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THE RANGE OF DEUTERONS IN HEAVY WATER AND OF PROTONS IN HYDROGEN

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ABSTRACT

The stopping power of heavy water for deuterons, and the range-energy relation for deuterons in heavy water, have been calculated for deuteron energies from 0 to 750 kv. The range of protons in hydrogen for proton energies from 0 to 1 MV has also been obtained. For proton energies below 60 kv the hydrogen canal ray data of Rüchardt were consulted. These data were not available at the time of writing LA-12, which is therefore superceded by this revised edition.

THE RANGE OF DEUTERONS IN HEAVY WATER AND OF PROTONS IN HYDROGEN

For the evaluation of various experiments it is necessary to know the range-energy relation for deuterons in heavy water, for deuteron energies from 0 to about 1 MV. An attempt has been made to calculate this relation theoretically. Since, in the course of the calculation, the stopping power of hydrogen (or deuterium) for deuterons or protons, was required, the range-energy for deuterons and protons in hydrogen was also obtained.

METHOD OF CALCULATION

The range R of a charged particle in a given medium is obtained from the loss in energy per cm. of path of the particle, $-dE/dx$, by performing the integration

$$R(E) = \int_0^E \frac{dE}{-\frac{dE}{dx}} \qquad (1)$$

When the stopping medium is heavy water, the energy loss per cm. will be a sum of two terms, one arising from the inelastic collisions of the charged particle with the electrons of the deuterium and the other from the inelastic collisions with the electrons of the oxygen. These contributions are treated separately.

If the velocity of the incident deuteron is large compared to the velocities of the electrons in the atoms of the medium, application of the Born approximation to the collisions is valid, and the energy loss per cm. has the form

$$-\frac{dE}{dx} = \frac{4\pi e^4}{mv^2} NB \qquad (2)$$

with

$$B = Z \log\left(\frac{2mv^2}{I}\right) \qquad (2a)$$

In this expression m is the mass of the electron, v is the velocity of the incident deuteron, N is the number of atoms per cm^3 of the medium, Ze is nuclear charge and I some average ionization potential of the atom. For deuterons being stopped in oxygen, the form (2a) for B is valid only for deuteron energies large compared to 2 MV. This expression, therefore, cannot be used in the range in which we are interested.

It has been demonstrated, however, that the Born treatment gives correct results all the way down to velocities of the incident heavy particle which are small compared with that of the atomic electron if the incident particle has a smaller charge than that of the atomic nucleus.(1) The difference from the high velocity case arises from the breakdown of a simplifying approximation which makes the summation over the electronic transitions relatively easy to carry out. More exact methods for the evaluation of this sum are therefore resorted to, and the final result expressed numerically instead of analytically.

(1) Mott, Proc. Camb. Phil. Soc., 27, 5[illegible] (1931).

Generally it is only necessary to apply these more refined methods to the calculation of the energy loss due to collisions with the K electrons. This has been done by Bethe(2). For the contribution B_K of the two K electrons to the stopping number B, defined in (2), he finds

$$B_K = 1.81 \log (3.63\,\eta) - C_K(\eta) \qquad (3)$$

where C_K is given graphically and

$$\eta = \frac{mv^2}{2Z_{eff}^2 Ry} = \frac{mE}{MZ_{eff}^2 Ry} \qquad (4)$$

Here $E = \frac{1}{2} Mv^2$ is the kinetic energy of the incident deuteron, Ry is the Rydberg energy, and Z_{eff} is the effective nuclear charge in the K shell. The quantity η is a measure of the square of the ratio of the velocity of the incident particle to the velocity of the K electron. The number 1.81 is the "effective number of K electrons" or the total oscillator strength for all transitions from the K shell to the unoccupied discrete levels and to the continuous spectrum, and is about right for the elements from carbon to aluminum. If the average excitation potential of the electrons outside the K shell is denoted by I', the total stopping number can be written

$$B = (Z - 1.81) \log \frac{2mv^2}{I'} + B_K \qquad (5)$$

OXYGEN

For oxygen, the effective nuclear charge in the K shell was taken as $8 - 0.3 = 7.7$, and the average excitation potential, I', of the L shell, guessed as about 50 volts on the basis of the empirical value of 40.3 volts for air and an estimate (from the Slater screening constants) of the effective nuclear charges in the L shells of air and oxygen. The "stopping cross section" per atom of oxygen, $\sigma_{ox.}$, or the energy loss per cm. divided by the number of atoms per cm^3

