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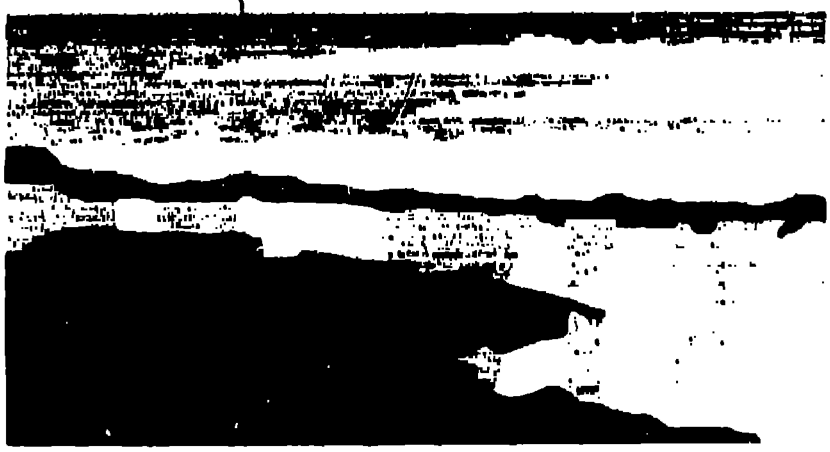

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THE GREEN'S FUNCTION METHOD FOR CRITICAL HETEROGENEOUS SLABS

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INTRODUCTION

Recently, the Green's Function Method (GFM) has been employed to obtain benchmark-quality results for nuclear engineering and radiative transfer calculations. was possible because of fast and accurate calculations of the Green's function and the associated Fourier and Laplace transform inversions.[1,2] Calculations have been prov in one-dimensional slab geometries for both homogeneous and heterogeneous media. heterogeneous medium is analyzed as a series of homogeneous slabs, and Placzek's lemma[4] is used to extend each slab to infinity. This allows use of the infinite medium Green's function (the anisotropic plane source in an infinite homogeneous medium) i solution. To this point, a drawback of the GFM has been the limitation to media with 1, where c is the number of secondary particles produced in a collision. Clearly, no physical steady-state solution exists for an infinite medium that contains an infinite so and is described by $c > 1$; however, mathematical solutions exist which result in oscill Green's functions. Such calculations are briefly discussing in Ref. 5. The limitatio media with $c \leq 1$ has been relaxed so that the Green's function may also be calculated media with $c \geq 1$.[9] Thus, materials that contain fissionable isotopes may be modeled

The obvious application of the GFM with $c > 1$ is the critical slab problem. Th critical slab problem has been solved using a variety of analytical and semi-analytical methods.[7] However, as the GFM is used for heterogeneous media with $c < 1$, it may be used for the heterogeneous critical slab problem. Reflected multiplying media have studied and have included some anisotropic scattering in the medium properties.[8-10] T analyses considered symmetric media with the multiplying medium in the center and fi or infinite reflectors on each side. The GFM with isotropic scattering will be employe analyze similar systems as well as asymmetric slab configurations. The analysis consi of determination of the critical width and associated flux profiles.

THE GREEN'S FUNCTION METHOD FOR FINITE MEDIA

The power of the GFM rests primarily in the ability to obtain solutions for finit media using the infinite medium Green's function. Multiple-slab systems are easily constructed as a series of single-slab problems that are connected by equating bounda angular fluxes. Using standard Green's function formulations for the one-group linea transport equation with isotropic scattering, the angular flux in a slab is expressed in t of the Green's function as

$$\psi(x,\mu) = \int_{-\infty}^{\infty} dx' \int_{-1}^{1} d\mu' \left[q(x) S_{ext}(x',\mu') + S_{tr}(x',\mu') \right] G(x-x',\mu|\mu') \quad ,$$

where $S_{tr}(x,\mu) = \mu\, \psi(0,\mu)\delta(x) - \mu\, \psi(\Delta,\mu)\delta(x-\Delta)$ (a result of Placzek's lemma), $S_{ext}(x,$ any inhomogeneous sources, and $q(x)$ is the characteristic function for the slab, define 1 inside the slab and 0 elsewhere. The effect of Placzek's lemma is to add boundary source terms to the infinite medium formulation. Upon inserting the expression for $S_{tr}(x,\mu)$ and separating the μ integral into positive and negative components we have

$$\phi(x,\mu) = \int_0^1 d\mu'\,\mu'\,\phi(0^-,\mu')G(x,\mu|\mu') - \int_0^1 d\mu'\,\mu'\,\phi(\Delta^-,\mu')G(x-\Delta,\mu|\mu') -$$

$$- \int_0^1 d\mu'\,\mu'\,\phi(0^+,-\mu')G(-x,-\mu|\mu') + \int_0^1 d\mu'\,\mu'\,\phi(\Delta^+,-\mu')G(\Delta-x,-\mu|\mu') + Q(x,\mu) \quad ,$$

where $Q(x,\mu)$ is the term for inhomogeneous sources. Both translational invariance $[G(x-x',\mu|\mu') = G(x,\mu|x',\mu')]$ and reciprocity $[G(-x,-\mu|-\mu') = G(x,\mu|\mu')]$ have been assumed. The interior angular flux in Eq. (1b) may be determined via quadrature if t angular boundary fluxes and the Green's function are known.

