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Inelastic Scattering Cross Sections of Lead and Uranium for 14 Mev Neutrons

Work done by:

H. H. Barschall
G. G. Everhart
H. T. Gittings
Arthur Hemmendinger
G. A. Jarvis
R. F. Taschek

Report written by:

H. T. Gittings

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Abstract

The inelastic scattering cross sections of lead and uranium for 14 Mev neutrons have been measured by means of threshold detectors whose thresholds were 12 Mev and 3 Mev.

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Inelastic Scattering Cross Sections of Lead and Uranium for 14 Mev Neutrons

Introduction

The purpose of this experiment was to measure the inelastic scattering cross section of uranium for 14 Mev neutrons and to determine the approximate energy loss during an inelastic collision. Duplicate runs were made in every case, using lead in place of uranium, as one would expect similar results except for the effects of fission.

The measurements were made with threshold detectors whose characteristics * are, ideally, zero detection cross section for neutrons below threshold energy and an approximately constant cross section above this energy. Actually, however, instead of a critical energy there is usually a transition zone one or two Mev wide. The true shape of this curve was used in analyzing the data. (See Fig. 1.) With detectors of this type the measured value of the inelastic scattering cross section is the sum of the cross sections for all processes in which the neutron is either scattered out or degraded in energy below the threshold of the detector, the most probable processes being (a) true inelastic scattering, (b) n,2n, (c) n, fission.

Disk scattering

The ideal experimental setup should be one in which the effects of elastic scattering are cancelled out. The first attempt at this was thin disk scattering as suggested by John Manley. A thin uranium disk

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* Appendix A.

with both faces covered with copper foil detectors was placed normal to the beam of 14 Mev neutrons. The copper foil on the side toward the neutron source should see all the primary neutrons plus all the elastically back-scattered neutrons. The foil on the face of the disk away from the neutron source should see the primary neutrons minus those elastically back-scattered and minus those inelastically scattered in the uranium. The effect of the primary neutrons alone was determined by a second run made with duplicate copper disks and without the uranium disk. For a primary beam of $\underline{I_o}$, the intensity on the side toward the source is $\underline{I_T}$ and the intensity away is $\underline{I_A}$.

Then

$$I_T = I_o\,(1 + N\sigma \text{ elastic})$$

$$I_A = I_o\,(1 - N\sigma \text{ elastic} - N\sigma \text{ inelastic})$$

Adding gives

$$N\sigma_{in} = 2 - \frac{I_T + I_A}{I_o}$$

$$\sigma_{in} = \frac{2\,I_o - I_T - I_A}{N I_o}$$

where N is the number of uranium atoms per cm^2 and the I's are obtained from the Geiger counting rate of the foils after irradiation. This is of course a crude picture but the method will not be discussed in more detail since it was not satisfactory; a correction factor in the foil sensitivity was larger than the main effect and very difficult to compute.

 [redacted]

This correction, called the obliquity factor, comes about in the following way. If the detector foil is in a uniform neutron flux, the size, shape, or orientation is of no importance, only the total number of atoms in the flux, provided of course the path lengths in the detector are very small compared to a mean free path. However in a non-uniform flux this is not the case. Let $\underline{n}$, the neutrons per cm^2 sec, be expressed as a function of a parameter, $\underline{\theta}$. Let $\underline{dA}$ be that differential area of the foil which sees a constant value of neutron flux and let it be expressed as function of $\underline{\theta}$. Let $\underline{P}$, the associated path length in the foil, be expressed as a function of $\underline{\theta}$. The rate of formation of radioactive atoms in the detector foil, $\underline{R}$, is then given by

$$R = \int_{\theta} \sigma n(\theta) \cdot \#\text{atoms/cc} \cdot P(\theta) \cdot dA(\theta) \qquad (1)$$

If $\underline{n}$ is a constant, this results in

$$R_c = \sigma n \cdot \#\text{atoms/cc} \int_A P\, dA = \sigma n \cdot \#\text{atoms/cc} \times \text{volume}$$

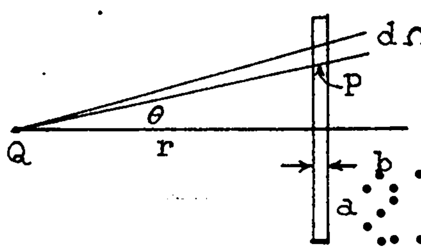
$$R_c = \sigma n \#\text{atoms} \qquad (2)$$

which is customarily used.