7. [illegible] and Livingston, Rev. Mod. Phys., 9, 1937, pp. 264-265.

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HYDROGEN

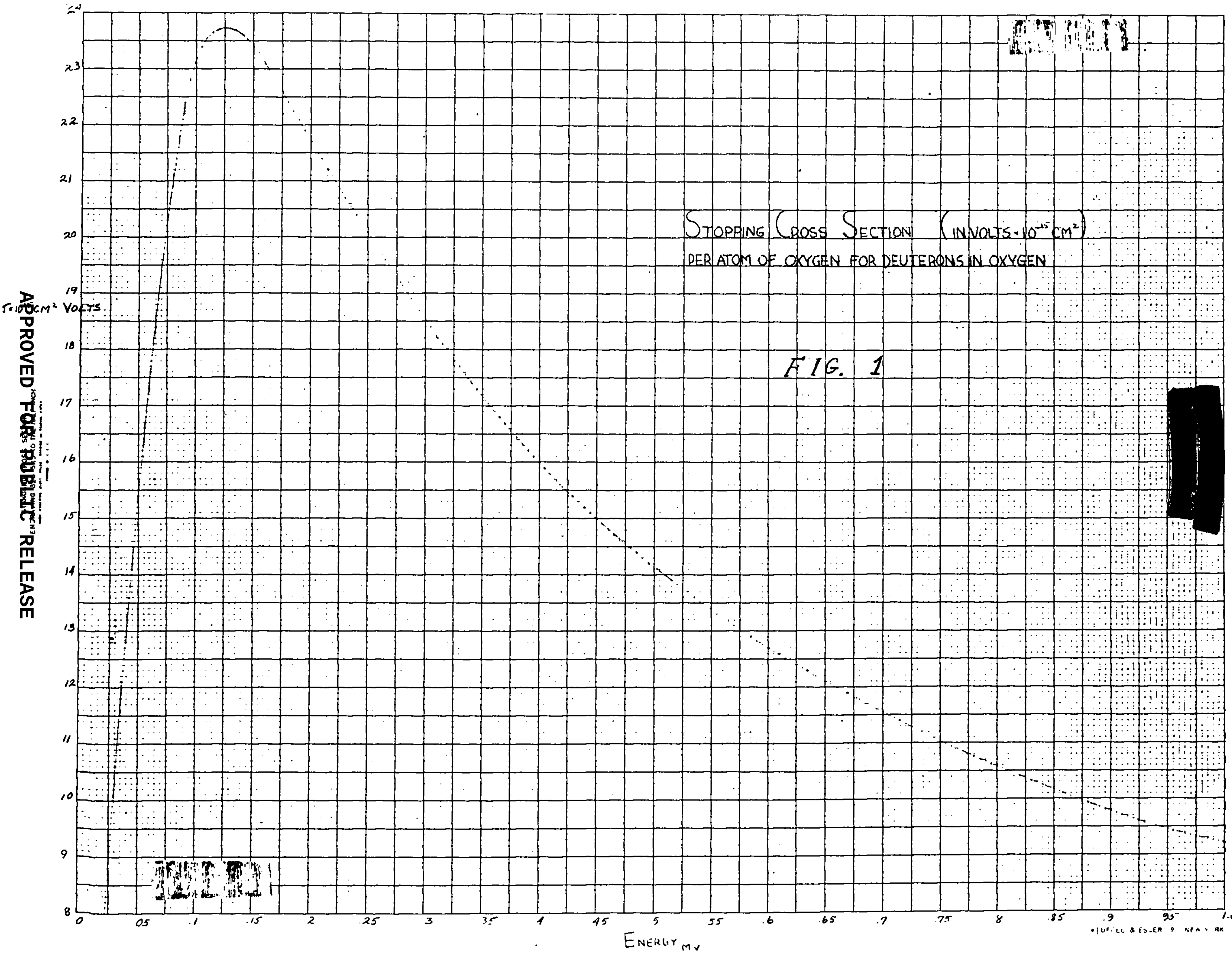
The energy loss in hydrogen (or deuterium) is due to the inelastic collisions of the deuteron with the single K electron of the atom. Since the total oscillator strength for all optical transitions from the K shell to higher states in both the discrete and continuous spectrum is unity, the best values for the stopping number B that could be obtained without making a special calculation for hydrogen, were (1/1.81) B_K, where B_K is given by (3). For Z^2_{eff} the value 1.173 was used, corresponding to an ionization potential of 1.173 Rydbergs for the removal of one of the electrons in the hydrogen molecule. The term log (3.63η) becomes asymptotically correct as the deuteron energy increases. However, the correction (1/1.81) $C_K(\eta)$ is subject to an error of the order of 10% or more since it was calculated for elements between carbon and aluminum. For this reason, the stopping cross section would be in error by about 3% at 125 kv, 1% at 250 kv, and less than 1% above 300 kv. At the lower energies there would be an additional error due to the failure to take account of the capture and loss of electrons. Below 70 kv we therefore took the experimental data of Rüchardt, which seemed more reasonable for hydrogen than for air and connected smoothly to the calculated curve above 300 kv. Thus we obtain the stopping cross section shown in Fig. 2, which we believe to be good to about 3 to 5% below 125 kv and to better than 1% above 200 kv.

