2.3 gives the location at which fallout will occur and must in some circumstances be treated with considerable care. Unfortunately, in the case of position the quantity of real importance is not the per cent error but the absolute error. So long as the fallout occurs a few miles away from inhabited areas no harm is done, whether that area be five miles from the detonation point or several hundred. It is primarily on account of Equation 2.3 that for hazardous shots very stable wind structures are required. There must be very little uncertainty in the time and space dependence of the winds and the integration must be carried out with precision. The fact that the wind structure is very frequently imprecisely known is the chief reason for "weather delays" on test operations.

Equation 2.3 is a vector equation which gives the range and bearing to the point at which a particle of size s originating at h and $\underset{\sim}{r}$ in the stabilized cloud will arrive after falling through the wind structure. It is very difficult indeed to solve Equation 2.3 for small particles descending from great heights, since the wind vectors at successively lower altitudes will change with both space and time. On the other hand the fall rate of small particles in still air is fairly constant.

If one is primarily concerned with close-in fallout, the deposition will necessarily result from the settling of large particles whose fall rates in still air are large compared with the vertical component of the winds. In addition one may expect only small changes in the wind structure from those present at the space-time origin. To offset these advantages, there is the fact that the fall rates of the larger particles change appreciably as the particles descend. Nevertheless, the restriction to close-in regions permits a fast and adequately precise solution to Equation 2.3 under any wind conditions. In this case the approximate expression is

$$\underset{\sim}{R}_s(h_1) = \underset{\sim}{r} + \underset{\sim}{\bar{V}}_s(h_1)\ t_s(h_1) \tag{2.3'}$$

where $\underset{\sim}{\bar{V}}_s(h_1)$ is the resultant horizontal wind velocity for particles of size s falling to the ground from height h_1. The time is given by Equation 2.2 integrated between h_1 and 0 and with $f_s(h)$ equal to the fall rate in still air. In general the velocity $\underset{\sim}{\bar{V}}_s(h_1)$ varies with particle size. It will be constant for that range of particles whose relative fall rates with respect to the fall rate of a particle of size s are independent of altitude. In cases of considerable directional shear the velocity $\underset{\sim}{\bar{V}}_s(h_1)$ may differ appreciably from the mean wind of the standard (constant fall rate) hodograph normally furnished by the weather people.

The need to obtain good precision in solving Equation 2.3 or Equation 2.3' places a more stringent condition on the solution to Equation 2.2 than does the determination of integrated dose. In consequence, the better models now use aerodynamic fall in place of the simpler Stokes Law in computing particle trajectories. Unfortunately, the law of aerodynamic fall is not analytic but contains an empirically determined drag coefficient. On the other hand, this disadvantage is very slight since the wind structure is also not analytic.

It is necessary now to specify the distribution function of Equation 2.1 in order to be able to follow the descent of the activity and to determine the surface densities. As mentioned before, the quantity of practical importance in fallout forecasting is the surface density of radioactivity as a function of the surface coordinates. Equations 2.2 and 2.3 permit us to determine the time and position at which a particle size group starting someplace in the cloud will arrive at the surface. Specification of the fraction of the activity carried by that group will permit us to determine the contribution by that group to the surface density at the point of arrival. The sum of all such contributions will be the total surface density in that particular region. If all the times of arrival and intensities are known, one may obtain the dose rate at any time and the integrated dose for the region.

We shall assume that the distribution function may be separated into two factors, one depending on the space coordinates of the stabilized cloud and the other on a suitable variable describing the particle size group. In the LASL model this latter variable is a linear dimension μ. Equation 2.1 may then be written

$$d^4Y = C'(h,r,\theta)\ dh\ r\ dr\ d\theta\ F_{h,r,\theta}(\mu)\ d\mu \qquad (2.1')$$

where the function $F_{h,r,\theta}(\mu)$ is the particle size distribution in a differential volume at h,r,θ in the stabilized cloud. Since the model assumes cylindrical symmetry, we may immediately write

$$d^3Y = 2\pi C'(h,r)\ dh\ r\ dr\ F_{h,r}(\mu)\ d\mu \qquad (2.1'')$$

In the LASL model the function $F(\mu)$ is assumed to be independent of radial position though it does depend parametrically on the height. It is also normalized such that, at any height, its integral over the full range of particle sizes is unity. The function $C'(h,r)$ is split once more into a factor depending on the height alone and a second factor depending on the radius and parametrically on the height. Explicitly, the function $C'(h,r)$ is given by

$$C'(h,r) = C_0(h)\ e^{-r^2/a(h)^2} \qquad (2.4)$$

where $C_0(h)$ is the central concentration $(r = 0)$ and $a(h)$ is a spread parameter almost, but not quite, equal to the standard deviation. Integration over μ and r gives

$$dY = \pi\ a^2(h)\ C_0(h)\ dh \qquad (2.1''')$$

This equation gives that part of the effective radiological yield in the layer of the cloud between h and $h + dh$. One of the leading practical problems confronting the LASL model as well as many of the others is the independent determination of $C_0(h)$ and $a(h)$, or the quantities analogous to them, such that the integral over h gives Y.

If the concentration is uniform in a layer, as other models assume, the form of Equation 2.1''' is exactly the same. One then takes $C_0(h)$ to be the uniform concentration and $a(h)$ to be the radiological radius.

Early work at LASL assumed an increase of the spread parameter $a(h)$ as the layer descended, but at present no evidence can be found for this increase, and in its current form the model assumes no change. It seems fairly clear that the dominant factor in the broadening of a fallout pattern is the wind structure. The "natural" broadening is small in comparison, and in cases of convergent wind flow could be negative.

For analytic and empirical reasons the LASL model assumes that the activity is distributed amongst the particles normally in the logarithm of the diameters. That is, the explicit form of the fall rate function is given by

$$F_h(\mu)\, d\mu = \frac{1}{\sigma(h)\sqrt{2\pi}} \exp - \frac{1}{2}\left[\frac{\ln \mu - \ln \mu_m(h)}{\sigma(h)}\right]^2 d(\ln \mu) \qquad (2.5)$$

where $\sigma(h)$ is the standard deviation of the logarithm about the logarithm of the mean diameter $\mu_m(h)$. Both $\mu_m(h)$ and $\sigma(h)$ are empirically determined and, like $a(h)$, depend upon the height. The variant of Equation 2.1 which embodies the features of the LASL model is therefore

$$d^3Y = C_0(h)\, dh\, 2\pi r\, e^{-r^2/a(h)^2}\, dr$$

$$\times \frac{1}{\sigma(h)\sqrt{2\pi}} \exp - \frac{1}{2}\left[\frac{\ln \mu - \ln \mu_m(h)}{\sigma(h)}\right]^2 d(\ln \mu) \qquad (\mathrm{I})$$

For convenience this equation will be referred to as Equation I. The equation specifies the amount of activity to be associated with particles in the range of diameters between μ and $\mu + d\mu$, in the ring between r and r + dr, and in the altitude interval between h and h + dh. This same amount of activity will settle on the surface into a ring centered at a position given by Equation 2.3, in which $\underset{\sim}{r} = 0$, and at a time given by Equation 2.2. Because of the finite intervals $d\mu$ and dh the ring will be slightly distorted in shape and the settling will occur in an interval dt about t. Any one point on the surface may receive contributions from many parts of the cloud. The fallout calculation is complete when all contributions at each point have been summed.

Chapter 3

FALLOUT EQUATIONS FOR THE ANALOGUE

The LASL model has been used as the basis for computation of fall-out patterns by the IBM 701. The machine computes radiation doses at suitable points on a polar grid. From these points one may plot iso-intensity or isodose lines. A few approximations have been made in preparing the code for the machine, but by and large the code adheres fairly closely to Equations I, 2.2 and 2.3. The machine has been used extensively to obtain best values for the parameters $\mu_m(h)$, $\sigma(h)$, a(h), and $C_0(h)$, essentialy by trial and error fitting to cases where the results were moderately well known.

There are no severe technical disadvantages to using the IBM 701 but there are several practical ones. One is the shortage of machine time on the LASL machines. Their main job is to compute problems of greater subtlety and distinctly more importance to the Laboratory. Secondly, dependence on the LASL 701's implies excellent communications between the Laboratory and the test sites. At present, communications to the Pacific Proving Grounds are not adequate for operational application of the 701 nor is it practical to acquire a 701 for on-site use. For these and other reasons it seems desirable to design a special-purpose computer for field use in solving operational forecasting problems. The machine must be small, rugged, easy to operate and maintain in the field, and simple enough to produce an adequate forecast in about one hour. Several such machines are presently under consideration. The Optical Analogue is one such machine.