A more convenient form of equation (1) in the case of a point source of neutrons is

$$R = \sigma \#\text{atoms/cm}^3 \int_{\Omega} n \text{ neuts/unit solid angle} \times P \times d\Omega \qquad (3)$$

As an example of this obliquity correction consider a point source $\underline{Q}$ neutrons per second and a circular disk detector.



$$P = \frac{b}{\cos\theta}$$

$$d\Omega = 2\pi \sin\theta\, d\theta$$

$$R = \sigma \#\text{atoms/cc} \int_{\Omega} \frac{Q}{4\pi} P\, d\Omega$$

$$R = \frac{\sigma \#\text{atoms/cc}\, Q\, b}{4} \log\left(1 + \frac{a^2}{r^2}\right)$$

[redacted]

 [redacted]

if a were small enough so that the flux could be considered parallel, the expected reaction rate would be R_o.

$$R_o = \frac{Q}{4\pi}\ \sigma\ A \times \Omega \times b; \quad \Omega = \frac{a^2}{r^2}\ ; \quad A = \#\,\text{Atoms/cc}$$

$$R_o = \frac{\sigma\, Q\, A\, b}{4} \cdot \frac{a^2}{r^2}$$

The obliquity factor $\frac{R}{R_o} = \frac{\log\,(1 + a^2/r^2)}{a^2/r^2}$, which increases to 1 as a/r gets small. Now in the case of disk scattering the obliquity factor would have to be obtained by considering each point of the uranium disk in the above manner and integrating over the volume, a complicated procedure when such other factors as secondary elastic or inelastic scattering in the uranium associated with the different path lengths are considered.

Method

The method finally used for the experiment was sphere scattering. Lead and uranium spheres of radius about one-half mean free path for energy loss were made with a small central cavity in which the detector foils were placed. In such a spherical geometry the effects of elastic scattering do not enter into the calculations since just as many neutrons are elastically scattered out as are elastically scattered into the detector. The foils were made in long thin strips which were rolled up tight to approximate a pseudosphere when placed in the cavity

[redacted]

and which could be unrolled and wrapped around a Geiger counter for good counting geometry.

Several sizes of uranium spheres were tried with copper detectors. Radii of 3, 4, and 5 cm all gave the same value of inelastic scattering cross section within the experimental error.

The experimental procedure (Fig. 2) consisted of placing the sphere with the internal detector in the neutron beam for about 10 minutes (approximately one half-life) and then counting the induced foil activity. A second run was then made with a duplicate detector but without the sphere. The ratio of the two foil activities was then a measure of what we have called inelastic scattering. 10 minute runs were chosen as a compromise since the foils would be up to about half of saturated activity and a number of runs could be made in a reasonable time.

Of course it was necessary to normalize the neutron flux during the two runs. This was done by using a monitor foil of the same material as the detector foil, placed in the beam in a standard geometry far enough from the sphere to minimize any back-scattering effects from the sphere.

After irradiation with the sphere in, the detector and monitor foils were counted simultaneously in two Geiger counters for about 10 minutes. The ratio of the total counts in such a case is the same as the ratio of the two foil activities at the end of irradiation. The same ratio was determined for the sphere-out run.

Computation

The transmission T of a neutron beam through a medium is given by

$$T = e^{-N\sigma r}$$

[redacted]

where N is the number of atoms per cc, r is the distance in centimeters and σ is the total cross section in general, but is only the inelastic in our case.

$$\sigma_{in} = \frac{-\log_e T}{Nr}$$

Using detector foils of equal mass, the transmission is the ratio of foil activities, Ad, with the sphere in and out, normalized to the same neutron flux by means of the monitor foil activities, Am.

$$T = \frac{\text{Ad in}}{\text{Ad out}} \times \frac{\text{Am out}}{\text{Am in}}$$

This ratio is evaluated from the counting data, using equal monitor masses.

$$T = \frac{(\text{Ad/Am})\ \text{in}}{(\text{Ad/Am})\ \text{out}}$$

When computed in this fashion the irradiation time and the counting time do not enter into the calculations, and so do not need to be known.

Results with 14 Mev Neutrons

The results to date are shown in the table below. The errors given are the mean deviations from the average. The geometrical cross sections are included for comparison. The cross sections are in barns.