RANGE

With the stopping cross section of hydrogen for deuterons, and hence also for protons, at hand, the range-energy relation for protons in hydrogen was calculated by numerical integration. Rüchardt gives experimental values for the ranges of slow protons (below 60 kv) in air and these can be converted into ranges in hydrogen by using his value of 0.4 for the stopping power of hydrogen relative to air at those energies. In this way the calculated range curve, shown in Fig. 3, was normalized to give the absolute ranges.

WATER

Finally, the stopping cross section per atom of oxygen was added to twice the stopping cross section per atom of hydrogen to obtain the stopping cross section per molecule of H_2O or D_2O expressed in units of 10^{-15} cm^2 volt. The resluts as shown in Fig. 4 may be in error by about 10% between 100 and 200 kv, 6% between 200 and 300 kv and 4% above 300 kv. The range as a function of the energy (Fig. 5) was calculated numerically and normalized by taking the range in H_2O at low energies as 1/1.9 times the range in air, where 1.9 is the value for the stopping power of water relative to air obtained from Rüchardt's data. The units used are molecules of D_2O per cm^2, one molecule of D_2O per cm^2 being equal to 3.325×10^{-20} milligrams per cm^2.


STOPPING CROSS SECTION (IN VOLTS · 10^{-15} CM^2)
PER ATOM OF OXYGEN FOR DEUTERONS IN OXYGEN
FIG. 1
$5 \cdot 10^{-15}$ CM^2 VOLTS
ENERGY MV
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
0
.05
.1
.15
.2
.25
.3
.35
.4
.45
.5
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.65
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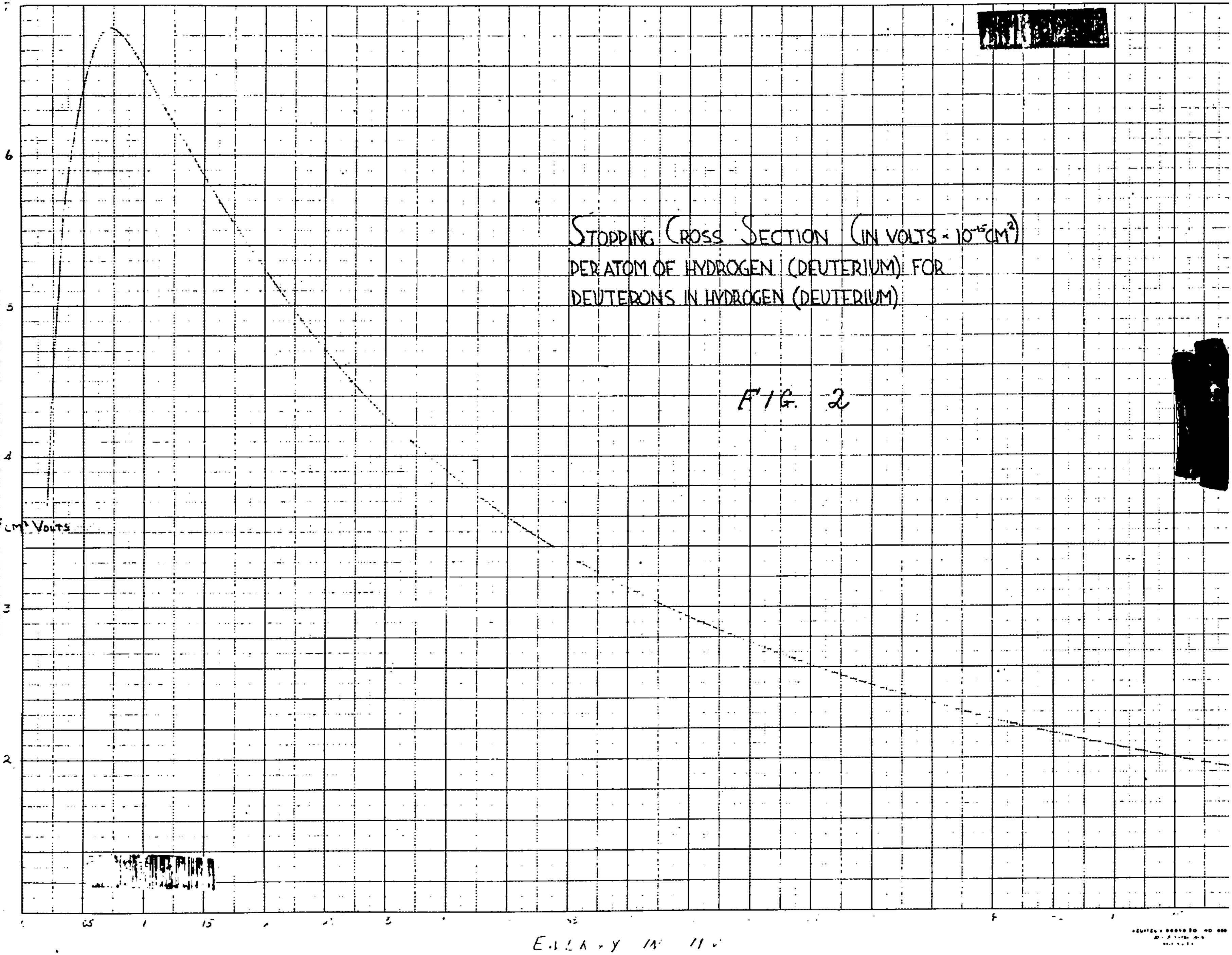