The purpose of the Optical Analogue is to produce a photographic plate on which the density of the developed image at various positions on the surface of the plate is directly related to the density of radioactivity at corresponding positions on the earth's surface. The machine is the first unit of a computer of which the second is a suitable microdensitometer. The plate may be projected with an overlay of the area in question for direct visual evaluation of the qualitative features of the pattern. With a little practice it should be possible for a skilled forecaster to make an adequate quantitative interpretation of the plate without the use of the densitometer.

So far as possible, we shall attempt to preserve in the Optical Analogue the main features of the LASL model, but some of these will have to be altered for the sake of simplicity in the machine. Also, the machine will be designed primarily for close-in forecasting. In the next test operation fallout in this region must be forecast in greater detail than long-range fallout. Within the general limitations which apply to all long-range forecasting, the long-range features may be handled adequately by the machine and also by single-point hand calculation.

Let us begin with a consideration of Equation I, which assigns activity to the various particle size groups at all parts of the stabilized cloud.

Since the LASL model assumes no change in the spread parameter $a(h)$ and since in our case there are no time and space changes in the wind structure, a particle originating at a position $\underset{\sim}{r}$ with respect to the central particles of the same size will land at the position $\underset{\sim}{R} + \underset{\sim}{r}$. That is, the trajectories of the offset particles are parallel to the trajectories of central particles of the same size. If all the particles in a thin layer at a given altitude were of the same size, the activity contribution from the layer would form a circular pattern around the central position and the surface density, according to the LASL model, would decrease as $e^{-r^2/a(h)^2}$ from the central position. Since the layer contains many particle sizes, the pattern formed by the layer consists of many circles with their centers extending out along the line of deposition. These circles overlap. The transverse distribution of activity from the layer is still proportional to $e^{-r^2/a(h)^2}$ but the distribution along the line is appreciably different. It is approximately proportional to $\frac{1}{\lambda} \exp - \frac{1}{2}\left[\frac{\ln \lambda - \ln \lambda_m}{\sigma(h)}\right]^2$ where λ is a coordinate along the line of deposition. In the presence of adjacent altitude layers whose patterns stretch out along different lines and whose parameters, including the mean speed, are all in general different, the superposed pattern retains very few of the characteristics of the individual component patterns. It is for this reason that the step-function activity distribution is successful in the other models. Since the step-function distribution will be satisfactory and since its use will vastly simplify construction and operation of the Analogue, we shall change the basic form of the LASL model accordingly. The explicit form of Equation I to be used in the Analogue becomes

$$d^2Y = C(h)dh\ \pi\ r^2(h)\ \frac{1}{\sigma(h)\sqrt{2\pi}}\ \exp - \frac{1}{2}\left[\frac{\ln\mu - \ln\mu_m(h)}{\sigma(h)}\right]^2 d(\ln\mu) \qquad \text{(II)}$$

in which $C(h)$ is now the uniform concentration in the layer at h and $r(h)$ is the radiological radius at h. Now $r(h)$ is no longer an integration variable but is the radius beyond which the activity is rigidly zero.

If one takes a beam of light of constant intensity and radius p and sweeps it at constant speed across a photographic plate, then the amount of light which falls at a point on the plate is proportional to the exposure time. If the point is a distance $x \leq p$ transverse to the direction of sweep, the exposure will be proportional to $\sqrt{1 - (x/p)^2}$. That is, the density of the image will not be uniform in the transverse direction. It will resemble the image produced by a Gaussian in the central third but not, of course, at the wings.

Equation II gives the fraction of the effective radiological yield to be associated with a particular particle size group originating in the interval dh at height h. These particles will settle out in time and will eventually cover a circle of radius $r(h)$ centered about a central position given by Equation 2.3. At this time, i.e., the fall time of the group from height h, the contribution from the layer may be measured in megacuries per square mile or in roentgens per hour. Practically, we are interested in cases where the radioactivity comes from fission fragments and we may therefore assume a decay rate proportional to $t^{-1.2}$. The contribution from the layer to the integrated dose may then be written

$$d^2I = Kt_\mu(h)^{-0.2}\, C(h)\, dh$$

$$\times \frac{1}{\sigma(h)\sqrt{2\pi}} \exp - \frac{1}{2}\left[\frac{\ln \mu - \ln \mu_m(h)}{\sigma(h)}\right]^2 d(\ln \mu) \qquad \text{(III)}$$

where $t_\mu(h)$ is the time of fall of particles of diameter μ from height h and K is a constant which gives the integrated dose in roentgens. The time $t_\mu(h)$ is given by Equation 2.2, and the change in exponent results, of course, from the integration of the decay rate.

Equation III is one of the basic equations of the Analogue inasmuch as it describes how the intensity of the light falling on the plate must be varied with time, altitude, and particle size as the beam sweeps out along the deposition line appropriate to the altitude h. The position of the beam at any time and the rate at which it sweeps are controlled by Equation 2.3. The radius of the beam is controlled by the normalization condition in Equation II, namely

$$Y = \int_0^{h_o} C(h)\,\pi\, r^2(h)\, dh \tag{II'}$$

where h_o is the radiological top of the cloud. Notice that the amount of light falling at any particular point on the plate depends not only on the intensity of the beam as determined from Equation III but also on the sweep rate, which is proportional to the mean speed of the wind, and on the radius of the beam.

In the next three chapters we shall consider in turn the methods by which the intensity, size, and position of the beam of light may be controlled in accordance with Equations III, II', and 2.3.

Chapter 4

CONTROL OF THE BEAM INTENSITY

The control of the beam intensity may be accomplished by means of a transmission filter whose density depends in a very complicated way on the altitude from which the particles fall, their sizes, and the time of fall. Only two of the three variables are independent. Specification of any two of the three will fix the third and will permit evaluation of Equation III. For any specification of the variables and the parameters the corresponding filter density is

$$D_{total} = - \log(d^2 I) + K_{total} \tag{4.1}$$

where K is a constant such that the density is always positive for all values of $(d^2 I)$ up to the highest value of interest and the logarithm is to the base 10. The total filter density may be made up of several filters, each one corresponding to a factor in Equation III. One of the major problems in the design of the machine is to separate the simple filters. There is, of course, no unique solution, particularly if one permits compromises. The following scheme is one of many possible ones, and the compromises contained in it appear to be acceptable.

The scheme selects the altitude and the time of fall as independent variables. With this selection one may either fix t and dt and scan the altitudes which contribute activity to the positions found from Equation 2.3, or alternatively fix h and dh and scan the time. In the Analogue it seems more convenient to do the latter. It is then obvious that one of the component filters should be a height filter. This filter should have a transmission corresponding to those factors in Equation III which do not depend on the time of fall. For each value of the height (and the layer thickness Δh) we shall then scan linearly in time from zero to some convenient upper limit which will be determined by the length of time we can expect the weather to hold, by the radial distance in which we are interested, by the lowest level of activity of concern, or by some combination of the three. The equation for the density of the height filter is

$$D_h = - \log \left[C(h)\ \Delta h \right] + \log \sigma(h) + K_h \tag{4.2}$$

where K_h is a constant such that the density is always positive in the range of interest. The standard deviation $\sigma(h)$ also appears in the exponential, as does the mean diameter $\mu_m(h)$. In this position they are inextricably bound to the time dependent factors of Equation III and cannot be included in the filter D_h.

The filter D_h may consist of a simple linear wedge. For each value of $C(h)$, Δh, and $\sigma(h)$ the wedge may be placed at a fixed position in the beam. In practice such a filter will consist of two wedges, one having a slope which is the negative of the other. If they are in series in the beam, the density will be constant in the region of overlap and one does not require a narrow beam. One of the wedges may be fixed in the beam and have dimensions no greater than the beam diameter. The density at one end should be zero. The other wedge may then be moved back and forth through the beam. Its length will depend only on the precision with which one wishes to establish the density. In order to keep the maximum transmission as high as possible at the zero setting of the filter, one would like the wedge to be as long as practical. If the wedge runs from $D = 0$ to $D = 3$ linearly in 6 inches and the beam diameter is 1/2 inch, then the minimum density will be $D = 0.25$. This minimum density may be reduced by decreasing the slope of the wedge (and also by increasing its length) or by decreasing the beam size.