Material	Al detector runs	Al detector σ in	Cu detector runs	Cu detector σ in	σ geometrical	σ fission	σ true inelastic
Lead	8	2.20 ± 0.17	11	2.29 ± 0.12	2.5	——	2.25 ± 0.15
Uranium	7	2.02 ± 0.21	15	2.50 ± 0.08	2.7	0.85	1.65 ± 0.10

[redacted]

[redacted]

In the table we see that the inelastic cross section of lead is essentially the same when measured by the copper and aluminum detectors. This suggests that the inelastically scattered neutrons which are degraded in energy below the copper threshold are also degraded below the aluminum threshold, or, a 14 Mev neutron loses over 11 Mev in its first inelastic collision in lead.

In the case of uranium there is a significant difference in the results of the measurements with the two detectors. This difference might be due to an average energy loss between the two thresholds but more probably is due to activation of the aluminum detector by the faster of the 28 fission neutrons. If this is so, then the results of the experiment are that the true inelastic scattering cross section (measured-minus fission) of uranium is twice as great as the fission cross section and the average energy loss due to an inelastic collision of a 14 Mev neutron in uranium is over 11 Mev.

<u>Correction of measurements with aluminum detector.</u>

Evaluation of the expected contribution from fission neutrons to the aluminum foil activation is given below.

The ratio $\underline{c}$ of detector activation $\underline{R}$ from fission neutrons to the detector activation $\underline{R_o}$ from primary beam neutrons is

$$c = R/R_o.$$

$\underline{R_o}$ is computed on the basis of the value of inelastic scattering cross section; $\underline{\sigma}$ determined with the copper detector,

[redacted]

$R_o = n_o$ neutrons/cm^2 sec x $e^{-N\sigma r}$ x σ_{Al} x # Al atoms/cc x Volume Al, where Al refers to aluminum.

R is the product of three computed functions.

a) The average number of fissions/cm^3 sec = F.

b) The average number of neutrons/fission detectable with aluminum = G.

c) The number of aluminum reactions/detectable neutron integrated over the uranium sphere = R_2 .

$$R = F \times G \times R_2$$

where

$$F^* = \frac{3\sigma_f n_o}{2a^3 A\sigma^2}\left[2a - \frac{1+A\sigma a}{A\sigma}\left(1-e^{-2A\sigma a}\right)\right]$$

$G^\dagger$ = 2.5 neutons/fission x 0.218 detectable neutrons/fission neutron

G = 0.55 detectable neutrons/fission.

$$R_2^{**} = 2\pi\rho\sigma_{Al}\,\#\text{Al/cc}\int_\rho^a \left[1-\frac{b^2-\rho^2}{2b\rho}\log\frac{b+\rho}{b-\rho}\right]\left(1-0.84\frac{b}{\lambda}\right)b^2\,db$$

$$C = R/R_o = FGR_2/R_o$$

$$C = \frac{3\pi\sigma_f\rho\left[2a-\frac{1+A\sigma a}{A\sigma}\left(1-e^{-2A\sigma a}\right)\right]\int f(b)\,db \cdot 0.55}{V e^{-A\sigma r} a^3 A\sigma^2}$$

C = 0.0645 = 6.45 %

ρ = 0.6 cm

V = 0.95 cc

r = 3.34 cm

a = 3.97 cm

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* Appendix B
** Appendix C
† Figure 1

[redacted]

$\sigma \quad = 2.50 \times 10^{-24}\ cm^2$

$\sigma_f \quad = 0.85 \times 10^{-24}\ cm^2$

$A \quad = 0.464 \times 10^{23}$ atoms/cm^3

$\lambda \quad = 9.2$ cm for 3 Mev neutrons.

$\int f(b) \quad = 0.76\ cm^3$

The average value of the transmission as measured was 0.731. If we consider that the value we should use is 0.731 (1 - σ) we get $\overline{T} = 0.684$. Putting this in the cross section formula gives

$$\sigma_{in} = -\frac{\log_e T}{N\ r} = \frac{0.380}{1.56 \times 10^{24}} = 2.44 \text{ barns.}$$