THE CRITICAL SLAB PROBLEM

The critical slab problem consists of a series of source-free slabs which scatte particles isotropically. At least one slab must be a multiplying medium. By letting x and $\mu \to -\mu$ for the exiting angular flux at the left boundary and $x \to \Delta^-$ for the exiting at the right boundary, the following equations are obtained for the boundary angular of the i^{th} slab:

$$\phi_i(0^+,-\mu) + \int_0^1 d\mu'\,\mu'\,\phi_i(0^+,-\mu')\left[G_i(0^-,\mu|\mu') \pm G_i(-\Delta,-\mu|\mu')\right] \pm$$

$$\phi_i(\Delta^-,\mu) \pm \int_0^1 d\mu'\,\mu'\,\phi_i(\Delta^-,\mu')\left[G_i(0,\mu|\mu') \pm G_i(-\Delta,\mu|\mu')\right] \pm$$

$$\phi_{i-1}(\Delta,\mu)e^{-\Delta/\mu} - \int_0^1 d\mu'\,\mu'\,\phi_{i-1}(\Delta,\mu')\left[G_i(0^+,-\mu|\mu') \pm G_i(\Delta,\mu|\mu')\right]$$

$$\phi_{i+1}(0^+,-\mu)e^{-\Delta/\mu} + \int_0^1 d\mu'\,\mu'\,\phi_{i+1}(0^+,-\mu')\left[G_i(0^+,-\mu|\mu') \pm G_i(\Delta,\mu|\mu')\right] = 0 \quad .$$

The exponential terms come from the separation of the Green's function into collide uncollided components. Again, the slabs "communicate" with one another via the boundary angular fluxes between the slabs. For a source-free medium, one solution $\phi_i(\mu) = 0$. This trivial solution occurs if the cumulative integral operator is invertible

However, the "critical" solution comes when the operator is singular, or non-invertible. Physically, with no source present, a slab configuration which is thinner than the critic configuration will have a zero neutron population (the trivial solution). If the thickness greater than the critical thickness, no steady state solution is available (the population continuously increases with time). Therefore, the goal is to determine the critical thick at which a steady-state flux distribution may be present without sources. The thicknes of all but one of the slabs are held constant, and the critical width of the slab under consideration is desired. There are $2N_s$ unknown functions - the boundary angular flu $\phi_i(0^+,-\mu)$ and $\phi_i(\Delta^-,\mu)$. Assuming a Gauss-Legendre quadrature rule (order L_m) for tl integrals, a cumulative solution vector is constructed from the boundary fluxes. The o of the vector is $2 \times N_s \times L_m$. A super matrix equation for Eq. (2) is constructed. As o form of the Green's function for $c > 1$ is complex, this matrix is also complex.[9] The critical solution comes when, for a given thickness of the variable slab, the determinan the super matrix is zero. A dual zero search is performed for the real and imaginary pa of the determinant, which provides the critical width of the variable slab.

Once the critical thickness is determined, the next step is to calculate the angula scalar fluxes for each slab. As usual, the magnitude of the flux distributions is arbitra a critical system (usually it is determined using a power level). With the GFM, the inte angular fluxes are determined via quadrature once the boundary fluxes are known. Normally, the operator matrix would be inverted to give the boundary fluxes; howeve matrix is singular and therefore can not be inverted. The boundary flux is determined an iterative process. For each slab, there are four boundary fluxes which must be calculated as seen in Eq. (2). The initial guess for one of the entering boundary fluxe $\phi_{i+1}(0^+,-\mu)$, is the normalization factor (a_0) which scales the scalar flux. The exiting boundary fluxes are then calculated from Eq. (2) with each slab being analyzed in sequence. This process is done iteratively until all boundary fluxes have converged t specified tolerance. Interior fluxes may then be calculated from Eq. (1b).

The Green's function method for a critical multi-slab system is demonstrated two adjacent slabs. The first slab is a multiplying medium with $c_1 = 1.5$, and the sec a reflector on the right side of the first slab with $c_2 = 0.9$ and a width of $\Lambda_2 = 1$. The critical width of the first slab was determined to be 0.9652. The scalar flux distribut along with a comparison using the ONEDANT[11] code, is provided in Fig. 1. Good agreement is obtained, with correspondence achieved by matching the left endpoint f Comparisons of the GFM results with results from symmetric reflected systems fou Refs. 8-10 yield the same critical widths.

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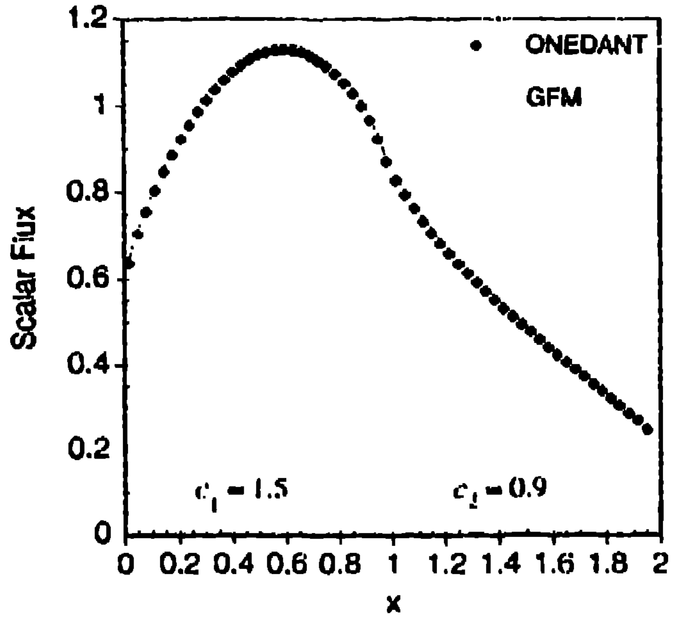

Fig. 1. Scalar flux distribution for two-slab critical system ($\Delta_1 = 0.9652$, $\Delta_2 = 1$