Apart from constants the remaining factors of Equation III are $t^{-0.2}$, the exponential, and the increment $d(\ln \mu)$. Since the second independent variable will be the time of fall, we must transform these factors from dependence on μ to dependence on time. In particular we must convert from activity per unit $d(\ln \mu)$ to activity per unit dt. This change is accomplished very simply by the relation

$$d \ln \mu = \frac{1}{\mu} \dot{\mu} \, dt \tag{4.3}$$

where $\dot{\mu}$ is the rate of change of the particle diameter with respect to the time of fall from the specified altitude h. In general the weighting factor $(\frac{1}{\mu} \dot{\mu})$ will depend upon the altitude. In practice, over the region of times and altitudes of greatest interest this factor is rather insensitive to the height. For altitudes between 15,000 feet and 100,000 feet and times out to 10 hours the greatest spread at any time is no more than about 25 per cent or about 0.10 density units. At altitudes below 15,000 feet the departure from the average becomes more noticeable, but even at 5,000 feet it is no more than 0.2 density units or approximately 60 per cent. In addition in the time range of interest the departure from the mean is very nearly constant in logarithmic units and one may obtain satisfactory compensation by an altitude correction for the setting of the height filter D_h. In consequence, it is possible with good accuracy to lump the factor $(\frac{1}{\mu} \dot{\mu})$ together with the factor $t^{-0.2}$ into a single function of the time alone. The equation for the density of this time filter is

$$D_t = - \log(t^{-0.2} \frac{1}{\mu} \dot{\mu}) + K_t \tag{4.4}$$

where K_t is a constant chosen to make the density positive for the earliest time of interest. The reason for leaving the exponential factor out of this filter will appear later. For the moment let us return to the time filter D_t.

The filter D_t must have a density which varies in time according to Equation 4.4. Any real time t will have a corresponding machine time t' given by

$$t' = K_1 t \qquad (4.5)$$

where K_1 is a constant of the machine which converts real time in hours to machine time in seconds or other convenient units. For $t = t_1$ (and hence $t' = t_1'$) the filter density is given by Equation 4.4. This density will occur at a position x_1 along the filter. For mechanical convenience it appears desirable to establish a constant ratio between x and t' such that

$$x = V_t t' \qquad (4.6)$$

The constant V_t is then the speed with which the filter passes through the beam. Actually one needs merely a one-to-one correspondence between x and t' and it is conceivable that better results would be obtained by establishing a non-linear relation between x and t'. A high traverse speed where the density changes rapidly with time and a lower speed where the change is slow would help the resolution. One could arrange the speed such that a linear wedge could be used. At present this type of speed control appears to be more complicated than the construction of a special filter designed for constant speed.

Since the slope of this filter is not constant but changes with position, one cannot create a region of constant density by overlapping a filter with reverse slope and traversing the second filter in the opposite direction with the same speed. However, if the rate of change of the slope is not too great throughout the time range of interest, one can obtain a fairly constant density over the diameter of the beam by this means without appreciable increase in the mechanical difficulties. The slope of the filter is such that the density will increase slightly in the center of the beam, a feature which may be undesirable in practice. On the other hand, this variation may be less than the variation between the two sides of the beam unless the beam can be made very small or the filter very long. Since the flux of light which can be passed through a filter is limited by the ability of the emulsion to stand heat and since the final spot will be formed with a parallel beam up to 3 inches in diameter, it seems desirable in this stage of the machine to use as large a beam as practical and therefore to use a pair of filters. The density of each should be one-half the value given by Equation 4.4 for each value of the time. They should each have the same traverse speed V_t but should move in opposite directions through the beam. For simplicity we shall continue to refer to this pair of filters as a single filter whose equation is Equation 4.4.

Let us consider the argument of the logarithm. The factor $t^{-0.2}$

represents the decay, but compared to the factor $(\frac{1}{\mu}\dot{\mu})$ it is insignificant. Between times of 0.1 hours to 10 hours it contributes only 0.4 density units. The other factor is much more powerful and varies approximately 2.5 density units in the same time interval. The exact range depends somewhat on the altitude from which the particles fall.

It is perhaps useful at this point to digress on the subject of aerodynamic fall, which was first applied to the fallout problem by the Rand Corporation.* We may then see why the factor $(\frac{1}{\mu}\dot{\mu})$ shows a weak dependence on height and also understand some of the complications of the exponential factor. The Rand theory introduces an empirically determined drag coefficient into the force equation. Balance of forces results in a terminal velocity for falling particles. The force to be balanced is the gravitational force

$$F_g = \frac{1}{6}\pi\mu^3(\rho_s - \rho_a)\, g \qquad (4.7)$$

where μ is the diameter of the falling spherical particle, ρ_s is the density of the particle, ρ_a is the air density and g is the gravitational acceleration. Since ρ_s is always much greater than ρ_a this equation is approximated by

$$F_g = \frac{1}{6}\pi\mu^3\rho_s g \qquad (4.7')$$

*Rand Corporation Report R-265-AEC (classified)

The balancing force is

$$F_D = \frac{1}{8} \rho_a \pi \mu^2 v^2 C_D \tag{4.8}$$

where v is the fall rate of the particle and C_D is the drag coefficient. The drag coefficient has been determined experimentally for spherical particles and is presented in the Rand study as a function of the Reynolds number

$$R = \frac{\rho_a \mu v}{\eta} \tag{4.9}$$

where η is the viscosity of the air. Since the viscous force (Stokes Law) is given by

$$F_s = 3\pi\eta\mu v \tag{4.10}$$

one may combine Equations 4.8, 4.9, and 4.10 to obtain

$$F_D = F_s \frac{RC_D}{24} \tag{4.11}$$

For very small particles (small values of the Reynolds number) the factor $RC_D/24$ approaches the constant value unity and the balancing force is just the viscous force. These particles fall according to Stokes Law. Large fast-falling particles, on the other hand, are strongly affected by aerodynamic drag and reach terminal speeds considerably less than the Stokes Law speeds. In the region of interest for our problem the useful range of the Reynolds numbers is $0.1 \leqslant R \leqslant 500$ and an analytic fit to the drag curve is

$$\frac{RC_D}{24} \doteq 1 + 0.119\, R^{0.735} \qquad 0.1 \leqslant R \leqslant 500 \tag{4.12}$$

The general equation of balance of forces is simply

$$F_g = F_D \tag{4.13}$$

and therefore in the range of interest

$$\frac{1}{6}\,\pi\mu^3\rho_s g \doteq 3\pi\eta\mu v \left[1 + 0.119\left(\frac{\rho_a \mu v}{\eta}\right)^{0.735}\right] \tag{4.14}$$

The only virtue of this equation is that it shows the approximate way in which the fall rate depends on the diameter, the air and particle densities, and the viscosity. For computing times of fall it is better and just as easy to use the drag curve directly since neither ρ_a nor η is quite analytic. The air density follows the barometric law primarily and is only slightly modified by the air temperature. The viscosity is proportional to the temperature, which in a normal atmosphere falls fairly evenly up to the tropopause and then rises again. The range of viscosities is not very great. In the Marshall Islands average winter atmosphere the viscosity varies about ±20 per cent from the average value between the surface and 100,000 feet.

In the case of small particles where the Reynolds number is very small, the balance equation results in

$$\mu^2 \doteq \frac{18\,\eta}{\rho_s\, g}\, v \qquad \mu \text{ and } v \text{ small} \qquad (4.14')$$

For any given small particle the fall rate is then inversely proportional to the viscosity and reflects the lapse rate. Such particles speed up in passing through the cold tropopause layer and slow down as they near the surface.

Large fast-falling particles follow the approximate equation

$$\mu^{0.73} \doteq \frac{1.493}{(\rho_s g)^{0.526}}\, \rho_a^{\;0.424}\, \eta^{0.153}\, v \qquad \mu \text{ and } v \text{ large} \qquad (4.14'')$$

For any given large particle the fall rate is very insensitive to the viscosity and the lapse rate and varies mainly with the air density. The speed decreases approximately exponentially as the particles descend.

If one takes a suitable average value for the viscosity and approximates the air density by an exponential variation with altitude, the approximate Equations 4.14' and 4.14" may be integrated to give an approximate relation between particle diameter, altitude and time of fall for the two cases of very small and very large particles. Let us assume that in general one may fit the data with an equation of the form

$$\mu \doteq \frac{a}{t^n} + \frac{b}{t^m} \qquad (4.15)$$

where both a and b depend on h. From integrals of the approximate Equations 4.14' and 4.14", one will obtain

$$\mu \doteq A\left(\frac{h}{t}\right)^{\frac{1}{2}} \qquad \mu \text{ small} \qquad (4.15')$$

and

$$\mu = B\left(\frac{1 - e^{-ch}}{t}\right)^{1.37} \qquad \mu \text{ large} \qquad (4.15")$$

where A and B are constants. In hand fitting to the form 4.15 at fixed altitudes it turns out that the power n is indeed +0.5 but that the power m changes slightly with altitude and has a central value near +1.5 for particles of density $\rho_s = 2.5$ gm/cm^3 and diameters up to 2,000 microns falling from altitudes between 5,000 and 100,000 feet. The coefficient a varies almost exactly with the square root of the altitude while the coefficient b varies as $(1 - e^{-0.0172h})^{1.5}$ if h is in thousands of feet. Again the power is $m = 1.5$. The constant $c = 0.0172$ is determined from an approximate barometric constant for the Marshall Islands average winter atmosphere multiplied by a factor 0.424, which appears as the power of the air density in Equation 4.14".