The mean deviation will be about the same, so

$$\sigma_{in} = 2.44 \pm 0.21 \text{ barns,}$$

which is in agreement with the value obtained with copper

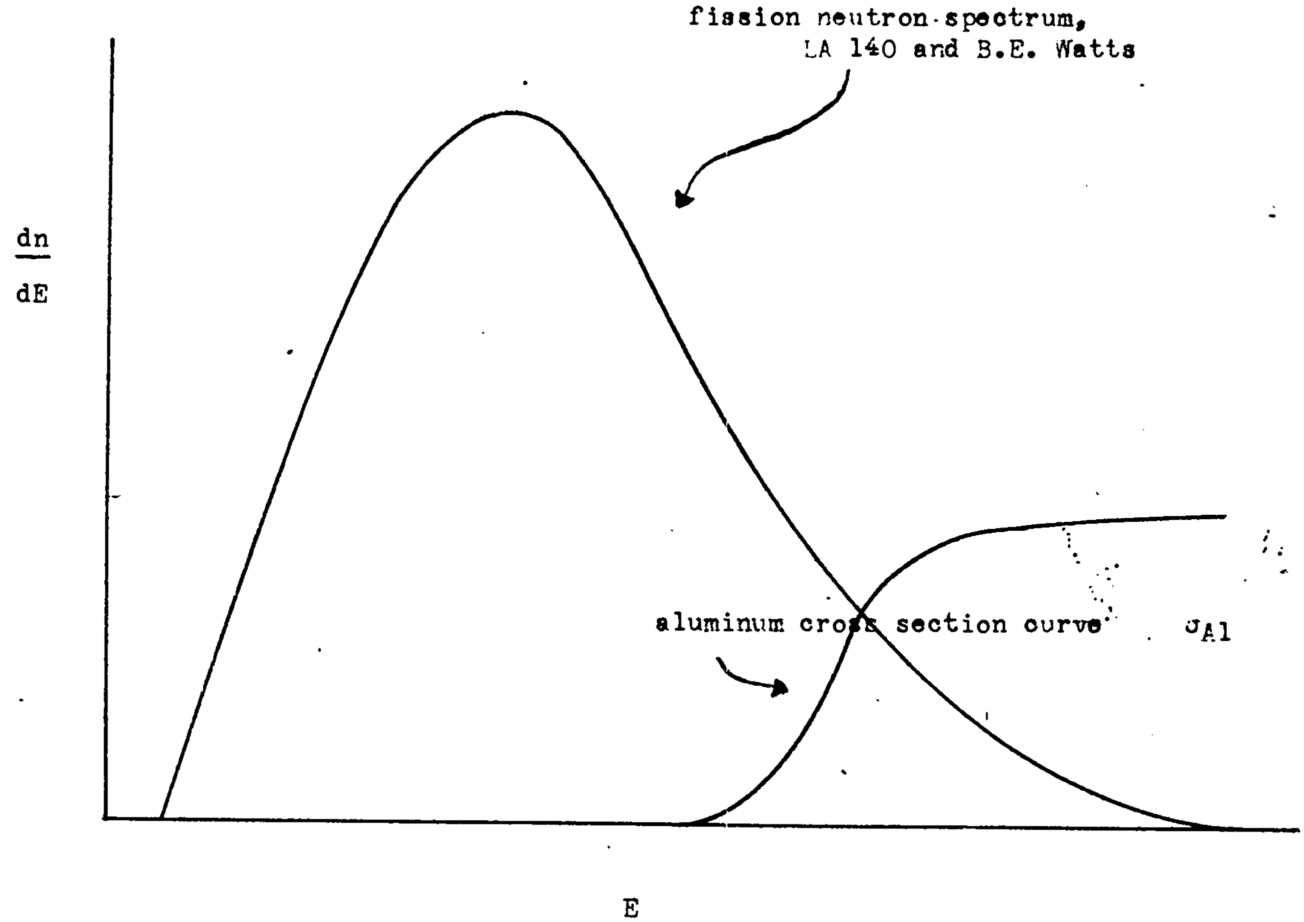
$$\sigma_{in} = 2.50 \pm 0.08 \text{ barns.}$$

Therefore the results of the experiment are probably correct as stated.

[redacted]

[redacted]

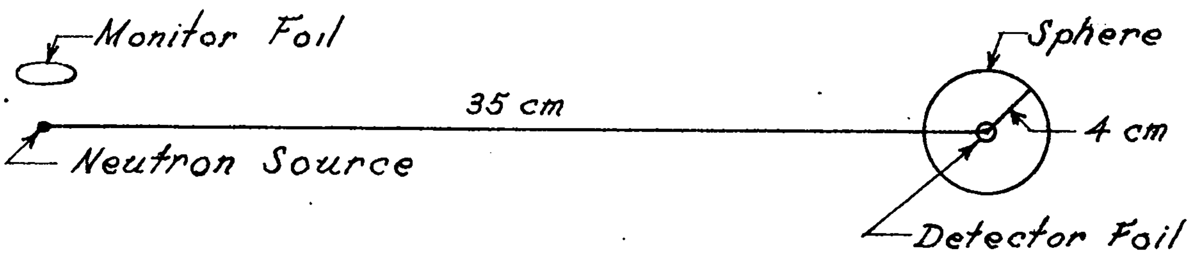
Fig. 1



The detection efficiency is the normalized, integrated product of these two functions, which is equal to 21.8 percent detectable neutrons per fission neutron.