FIG. 2

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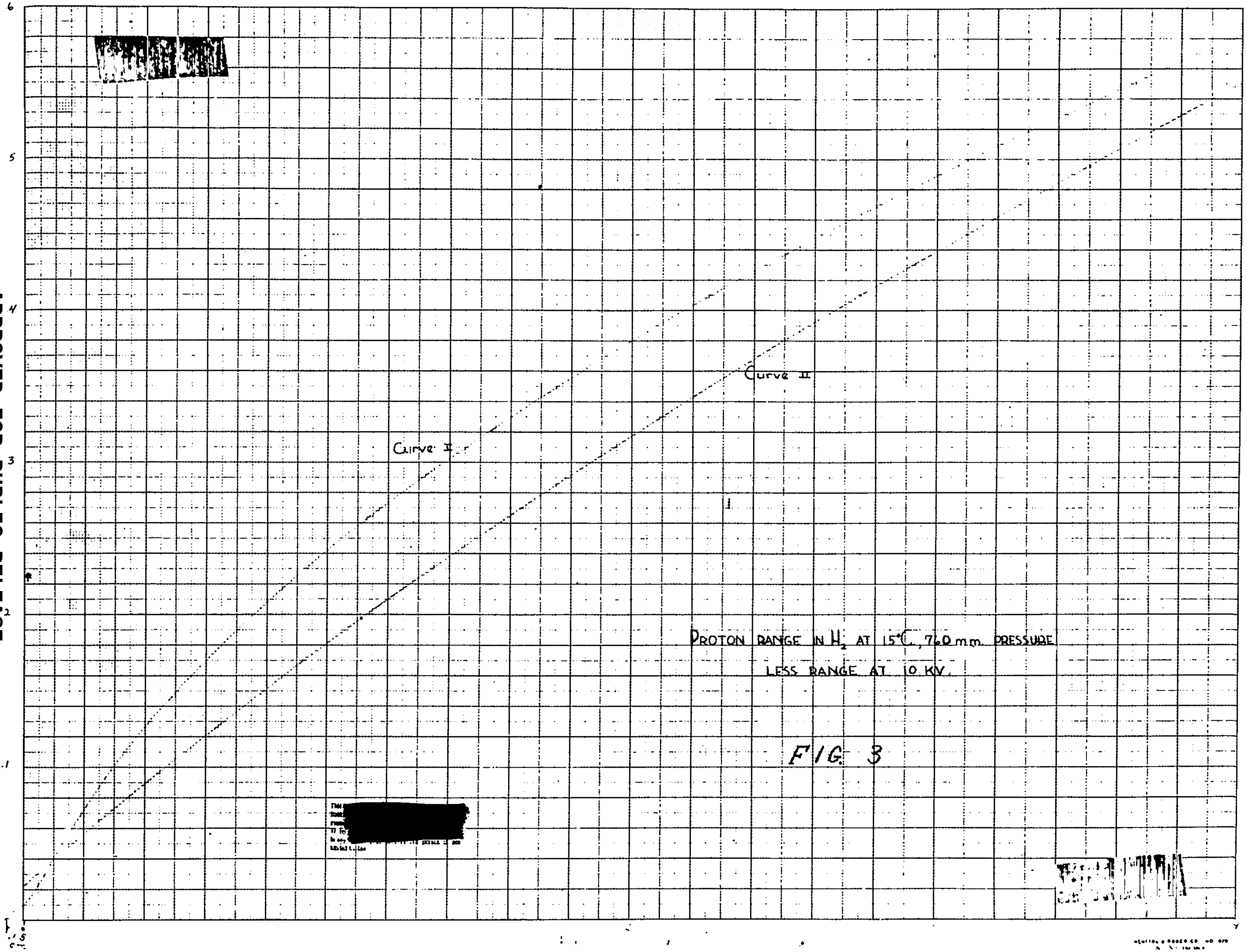
Curve I
Curve II
PROTON RANGE IN H_2 AT 15°C, 760 mm. PRESSURE
LESS RANGE AT 10 KV.
0
1
2
3
4
5
6