With these constants the form 4.15 becomes

$$\mu = \frac{Ah^{1/2}}{t^{1/2}} + \frac{B(1 - e^{-ch})^{3/2}}{t^{3/2}} \tag{4.16}$$

The factor $(\frac{1}{\mu}\,\dot{\mu})$ is therefore given by

$$\frac{1}{\mu}\,\dot{\mu} = -\frac{1}{2t}\left[\frac{1 + 3r/t}{1 + r/t}\right] \tag{4.17}$$

where r is the ratio of coefficients and is given by

$$r = \frac{B}{A}\,\frac{(1 - e^{-ch})^{3/2}}{h^{1/2}} \tag{4.18}$$

From these relations it is easy to see why the factor $(\frac{1}{\mu}\,\dot{\mu})$ is insensitive to the altitude h. The numerical value of r is approximately unity at $h = 100{,}000$ feet. Equation 4.18 shows that r does not drop to the value 0.5 until one reaches approximately 25,000 feet. In addition one may see in Equation 4.17 that for $t \approx 3r$ or greater the factor $(\frac{1}{\mu}\,\dot{\mu})$ becomes increasingly insensitive to any value of r. On the other hand one may see also that $(\frac{1}{\mu}\,\dot{\mu})$ becomes increasingly insensitive to r when $t \approx r$ or less.

The range of times for which the Analogue is designed is $0.1 \leqslant t \leqslant 10$ hours. In this range if one takes the average value of $-\log\,(\frac{1}{\mu}\,\dot{\mu})$ for $0.3 \leqslant r \leqslant 1$ there will be no error in transmission any larger than 25 per cent. For smaller values of r (lower altitudes) the departure from this average curve may be greater than 25 per cent. On the other hand, the correct transmission is lower over the full range in time and one may therefore compensate by a correction in the height filter D_h. If more precision is desired, one may of course adopt the correct curve for 100,000 feet and apply small compensations in D_h for all lower altitudes. Down to 50,000 feet or lower this compensation is likely to be less than the precision with which the filter D_h can be set.

Let us return now to the exponential factor. We must have a filter whose density is given by

$$D_\mu = + \frac{1.1515}{\sigma^2} (\log \mu - \log \mu_m)^2 \tag{4.19}$$

where the logarithms are to the base 10. The term μ_m is a mean particle diameter and is a function of the height, and σ measures the spread in the log normal distribution about the natural logarithm of μ_m and may also be a function of the height. The appearance of σ as a multiplicative factor rather than an additive term presents an unpleasant complication in principle but in practice causes little difficulty. The U.S. Weather Bureau particle size analysis* on which the log normal distribution is based indicates that σ is very nearly constant and has

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*The parameters σ and μ_m for the log normal distribution were determined at LASL by fitting to the data in "A Method of Fallout Prediction," by Nagler, Machta, and Pooler, U.S. Weather Bureau, June 1955 (classified).

the numerical value $\sigma = 1$. We shall consider later what methods may be used to obtain approximately correct filtering for $\sigma \neq 1$.

Equation 4.19 says that for fixed values of σ the density D_μ is constant for fixed ratios of μ/μ_m. Unfortunately the density D_μ must have certain values at certain times, and because of the complicated relation between particle diameter and time of fall the transmission of the filter is not normal in the logarithm of the time of fall. Because of this fact the transmission curves, on a time base, will have different shapes depending on μ_m and also on the height. Nevertheless, it is possible to construct for the Analogue a single filter which will adequately approximate the correct transmission as a function of time for all combinations of μ_m and h which will be of interest. The main reason for the success of the single filter is the fact that the Analogue must handle only the limited time range $0.1 \leq t \leq 10$ hours. This range may be further restricted by the fact that for very large yields the useful lower limit of time is more nearly 0.5 hours while for very small yields the upper limit may be reduced to 2 or 3 hours. Since the large yields are clearly the more important practical cases, the Analogue will be designed to give more precision at this end of the yield range. It should still give results which are good to a factor 2 or better for clouds which reach a peak height of only 20,000 feet.

The U.S. Weather Bureau analysis indicates that the mean particle diameter may be represented by an equation of the form

$$\alpha = A - B \log \mu_m \tag{4.20}$$

where α is the relative height of the layer of interest, that is

$$\alpha = \frac{h}{h_o} \tag{4.21}$$

where h_o is the height of the top of the cloud. The cloud may be divided into any number of layers, each layer centered about a relative height given by Equation 4.21. According to the form 4.20, the low layers have large values of μ_m while the high layers have small values of μ_m. In the Weather Bureau model at $\alpha = 0.094$ the diameter of the mean particle is 1024 microns while at $\alpha = 0.906$ the diameter is only 66.7 microns. The relation between α and μ_m need not be specified in order to design the Analogue. Any value of μ_m may be assigned to any altitude. The Weather Bureau model merely illustrates the general order of magnitude and the trend of μ_m with altitude.

If one believes this trend, then for high layers we shall, because of the time limits, be interested only in particles larger than the mean particle. A 67-micron particle will take more than 20 hours to fall from 100,000 feet, for example, and will take approximately 5 hours from 18,000 feet. On the other hand, a 1,000-micron particle takes only a little over 0.1 hours to fall from 10,000 feet, which is about as high as a layer corresponding to $\alpha = 0.09$ will ever be. Consequently for low layers we shall be principally interested in particles smaller than the mean size. This means, therefore, that in preparing a composite filter to represent Equation 4.19, the first part, where the density is decreasing with decreasing particle diameter, should have a shape determined primarily by the high layers. The shape of the saddle will be determined by intermediate layers, the shape of the tail by low layers.

In order to see how to design the filter, let us consider that it is already built and that we have a certain value of the density at each position along the length of the filter. There will be one position along the length at which the density is zero or at least is minimum. This position of the filter is supposed to be in the beam at a time corresponding to the time of fall of a particle of diameter μ_m. In hours, this time may vary widely from something less than 0.1 hours to something considerably greater than 10 hours, depending on the value of μ_m and on the height of the layer. The same general statements are true for values of the density other than the minimum. In such cases we are interested not in the particle of diameter μ_m but in a particle whose diameter is in a fixed ratio to μ_m.

Let us suppose that the position of minimum density is a distance x_{min} from the start of the filter. This point will be in the beam at a certain machine time $t'_{min}(\mu_m, h)$. If the filter passes through the beam with constant speed, then this constant speed must be

$$V_{min}(\mu_m, h) = \frac{x_{min}}{t'_{min}(\mu_m, h)} \tag{4.22}$$

From Equation 4.22 one may therefore compute a set of traverse speeds for any values of μ_m and h such that one obtains the minimum density at the correct machine time t' corresponding to the real time t.

In exactly the same way one may determine a set of traverse speeds such that the filter will be at the position x_1, corresponding to density D_1 at the correct time t_1' from the equation

$$V_1\left[(\frac{\mu}{\mu_m})_1,h\right] = \frac{x_1}{t_1'\left[(\frac{\mu}{\mu_m})_1,h\right]} \tag{4.23}$$

Unfortunately, because of the intricate relation between particle size and time of fall, the speeds for fixed μ_m and h as determined from Equation 4.23 will not in general be the same as those determined from Equation 4.22. Since either t'_{min} or t_1', or both, may lie outside the range of interest, it is perfectly possible that either or both of the speeds V_{min} and V_1 will not be relevant.

For any fixed values of μ_m, h, and σ one may compute the curve of density versus time of fall from aerodynamic fall theory and Equation 4.19 for the range of time of interest. Suppose now that one computes a second curve for values of μ_m, h, and σ not too different from the first set. On a semi-log plot the two curves will look very much alike except that the second curve will be shifted in time with respect to the first. Because of the slight difference in shape the time shift will vary with density. There will, however, be some value of the shift for which the area between the two curves, over the region of overlap, will be a minimum. This shift gives the best value of the ratio of traverse speeds for a single composite filter designed to approximate two exact filters. The composite may, of course, be exact for either case or approximate for both. In the Analogue the composite filter is made up not from two curves but from a large number and is exact for none. The reason for this is that no single curve is pertinent outside of the time range $0.1 \leqslant t \leqslant 10$ hours. Since this time range will correspond to a limited range of x in every case, the density at the ends of the range must be modified to provide a good fit for curves using adjacent parts of the filter.