[redacted]

Fig. 2



Experimental Set-up

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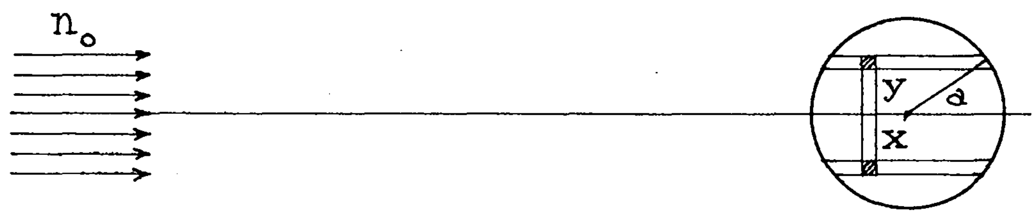
Appendix A

Detector	Reaction	Threshold	Half-life	Cross Section	Size
Cu^{63}	n-2n	~ 12 Mev	10.4 min	0.25 barns	12" x 1/4" x 0.003"
Al	n-p	~ 3 Mev	10.2 min	0.052 barns	12" x 1/2" x 0.008"

[redacted]

Appendix B

To evaluate the total number of fissions in a uranium sphere in a uniform flux of neutrons,



First assumption

Any effects of elastic scattering cancel out due to the spherical geometry.

The locus of differential volumes which see the same neutron flux is a ring of volume $dv = 2\pi y \; dy \, dx$. The number of fissions in dv/sec = df.

$$df = \sigma_f \, n \, x \, \#A/cc \; dv$$

The neutron flux n falls off due to inelastic scattering and fission as

$$n = n_o \, e^{-A\sigma t}$$

where t is the thickness of material.

$$df = \sigma_f \, n_o \, e^{-A\sigma}\left(\sqrt{a^2-y^2} - x\right) \#A \, 2\pi y \; dy \; dx$$

$$f = \sigma_f \, n_o \, A \, 2\pi \int_{-a}^{a} e^{A\sigma x} \int_{0}^{\sqrt{a^2-x^2}} y \, e^{-A\sigma\sqrt{a^2-y^2}} \, dy \; dx$$

To obtain the average number of fissions per second cm^3 we take

$$F = \frac{f}{4/3\ \pi\ a^3}$$

or $$F = \frac{3\sigma_f n_o}{2a^3 A \sigma^2}\left[2a - \frac{1 + A\sigma a}{A\sigma}\left(1 - e^{-2A\sigma a}\right)\right]$$

which is given as equation (a). As a check on the validity of the equation, we can see if it reduces to the simple form of

$$F_o = n_o \sigma_f A$$

which we know holds true for small values of $\underline{a}$. The procedure is to take $\lim_{a \to o} F$, the necessity being that $\underline{F}$ is indeterminate for small a.

Write

$$F = F_o \times \frac{3}{2a^3 A^2 \sigma^2}\left[2a - \frac{1 + A\sigma a}{A\sigma}\left(1 - e^{-2A\sigma a}\right)\right]$$

Differentiating three times to get rid of the $\underline{a^3}$ in the denominator we get

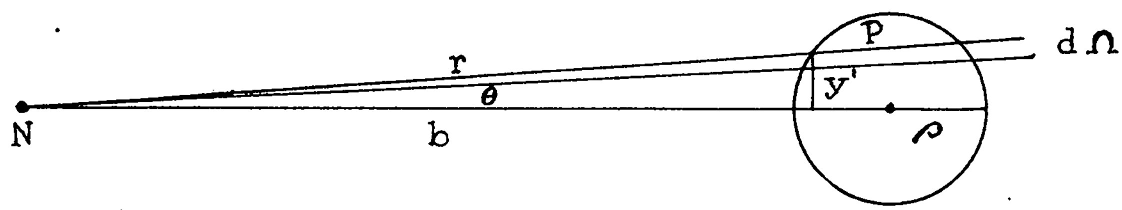
$$\lim_{a \to o} F = \lim_{a \to o} \frac{3}{2} F_o \left[\frac{4A^3\sigma^3}{6A^3\sigma^3}\right]$$

$$\therefore \lim_{a \to o} F = F_o$$

[redacted]

Appendix C

To determine the rate of formation of radio-aluminum atoms due to the presence of fission neutrons of Q per cc with energy above 3 Mev, consider first a point source of neutrons, N neutrons per unit solid angle, at a distance b from the center of the aluminum detector radius a.