FIG. 3

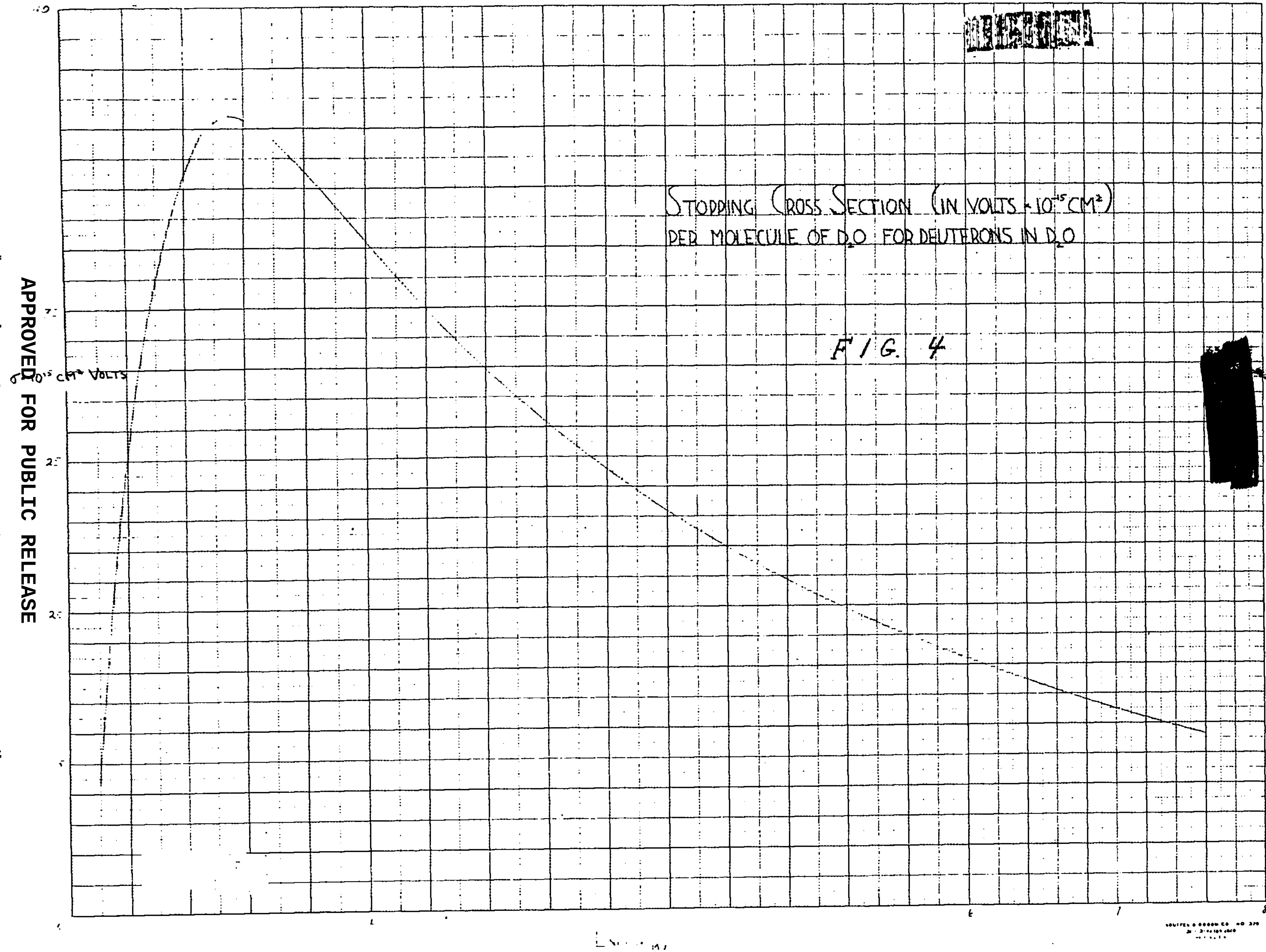

FIG. 4

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