The method of constructing the best composite filter is, then, to establish speed ratios by least-area fitting as indicated above. By assigning a speed to one curve, one obtains the speeds for all the others. With these speeds and the correct times of fall one calculates for each density a set of values of x. The average of the set is then assigned the density in question. Since only times in the range $0.1 \leqslant t \leqslant 10$ hours are used, the average values of x are determined only by the curves which will use that part of the composite filter. At this point the average values of x are merely relative. The real values are determined by mechanical and optical considerations.

A composite filter formed in this way from the even-numbered layers of two sixteen-layer clouds with values of $h_o = 100,000$ feet and $h_o = 40,000$ feet results in errors in transmission no more than about 15 per cent. Fitting to this composite of curves for a cloud with

h_o = 20,000 feet results in errors in transmission no greater than 25 per cent. In consequence it appears perfectly feasible to use a single composite filter for all cases of interest. The only disadvantage to such a scheme is that it requires a wide range of traverse speeds to accommodate the highest layers of a high cloud and the lowest layers of a low cloud.

Since the shape of the filter curve for constant values of μ_m changes very little with altitude, one may at any time and with good precision alter the mean particle size at any altitude by merely changing the traverse speed. It is not necessary to use the Weather Bureau values of μ_m.

It is also possible to accommodate values of σ other than unity by changing the traverse speed provided σ does not vary too much. A transmission error of about 50 per cent will result when σ is permitted to vary a factor $\sqrt{2}$ in either direction. If experience indicates that better results may be obtained with considerably different values of σ, it will be desirable and fairly easy to construct a new filter D_μ.

As with D_h and D_t it is desirable to replace the single filter D_μ with a pair of filters, one with slope the negative of the other, and to traverse them in opposite directions through the beam. The filters may then be appreciably shorter for the same resolution.

The transmission filter for control of the beam intensity is made up of three filters, D_h, D_t, and D_μ, which are in series in the beam.

The first and simplest is D_h (Equation 4.2), which includes the activity concentration in each altitude layer, the thickness of the layer, the spread parameter $\sigma(h)$ where it appears as a normalization constant, and any corrections which are not strongly time dependent. The second filter D_t (Equation 4.4) is also simple but must move through the beam at constant speed. Since this filter contains time factors alone, it will always move at the same speed regardless of altitude or particle size distribution. The third filter D_μ (Equation 4.19) is complicated in design but fairly simple when built. It requires a wide range of traverse speeds and the speed must be reset for each traverse. The speed is, however, constant for each traverse. This filter reflects the distribution of activity with particle size within the layer at the altitude h.

Chapter 5

CONTROL OF THE BEAM SIZE

The diameter of the circular spot of light at the photographic plate must correspond to the real physical diameter of the layer of thickness Δh at h. The real dimensions are controlled by the normalization condition of Equation II'

$$Y = \int_0^{h_o} C(h)\ \pi\ r^2(h)\ dh \tag{II'}$$

In the incremental form appropriate to continuous scanning in time this equation implies, for the jth layer

$$\Delta Y_j = C_j\ \pi\ r_j^{\ 2}\ \Delta h_j \tag{5.1}$$

where r_j is the radius of the jth layer and is fixed for each discrete altitude h_j. The spot size is therefore constant during each time scan, as were C_j and Δh_j and the other height-dependent quantities mentioned in Chapter 4. The machine radius is of course related to the real radius by

$$r' = K_2 r \tag{5.2}$$

where the constant K_2 is the second machine constant converting real nautical miles into inches at the photographic plate. The first machine constant K_1, mentioned in Chapter 4, converted real time in hours to machine time in seconds.

The two constants K_1 and K_2 are related to each other only by the requirement that the exposure of the photographic plate not suffer from reciprocity failure in the dose range of interest. This is a necessary condition on the constants. So far as possible, while meeting this condition, we wish to make the machine time short so that a forecast can be made quickly and we wish to make the space resolution at least as good as is compatible with the uncertainties in model assumptions.

Since the model assumptions are presently quite poorly established, it turns out that the desired degree of space resolution is determined primarily by atoll geography. Both Eniwetok and Bikini are roughly round and have an average diameter of approximately 20 miles. The fleet, if at sea, is customarily 30 to 35 miles from the shot position. Consequently it appears that a range of about 40 miles around the shot point is as far out as will concern us for close-in fallout.

An average island in either atoll has an average diameter of about 1/4 mile, though the major islands are appreciably bigger and except for Engebi and Namu are long and thin. If the resolution on the photographic plate is 10 lines per millimeter or better, then an island

1/4 mile across will be recognizable by its shape if the image is about 1/2 mm across. Since to cut down machine time we may well use fast, coarse-grained emulsions, we should not plan on higher photographic resolution.

The above considerations lead to the choice of a standard 8 x 10 inch photographic plate with a scale of 1 inch = 10 nautical miles. If the minimum practical layer diameter is 1 mile, then the minimum spot size will be 0.1 inches or 2.54 mm. At present we consider that the maximum effective radiological diameter will be less than 25 miles, though observed late-time dimensions have been considerably greater. The upper limit of 25 miles requires an upper limit of $2\frac{1}{2}$ inches on the spot diameter.

There are two rather general ways to put spots of the proper size on the photographic plate. The first is to arrange an optical system of appropriate aperture, focal length, and magnification which will form an image of an illuminated or luminous disc whose size can be controlled. The second is to arrange an optical system of appropriate aperture and focal length which will produce a beam of parallel rays whose diameter is subsequently controlled by a variable iris. Either system is satisfactory in principle and our choice is largely determined by optical and mechanical considerations elsewhere in the system.

The details of the spot position control will be taken up in the next chapter, but the general method affects our choice of the size control system. The spot will be deflected off a rotating mirror. In order to keep the spot fairly round, the light path from the mirror must be long compared with the size of the plate. Consequently there are some difficulties in arranging an optical system with a magnification of one or less. If the magnification is greater than one, the precision with which the diameter of the object spot must be established increases. Also, with a flat plate the image spot will defocus unless the f/number of the system is very large and the depth of focus long. At the edge of the plate the spot will be wider and less intense than it should be, because of both defocussing and oblique incidence of the rays.

With the parallel beam system the spot will be elliptical at the edge of the plate but the short axis of the ellipse, transverse to the direction of sweep, will be the correct spot diameter. The intensity normal to the beam will be correct and the integrated light at each position on the plate will be correct. We also avoid magnification problems and ease the requirement on the precision with which the diameter of the iris must be set. On the other hand we must have a refracting lens at least $2\frac{1}{2}$ inches in diameter and an iris which will open up to that size. Both are readily available.

The control of the beam size is therefore accomplished by producing a parallel beam at least $2\frac{1}{2}$ inches in diameter and stopping it down with a variable iris. The variation in intensity of the beam is then not affected by the size control system.

Chapter 6

CONTROL OF THE BEAM POSITION

As mentioned in Chapter 5, the general method of controlling the beam position is by deflecting it from a rotating mirror. The necessary relations to be met by the beam position control are contained in Equation 2.3 or, in our case, Equation 2.3'

$$\underset{\sim}{R}(\mu,h_1) = \underset{\sim}{r} + \overline{\underset{\sim}{V}}(\mu,h_1)\ t(\mu,h_1) \qquad (2.3')$$

where $\underset{\sim}{R}(\mu,h_1)$ is the terminal position of the particle of diameter μ falling from height h_1, $\underset{\sim}{r}$ is the initial radial position of the particle at height h_1, $\overline{\underset{\sim}{V}}(\mu,h_1)$ is the resultant wind from the surface to height h_1 properly weighted for a particle of fall rate $f(\mu,h)$, and $t(\mu,h_1)$ is the time of fall of the particle of diameter μ from the height h_1. The terminus of the vector $\underset{\sim}{R}$ traces out the line of deposition of particles which fall from the initial position $\underset{\sim}{r},h_1$. Since our model assumes no lateral spread during the settling of the particles from a layer, we may confine our attention to the line of deposition of the central particles ($\underset{\sim}{r} = 0$). The trajectories of all particles in the layer are parallel to the trajectory of the central particle of the same diameter. The path of deposition may be considered as the superposition of a set of discs of the same radius. The center of each disc is on the line of deposition of the central particles and each disc represents a particle size group. The line of deposition is actually calculated from the equation

$$\underset{\sim}{R}(\mu, h_1) = \sum_{j=1}^{i} \underset{\sim}{\overline{V}}_j \frac{\Delta h_j}{f_j(\mu)} \tag{2.3''}$$

where the vectors $\underset{\sim}{\overline{V}}_j$ are the mean wind velocities in the incremental altitude layers Δh_j, $f_j(\mu)$ is the average fall rate of a particle of diameter μ in the layer centered at h_j, and i is the number of wind layers between the surface and the altitude h_1. The standard hodograph prepared by the weather people is a simplified version of Equation 2.3" in which

$$f_j(\mu) = 5 \text{ kilofeet/hour} \tag{6.1}$$

the values Δh_j are layer thicknesses in thousands of feet, and i designates the highest layer of the observation or forecast. Since, for constant fall rates, the times in each layer depend on the increments Δh alone and do not depend on the altitude, the vector $\underset{\sim}{R}$ will have the same direction for all particles starting in each layer. The line of deposition for the ith layer is a straight radial line obtained by summing Equation 2.3" from 1 through i subject to the condition

$$f_{\mu} = \text{constant} \qquad 1 \leq j \leq i \tag{6.2}$$

For constant fall rates the time of fall of particles from the layer at h_i is simply proportional to the altitude h_i.