For different angles θ the flux incident on the sphere is different by $1/r^2$ and the path length P is different in the sphere and hence the reaction probability is also different. Let the number of reactions per second in the differential solid angle $d\Omega$ be dR_1,

$$dR_1 = \sigma_{Al} \times N \, d\Omega \times \# \text{Al/cc } P.$$

First to express P as a function of θ,

$$(x-b)^2 + y^2 = \rho^2$$

$$y = x \tan\theta$$

$$x = b\cos^2\theta \pm \rho\cos\theta \sqrt{1 - k^2\sin^2\theta}$$

where $k = b/\rho$.

$$y = b\sin\theta\cos\theta \pm \rho\sin\theta\sqrt{1 - k^2\sin^2\theta}$$

$$P = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$P = 2\rho\sqrt{1 - k^2\sin^2\theta}$$

[redacted]

To get $d\Omega$ as a function of θ,

$$d\Omega = \frac{dA}{r^2} = \frac{2\pi y' \; r d\theta}{r^2} = 2\pi \frac{y'}{r} d\theta$$

but $y'/r = \sin\theta$.

$$d\Omega = 2\pi \sin\theta \, d\theta$$

$$dR_1 = \sigma_{Al} \times N \times \#Al/cc \times 2\rho\sqrt{1 - k^2 \sin^2\theta} \times 2\pi \sin\theta \, d\theta$$

$$R_1 = \sigma_{Al} N \# Al \; 4\pi\rho \int_0^{-1\frac{a}{b}} \sin\theta \sqrt{1 - k^2 \sin^2\theta} \; d\theta$$

$$R_1 = 2\pi\rho\sigma_{Al} N \# Al \left[1 - \frac{b^2 - \rho^2}{2\rho b} \log \frac{b+\rho}{b-\rho} \right]$$

ρ is determined by setting $4/3\pi\rho^3$ = volume

$$\text{Volume} = \frac{\text{mass}}{\text{density}} = \frac{2.56}{2.7} = 0.95 \text{ cc}$$

$$\rho = 0.61 \text{ cm}$$

This will hold for small b but for large b it does not take into account the inelastic scattering in the uranium. However for these large b the change in flux due to the factor $e^{-r/\lambda}$ is small compared to the change due to $1/r^2$, and so this form is still a fair approximation. This is the contribution of a point source. The total source strength for given b is $N \times 4\pi b^2 \, d\,b$.

$$dR_1' = 8\pi^2\rho b^2 \sigma_{Al} N \# Al \left[1 - \frac{b^2 - \rho^2}{2b\rho} \log \frac{b+\rho}{b-\rho} \right] d\,b$$

[redacted]

We must put in the exponential decay factor now since we have varying b. This is $e^{-b/\lambda}$ where λ is the mean free path for energy loss, since again we assume elastic scattering cancels out. For our range of values of b we can set this to $(1 - 0.84\, b/\lambda)$*. $N = Q/4\pi$.

$$R_1' = 8\pi^2 \rho\, \sigma_{Al}\, N\, \#Al \int_\rho^d \left[1 - \frac{b^2-\rho^2}{2\rho b} \log \frac{b+\rho}{b-\rho}\right]\left(1 - 0.84\frac{b}{\lambda}\right) b^2\, db$$

$$R_1' = 2\pi\rho\, \sigma_{Al}\, Q\, \#Al \int f(b)\, db$$

$$R_2 = \frac{R_1'}{Q} = 2\pi\rho\, \sigma_{Al}\, \#Al \int f(b)\, db$$

This integral is too messy to evaluate analytically, so we plot the integrand and measure the area, giving

$$\int_\rho^d f(b)\, db = 0.76\ \mathrm{cm}^3.$$

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* This is the best fitting straight line for: $0 < b < \lambda/3$.

[redacted]