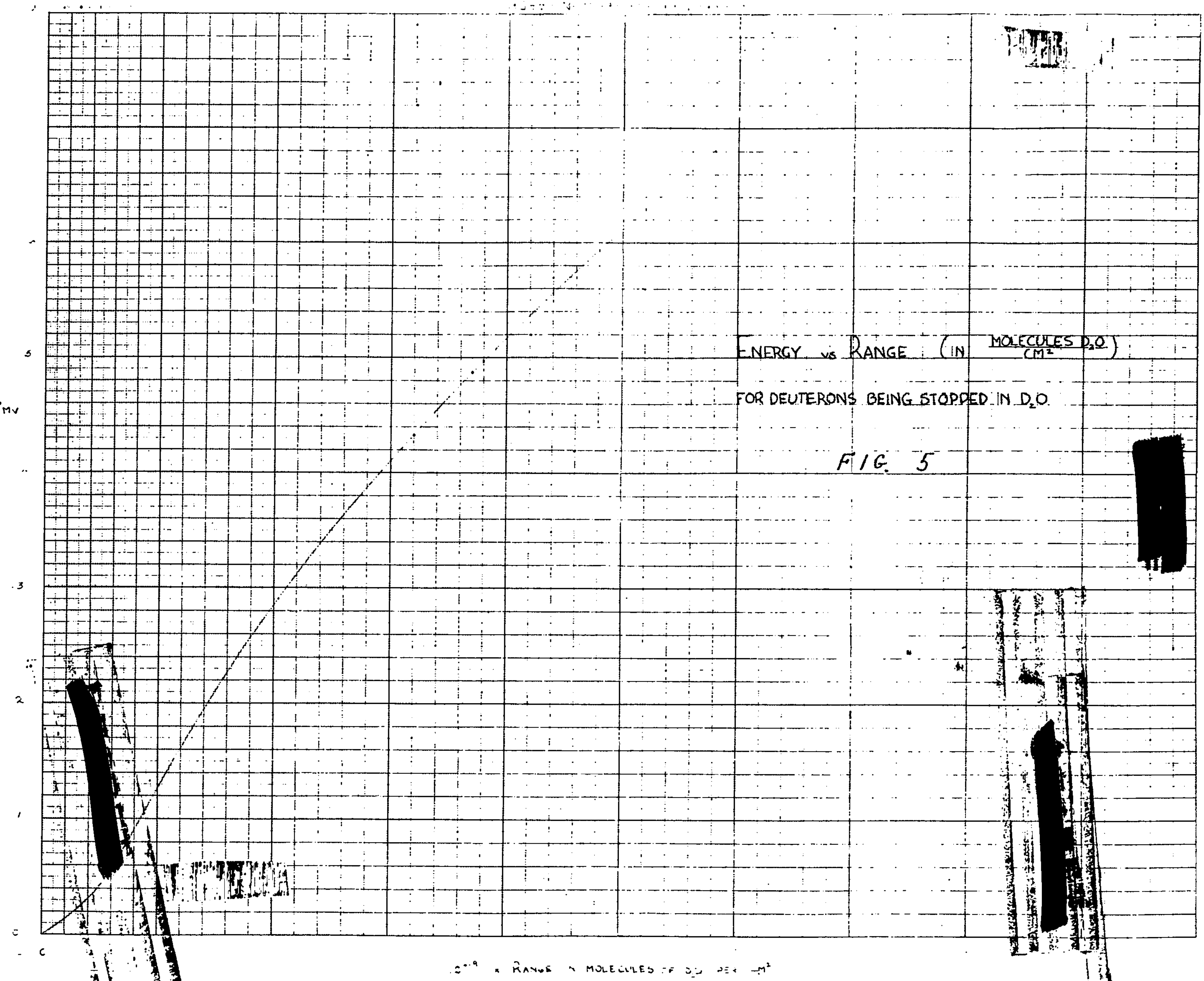

FIG. 5

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