In our case the fall rates are not constant because of the variation of viscosity and air density with altitude. Consequently, the time a particle spends in a layer depends not only on the altitude interval but also on the altitude itself. The result is that in general the line of deposition of central particles falling from a layer at a given altitude is not a straight radial line. To the extent that the ratio of the fall rates of the particles is independent of altitude, the line of deposition will be straight. If the range of particle sizes is not too large, the straight line approximation is acceptable. It breaks down rather quickly for particles falling from high altitudes through wind structures with pronounced directional shears. In such cases the bearing to the line of deposition may swing almost 180 degrees as one moves to successive positions along the line. In the Marshall Islands this difficulty, if it is going to occur, will take place for altitudes above the tropopause. We have not found a real case in which the difficulty is serious for lower altitudes.

In principle one may fairly easily construct an auxiliary computer whose output will control the beam position as a function of time of fall according to Equation 2.3", where now we have specified the relation between particle diameter and fall rate in the individual layers. For the present, however, we believe that satisfactory results may be obtained without this additional complication in the machinery. This belief is based upon an examination of the results of the straight line approximation used in a large number of observed Marshall Islands wind structures.

The selection of a suitable mean speed and bearing for the straight line approximation to the line of deposition will be made subjectively upon examination of the wind structure. To assist in this selection, we shall furnish to the weather people a set of weighting factors from which they can prepare perhaps two or three adjusted hodographs in place of the usual constant fall rate hodograph. The weighting factors will, of course, be simply the times which representative particles spend in fixed altitude layers as they descend through the atmosphere. The hodographs so drawn are then the actual deposition points of the central representative particles as a function of the altitude from which they fall. The lines joining points of constant altitude are the lines of deposition from those altitudes.

Each point of the adjusted hodographs may be characterized by an altitude, a time, a distance from the origin, and a bearing from some reference direction. The distance divided by the time is the resultant wind speed for the representative particle falling from the altitude in question. Because of aerodynamic fall both the bearing and the resultant wind speed will be different for particles of different sizes falling from the same altitude. Our problem is to select a single bearing and speed which will best represent the deposition from the layer over the space and time range of interest.

If the resultant wind speeds for the representative particles are low and even the smallest representative particle falls within a 40-mile radius, then the largest representative particle will fall very close in indeed and we shall be little concerned with its true bearing. We shall then favor the mean speed and bearing for the small particle. On the other hand, if the wind structure is such that the resultant wind speeds are large, we shall disregard the small particles and favor the speed and bearing for the large. One could, of course, examine the wind structure, select the single most appropriate representative particle, and compute the speed and bearing for that particle. This seems a cumbersome and unnecessary procedure.

We should point out that the beam intensity control system is designed to produce the correct variation of intensity with time of fall and that equal intervals in time correspond to equal lengths along the true line of deposition. In using the straight line approximation we are assuming that equal increments in radial distance adequately represent equal increments in length along the line of deposition. Very near the origin this is likely to be a poor approximation, but we accept it because it is made in a region of little practical importance. Actually the beam will be at the correct position at $t = 0$ and at one other subsequent time. At all other times it will have the correct intensity (within the approximations of the intensity control system itself) but will be off in distance and bearing to the extent that the true line of deposition differs from the straight line approximation.

In the Analogue we shall deflect a parallel beam of light from the face of a rotating mirror onto a flat photographic plate. There are two practical axes of rotation for the mirror. The first is an axis normal to the central ray of the beam. The other is an axis colinear with the central ray.

Let us consider that at zero time in both cases the mirror is so located in the beam that the central ray is reflected at 90° from the incident beam and that the reflected beam is normally incident on the plate. The point at which the central ray of the beam strikes the plate is the origin of a polar coordinate system R', ϕ on the plate and corresponds to the polar coordinate system R, ϕ on the surface of the earth. The relation between R' and R has been discussed in Chapter 5. Let L be the distance between the point at which the central ray strikes the mirror and the point $R' = 0$ on the plate, and let θ be the angle of rotation of the mirror shaft measured from its initial position.

If the axis of the mirror shaft is normal to the plane of the incident and reflected rays, then the point at which the central ray strikes the plate will be given by

$$R' = L \tan 2\theta \tag{6.3}$$

If the axis of the mirror shaft is colinear with the incident ray, the corresponding position equation is

$$R' = L \tan \theta \tag{6.4}$$

In either case, in order to maintain a small eccentricity in the ellipse at the edge of the plate and to maintain a fairly constant sweep speed, we wish to make L large compared with the plate dimensions.

There is no compelling reason to pick one shaft arrangement in preference to the other. We have selected the second, whose position equation is 6.4, in preference to the first for three main reasons. In the first place, the normal to the mirror is always at 45° to the incident beam and the mirror therefore has a long dimension which is a factor $\sqrt{2}$ times the maximum beam diameter ($2\frac{1}{2}$ inches). In the other case the long dimension would have to be increased to handle the full beam at the incident angle whose tangent is $1 + R'_{max}/2L$. Secondly, Equation 6.3 applies only if the axis of rotation passes through the plane of the mirror, and the mounting of the mirror on the shaft therefore becomes a little less straightforward. Thirdly, our feeling is that since shaft speeds will be fairly low, the lower the speed the more trouble we shall encounter with spurious vibrations. Shaft speeds for the second arrangement will be twice as fast as for the first to obtain the same sweep speed on the plate. Since the beam is supposed to be of constant intensity or at least circularly symmetric it does not concern us that the beam will rotate as it sweeps.

Adopting the colinear shaft axis the sweep speed of the beam will be given by

$$\frac{dR'}{dt'} \doteq L\omega = V' \tag{6.5}$$

where we have included the approximation $L \gg R'_{max}$. Here t' is of course the time in machine units and V' is related to the mean resultant wind speed by

$$V' = \frac{K_2}{K_1} \bar{V} \tag{6.6}$$

The constants K_2 and K_1 are the machine constants referred to in Chapters 4 and 5. The angular speed of the shaft, ω, is simply proportional to $\bar{V}$. Our present estimate is that $\bar{V}$ will lie between 0 and 50 knots, with the upper limit being rather unlikely for the Marshall Islands except in the event of deep easterlies.

Since the bearing angles scale directly, we shall mount the plate holder in a ring and turn the ring to the appropriate angle as read from the hodograph.

For each altitude layer we shall therefore make two settings. The bearing angle will be set by rotating the plate holder to the angle given by the hodograph, and the mirror shaft speed will be set from a curve (or table) of shaft speed versus mean wind speed, where the wind speed too is taken from the hodograph.

There is a wide variety of ways in which the beam position control may be engineered. The method described above appears to be adequate for the intended application. In the light of our present understanding of the fallout phenomena no more elaborate scheme is justified. On the other hand, engineering considerations may lead to an even simpler and equally satisfactory design. It is perhaps worthwhile to point out that the beam position control is in principle completely independent mechanically and optically from other elements of the Analogue and that this element of the machine may be improved at any time without significantly affecting the design of the remainder.

Chapter 7

SOME ENGINEERING ALTERNATIVES

The main engineering problems in the machine as described in the preceding chapters are the speed controls and the timing. There are two filters, the beam position control system, and a mechanical shutter, all of which must be controlled in specified time relations to each other.

By itself the mechanical shutter presents no problem. Such shutters are very fast on the time scale of the Analogue. The other three moving parts are likely to have considerably more inertia and will require varying amounts of time to reach the speeds they should have at zero time. In addition to having the proper speeds they must also, of course, be in the proper positions.

In this respect the filter D_t described in Chapter 4 is probably the most tractable since it always moves through the beam at the same rate. Consequently, it should be relatively simple to adjust the drive of the filter to bring it up to its constant speed in a reproducible length of time. A pin at the zero time position on the carriage could supply the trip for the mechanical shutter.

Both the beam positioning mirror described in Chapter 6 and the filter D_μ described in Chapter 4 may have a variety of speeds at zero time and will therefore in principle require variable lead times. The more troublesome of these two items is the filter D_μ, and it appears desirable to suggest an alternative method for operating this filter.

The scheme proposed in Chapter 4 was that a single composite filter should be made for traverse through the beam at a wide range of constant speeds. The actual range calculated for a 16-layer cloud with the Weather Bureau median particles and maximum cloud heights between 20,000 feet and 100,000 feet was about 2500 to 1. This range can be achieved with available variable speed motors and reduction gears, but there are further unpleasant implications. Let us consider a specific variable speed drive used on a commercially available oscillograph recorder. This drive is continuously variable from 3 inches/sec up to 50 inches/sec. If, in the Analogue, we use a machine constant K_1 such that 2 seconds are equivalent to 1 hour, and assume that the running time of the filter D_μ at its fastest speed must be 6 seconds, then the length of the filter must be 25 feet. This length is rather longer than is handy. Furthermore, at the other extreme, for the minimum filter speed of 0.02 inches/sec, which can be obtained with the same drive through the use of two 15 to 1 reduction gears, we find that for a running time on the machine that is equivalent to 20 hours we use only 0.8 inch on the filter. Also, the position of minimum density is only about 0.4 inch from the start. Since roughly 3/4 of the layers of the cloud are represented by the first foot or two of the filter and 1/4 of the layers by the first few inches, the use of this 25-foot filter seems highly inefficient.

One solution to this problem is to abandon the single filter and replace it with three shorter filters, each one used over the same motor drive range but designed such that the filtering still has the correct time dependence. There is, however, a more elegant solution which appears to be feasible and has several very desirable features. The idea comes directly from the method of determining speed ratios and constructing the composite filter D_μ as described in Chapter 4.

Let us restrict our attention to the time range $0.1 \leqslant t \leqslant 10$ hours. Each individual semi-log plot of the filter density against time of fall for the various layers in the cloud extends two decades along the logarithmic axis, from $t = 0.1$ to $t = 10$. Consequently, if the composite filter were laid out on a semi-log scale rather than a linear scale, we should, for each layer, be interested in the same physical length of the filter, though we should be interested in different parts of the filter for different layers. Furthermore, whatever the part which concerned us, the time scale along the length would be the same. Using the same value of the machine constant K_1, we would always want to traverse the first decade from $t = 0.1$ to $t = 1$ in 1.8 seconds and the second decade from $t = 1$ to $t = 10$ in 18 seconds.

This scheme for traversing the filter D_μ through the light beam requires a logarithmic drive, which in itself is somewhat more complicated than the constant speed system. It has, however, three excellent features. In the first place it is always the same. The filter carriage always travels in exactly the same way, outside the time region of interest as well as inside. One no longer has a variable lead time for the filter D_μ. To get the portion of the filter corresponding to the layer of interest, one merely shifts the filter on the carriage in the same way that we shifted the D_μ versus t transparencies to find the speed ratios.

The second advantage to this scheme is that the physical size of the filter may be quite small, perhaps a foot or so in length. The third advantage is that the precision of the filter may be uniform along its length. We are not forced to compress an important region of the filter, the region of decreasing density, into a very small physical length in order to accommodate the region of rising density.

The equation of motion for the filter carriage in this scheme is

$$x = S \log \frac{t'}{t_o'} \tag{7.1}$$

where x is the carriage displacement, S is the length of the filter corresponding to one time decade, t' is the time in machine units, and t_o' is the machine time corresponding to the minimum real time of interest. The carriage speed as a function of displacement is given by

$$\frac{dx}{dt'} = \frac{0.4343\ S}{t_o'}\ e^{-x/0.4343\ S} \qquad (7.2)$$

One may equally well define the position $x = 0$ as the position of the carriage at a reference time other than the minimum time of interest. Equation 7.1 then gives negative values of x for times less than the reference time but the speed equation still applies. There will, how ever, be only a limited range of x and t' over which the speed will follow Equation 7.2. We have chosen to define one limit at $x = 0$, $t' = t_o'$. The other limit depends on the maximum real time of interest. Temporarily we shall merely call this limit x_{max}.

There are several ways in which one may arrange a driving system following Equation 7.2. One likely possibility is to use a variable pitch screw on a constant speed shaft. In this arrangement the angular position of the shaft is given by

$$\theta = \omega(t' - t_o') \qquad (7.3)$$

where ω is the angular speed of the shaft and $\theta = 0$ at $t' = t_o'$. By solving Equations 7.1 and 7.3 simultaneously one obtains

$$x = S \log \left(1 + \frac{\theta}{\omega t_o'}\right) \qquad (7.4)$$

Equation 7.4 then gives the axial distance to all points of the groove as a function of the angle of rotation of the shaft. In our case we cannot define the pitch in the usual way since it is decreasing continuously as one goes to increasing values of x. To see how the pitch changes we may differentiate Equation 7.4 and make the substitution

$$\theta = 2\pi R \tag{7.5}$$

where R is the number of revolutions of the shaft. The result is

$$\frac{dx}{2\pi\, dR} = \frac{0.4343\ S}{1 + 2\pi r/\omega t_o'} \; \frac{1}{\omega t_o'} \tag{7.6}$$

For $t' \gg t_o'$ both θ and R become large and the approximate form of Equation 7.6 becomes

$$\frac{dx}{dR} \doteq \frac{0.4343\ S}{R} \tag{7.6'}$$

For mechanical reasons we shall eventually arrive at some minimum practical value of the pitch, or Δx per revolution. At this position the number of revolutions through which the shaft has turned will be

$$R_{max} \doteq \frac{0.4343\ S}{\Delta x_{min}/rev} \tag{7.7}$$

We have specified the end position of the carriage by x_{max}. We may find the shaft speed from Equation 7.2 in terms of Δx_{min} and x_{max}. The result is

$$\frac{\omega}{2\pi} = f = \frac{0.4343\ S}{t_o{}'\ \Delta x_{min}}\ e^{-x_{max}/0.4343\ S}\ \text{rps} \tag{7.8}$$

We may also calculate the radius of the shaft if we specify the smallest angle which the groove may make with an element of the surface of the shaft. If this minimum angle is Ψ, then the radius of the shaft is given by

$$r = \left.\frac{dx}{d\Theta}\right|_{\Theta=0} \tan \Psi \tag{7.9}$$

Evaluating $\frac{dx}{d\Theta}$ at $\Theta = 0$ and substituting in Equation 7.9 gives

$$r = \frac{0.4343\ S}{\omega\, t_o{}'} \tan \Psi \tag{7.10}$$

A further substitution for ω from Equation 7.8 gives

$$r = \frac{\Delta x_{min}}{2\pi}\ e^{+\,x_{max}/0.4343\ S}\ \tan \Psi \tag{7.11}$$

In this development we have not designated the five free parameters S, x_{max}, Δx_{min}, $t_o{}'$ and Ψ. Let us look at the results from the following assumed values

$$
\begin{aligned}
S &= 2 \text{ inches (1 time decade)} \\
x_{max} &= 2.38\ S = 4.76 \text{ inches} \\
\Delta x_{min} &= 0.1 \text{ inch} \\
t_o' &= 0.2 \text{ second} \\
\Psi &= 45^\circ
\end{aligned}
\tag{7.12}
$$

From the appropriate equations one obtains

$$
\begin{aligned}
r &= 3.85 \text{ inches} \\
f &= 0.181 \text{ rps} = 10.86 \text{ rpm} \\
t'_{max} &= 48 \text{ seconds} \\
\left.\frac{dx}{dt'}\right|_{x=0} &= 4.343 \text{ inches/sec} \\
R_{max} &= 8.686 \text{ revolutions}
\end{aligned}
$$

Under these conditions we find that what we have been calling a shaft is in fact a slowly turning drum whose length is less than its diameter. The choice of x_{max} is such that, if one can afford the 48 seconds running time and accept the increasing inaccuracies of the composite filter, one may extend the time range to 24 hours.

The length x_{max} is, of course, only the length over which the speed is logarithmic. Some additional length will be required to bring the carriage up to speed ahead of the logarithmic region. The additional length of the drum and the time to reach the required carriage speed at $x = 0$ will depend mainly on the acceleration the system will withstand. Both the time and distance should be made as short as possible.

There remain a few words to be said about an alternate beam position control system. If the beam is parallel, one may remove all the ellipticity of the spot and the requirement for a long writing arm by using linear rather than rotational motion for the deflecting mirror. From the engineering point of view it is just as easy to arrange the linear motion, either with a constant pitch screw on a variable speed shaft, or with a constant speed variable angle wedge. In this latter arrangement the sweep speed of the beam is proportional to the tangent of the angle of incline.

The linear motion system seems better than that described in Chapter 6 and just as simple. It is therefore the recommended design.

Chapter 8

SCALE CHANGES

The prime purpose of the Analogue is to solve the close-in, early-time part of the fallout problem. We have assumed that 40 nautical miles and approximately 10 hours will include the most important features. We have designed the various components with this range in space and time in mind.

We have, however, designed the beam intensity control system with the capability of time coverage out to 24 hours, though with some sacrifice in accuracy for the filter D_μ.

If for any reason one wishes to extend the space coverage as well as the time, two very simple changes may be made in the machine settings. For example, if one wishes coverage out to 80 nautical miles, all that is necessary is to reduce the variable iris setting by a factor 2 and at the same time reduce the beam sweep speed a factor 2. Film densities will be the same as before.

One may also accommodate clouds with effective radiological diameters greater than 25 miles by reducing the scale.

These comments apply to the machine constant K_2 alone. Changes in the machine constant K_1 require changes in the speeds of the shafts driving the filters D_t and D_μ. These speeds are not continuously variable. It may develop that the Analogue should have this capability, but in our present conception it does not. This type of control would be useful if, with no other change in the system, one wished to examine a very wide range in integrated doses. One would then like to control the exposure time of the plate by varying the constant K_1 to place the region of best contrast of the emulsion in the dose region of interest. For operational forecasting we expect, however, that we shall always be mainly concerned with the dose range from approximately 4 roentgens up to about 400, and that regions of higher and lower integrated dose will be relatively uninteresting. The final, and fixed, value of the machine constant K_1 will be chosen to emphasize the single most important dose range. We have still, of course, the capability of extending the dose range through the use of neutral density filters and through variation of the projection lamp power, but these methods, we expect, will permit us to examine only the higher doses.

Chapter 9

THE COMPLETE ANALOGUE

The purpose of this short chapter is to review the list of input information required and the ways in which these assumptions, data, and forecasts are used for operating the complete machine.

There are actually only two categories of information, that which includes the model parameters and that which includes the features of the weather. In the first category are the predicted effective radiological yield Y, the assumed values for the median particle size μ_m, the standard deviation σ and the average concentration C as functions of height, and the predicted radiological dimensions. In the second are the wind structure, the lapse rate, the relative humidity, and any other interesting features of the weather.

To some extent the two groups of information are related. For example, the maximum forecast cloud height may in some cases be markedly affected by the weather conditions, though in most cases the height and lateral dimensions may be forecast with adequate precision on the basis of climatic data alone.

Prediction of the effective radiological yield depends upon prediction of the fission and fusion yields, on the burst height, and on the type and distribution of materials available for activation and scavenging. The effective radiological yield determines the weight and to some extent also the shape of the height distribution function $C(h)$.

It is necessary. in order to set the transmission of the filter D_h and the iris radius r', that we know the incremental weight for each altitude layer. That is, we must know the average concentration C, the radiological radius r, and the layer thickness Δh as functions of altitude. To set D_h we must also know the standard deviation σ and the layer height correction δ as functions of altitude. Among these quantities Δh is arbitrary, except that it cannot exceed some maximum value, and δ may be tabulated in advance. The numbers C and r must, however, be calculated from an estimate of the fraction of activity in each altitude layer and the total effective radiological yield.

In order to obtain the proper setting for the filter D_μ, we must assume values for μ_m and σ for each of the altitude layers and we must predict the real height of each layer.

So far we have used little. if any, specific weather information. The wind information is required primarily for the beam position control. We must supply to the machine the effective bearing and mean resultant wind speed for each altitude layer.

Except for the position control system, tables of settings can usually be prepared on the basis of a 24-hour wind forecast. The procedure would be first to predict the maximum cloud height h_o and select some suitable number of layers n of equal thickness

$$\Delta h_j = \frac{h_o}{n} \tag{9.1}$$

Then calculate the altitudes h_j from

$$h_j = \frac{h_o}{2n}(2j - 1) \qquad 1 \leq j \leq n \tag{9.2}$$

For each value of h_j select values of r_j and C_j. The correction δ_j may be obtained from a single universal curve of δ versus h. For the Weather Bureau model the value of σ_j is unity for all altitudes. If other values of σ_j are desired, they too should be tabulated against h_j. From the tabulated values of C_j, Δh_j, σ_j, and δ_j compute the densities D_{hj} for each layer number j from 1 through n. The actual setting of the filter in inches or other convenient units may be obtained from the curve of wedge density versus distance along the wedge. Tabulate these settings against the layer number j.

Having assigned radii r_j, it is now necessary to pick the machine constant K_2 and compute the iris settings r_j'. Tabulate the settings r_j' against the layer number j.

For each value of h_j select μ_{mj} and σ_j. The correct settings for the filter D_μ may be obtained from a family of curves. Tabulate these settings against the layer number j.

Proper settings for the filters D_μ and D_h and the iris are now available for use in the machine. The two remaining settings. for the mean speed and bearing, must be taken from the latest complete local forecast. From the weighted hodographs, select the best bearing angles ϕ_j and compute the mean resultant wind speed $\overline{V}_j$ for each altitude h_j. The ϕ_j values may be tabulated directly against the layer number j. The speed settings may be obtained from a universal curve corrected for the machine constant K_2. Tabulate these settings against the layer number j.

For each layer there may be as many as five independent settings depending on the model and the wind structure. They are, however, very simple settings and with a little practice on the part of the operator should not take much time. If the wind information is based on a balloon run, the low layers may be run as the balloon rises.

There will doubtless be many things about the prototype Analogue and its operation which field experience will lead us to correct or amend. Our main concern at the present time is to get a machine into operation in time to check its performance on a few sample problems prior to its first operational use.

APPENDIX

A PROCEDURE FOR SINGLE-POINT HAND CALCULATION

A single-point hand calculation based on the Analogue design principles may be made in the following way:

1. Plot the weighted hodographs to scale on a suitable map. These are the weighted hodographs required for selection of the mean wind speeds and mean bearings as functions of altitude.

2. Locate the position of interest R,ϕ on the map.

3. Draw a straight line from the origin through the position R,ϕ.

4. Determine the "best" height h and mean wind speed $\bar{V}$ for that height from the intersection of the line (Step 3) with the weighted hodographs. One of these hodographs, or an interpolated hodograph, will be the "best" hodograph for the position R,ϕ.

5. Draw a circle of radius r equal to the predicted radiological radius for the "best" height h around the position R,ϕ.

6. Draw two straight lines from the origin tangent to this circle (Step 5). The bearings of these lines will be $\phi \pm \sin^{-1} r/R$.

7. The intersections of the tangent lines (Step 6) with the "best"

hodograph will give two heights above and below the "best" height. The difference in heights is Δh.

8. Multiply the predicted concentration C for the "best" height by Δh (Step 7).

9. Read T_h from the graph of T_h versus $C\Delta h$. This graph is constructed from

$$T_h = D_{h\ max} - D_h$$

where D_h is the density of the height filter in the Analogue.

10. From the mean speed $\bar{V}$ (Step 4) calculate the times to the positions $R-r,\phi$ and $R+r,\phi$. Subtract these times to get Δt.

11. From the graph of T_t versus t find T_t for the two times (Step 10). This graph is constructed from

$$T_t = D_{t\ max} - D_t$$

where D_t is the density of the time filter in the Analogue.

12. From the graph of V_μ' versus h/h_o find the filter speed V_μ' for the "best" height h. This graph consists of a family of curves with h_o as a parameter.

13. Locate the position $t = 1.0$ on the graph of T_μ versus x by displacing a log scale time base such that on the graph the time $t = 1.0$ is at the position $x = V_\mu'$. This graph is constructed from

$$T_\mu = D_{\mu\ max} - D_\mu$$

where D_μ is the density of the particle size filter in the Analogue.

14. Using the time scale resulting from Step 13, read T_μ for the two times found in Step 10.

15. Add T_t and T_μ for the two times, average the sums, and add the average to T_h (Step 9) to find T_{total}.

16. Multiply the antilog of T_{total} by Δt (Step 10). The product is proportional to the integrated dose at the position $R, \emptyset$.

In some cases there will be more than one "best" height. This situation may occur, for example, in the presence of large directional shears. The calculation must be repeated for each "best" height and the results of Step 16 added arithmetically.

To the extent that the best result at a point may be regarded as that given by the Analogue, the hand calculation may be improved by subtler averaging than the simple arithmetic average.

It will also frequently be possible, particularly for distant points, to improve over the Analogue by using the exact curves of T_μ versus t in place of the composite used in the Analogue. In such cases one should also attempt to employ a weather forecast which includes space time variation of the winds.

The actual numerical value of the dose may be obtained from the same calibration as is used for the Analogue.