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los alamos
scientific laboratory
of the University of California
LOS ALAMOS, NEW MEXICO 87544

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D. W. Muir

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Division of Controlled Thermonuclear Research.

SENSITIVITY OF FUSION REACTOR AVERAGE CROSS SECTIONS
TO THERMAL BROADENING OF THE 14-MeV NEUTRON PEAK

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ABSTRACT

Ion motion in a deuterium-tritium thermonuclear plasma can cause significant broadening of the 14-MeV peak in the fusion neutron energy spectrum. The shape of the peak is shown to be nearly Gaussian; the full width at half maximum FW_{DT} being related to the plasma thermal energy kT by the relation $FW_{DT} = 5.59 \sqrt{kT}$, where both FW_{DT} and kT are expressed in MeV. Numerical examples related to fusion reactor technology illustrate the sensitivity of spectrum-averaged cross sections to the plasma temperature.

I. INTRODUCTION

In recent years numerous studies have explored the possible characteristics of controlled fusion reactors.[1] Most of the concepts discussed for the first generation of these reactors are based on the D-T cycle. In this cycle, the plasma consists of an equimolar mixture of deuterium and tritium, and the reaction $T(d,n)^4He$ accounts for nearly all of the energy production. An important part of these fusion reactor studies is the calculation of the transport of the neutrons released in the D-T reaction throughout the material regions adjacent to the plasma. In addition to the space-energy distribution of the transported neutrons, one needs to know the nuclear cross sections for the production of important neutron effects, such as tritium breeding, energy deposition, gamma-ray production, activation, transmutation, helium production, and others. The cross sections for the processes listed above often exhibit threshold behavior near 14.1 MeV, the nominal energy of the source neutrons, so that it is important to know the detailed shape of the energy spectrum near the peak.

As noted by Nagle et al.,[2] the thermal motion of the interacting ions in a fusion plasma can impart a considerable energy spread to otherwise monoenergetic reaction products. For a stationary plasma with a Maxwellian distribution of ion velocities, Lehner[3] has shown that the energy distribution of the products is nearly Gaussian. In the following, we present a somewhat more transparent derivation of this Gaussian shape, and we present numerical examples which illustrate the sensitivity of threshold neutron reaction rates to the temperature of a D-T fusion plasma.

II. THE GAUSSIAN DISTRIBUTION

The probability that an ion with mass m_1 will have a velocity in the range $(\bar{v}_1, \bar{v}_1 + d\bar{v}_1)$ <u>and</u> that another ion with mass m_2 will have a velocity in the range $(\bar{v}_2, \bar{v}_2 + d\bar{v}_2)$ is just the product of two independent Maxwell distributions. This probability product can be recast in terms of the relative velocity of $\bar{v}$ and the center-of-mass velocity $\bar{V}$ of the two-ion system. Somewhat surprisingly, the resulting distribution $N(\bar{V},\bar{v})$ is also the product of two <u>independent</u> Maxwell distributions.[4]

$$N(\bar{V},\bar{v})d^3Vd^3v = P(V)d^3V \cdot p(v)d^3v \tag{1}$$

where

$$P(V) = \left[\frac{m_1 + m_2}{2\pi kT}\right]^{3/2} \exp\left[-\frac{(m_1 + m_2)V^2}{2kT}\right] \tag{2}$$

and

$$p(v) = \left[\frac{\left(\frac{m_1 m_2}{m_1 + m_2}\right)}{2\pi kT}\right]^{3/2} \exp\left[-\frac{\left(\frac{m_1 m_2}{m_1 + m_2}\right) v^2}{2kT}\right], \quad (3)$$

where V is the magnitude of $\overline{V}$, v is the magnitude of $\overline{v}$, k is Boltzmann's constant, T is the plasma temperature.

Let us introduce $\sigma(v)$, the cross section for the fusion reaction of interest. The probability distribution $R(\overline{V},\overline{v})$ for ion pairs which actually undergo a fusion reaction is just

$$R(\overline{V},\overline{v})d^3V d^3v = P(V)d^3V \cdot q(v)d^3v , \quad (4)$$

where

$$q(v) = k\sigma(v)vp(v) \quad (5)$$

and where k is chosen such that $\int q(v)d^3v = 1$.

We now proceed to a discussion of those properties of the distribution $R(\overline{V},\overline{v})$ which affect the shape of the fusion neutron energy spectrum.

The average neutron energy will be increased somewhat because of the kinetic energy E_v associated with the relative motion of the reacting ions. A calculation of the average value of E_v will require a numerical integration over the non-analytic function q(v), Eq. (5). However, this contribution (typically a few times kT) is much smaller than the nuclear energy release or Q-value (17.590 MeV in the case of the D-T reaction). Thus, for plasma temperatures in the range of 10-50 keV, it is adequate to set the average contribution equal to a constant, say 50 keV, and to lump it together with the nuclear Q-value. This procedure results in a center-of-mass neutron energy of 14.07 MeV for the D-T reaction. In addition to this effect, which slightly shifts the location of the peak, there will be a broadening of the spectrum due to the distribution q(v) of relative velocities. As shown in the following, this broadening is negligible in comparison with the effect of the distribution P(V) of center-of-mass velocities.

In order to evaluate the effect of P(V), we return to the expression for the probability distribution $R(\overline{V},\overline{v})$ given in Eq. (4). Since the distribution of relative ion velocities q(v) is isotropic, we can assume, without loss of generality, that the product neutron is emitted along the z-axis in the center-of-mass system. Let us introduce the symbol E_0 for the center-of-mass energy of the neutron and v_0 for the associated velocity. Then

$$E_0 = \frac{1}{2} mv_0^2 ,$$

m being the mass of the neutron. The energy E of the neutron in the laboratory system is

$$E = \frac{1}{2} m \left[(v_o + V_z)^2 + V_x^2 + V_z^2\right]$$

$$E \simeq \frac{1}{2} m \left[v_0^2 + 2v_0 V_z\right] = E_0 + mv_0 V_z . \quad (6)$$

Following the work of Bell,[5] we have used the fact that the neutron speed v_0 is much greater than the center-of-mass speed V. From this last result, we have

$$V_z = \frac{E - E_0}{mv_0}$$

$$dV_z = \frac{dE}{mv_0} .$$

We denote by S(E) the desired energy distribution. From Eq. (6), to first order E is a function only V_z, so that

$$S(E)dE = P_z(V_z)dV_z ,$$

P_z is obtained by factoring P(V) from Eq. (2), which gives

$$S(E) = \frac{1}{mv_0} \left(\frac{m_1 + m_2}{2\pi kT}\right)^{1/2} \exp\left[-\frac{(m_1 + m_2)(E-E_0)^2}{2m^2 v_0^2 kT}\right]$$

$$S(E) = \frac{1}{W\sqrt{\pi}} \exp\left[-\left(\frac{E - E_0}{W}\right)^2\right] \quad (7)$$

where

$$W = \left(\frac{4mE_0 kT}{m_1 + m_2}\right)^{1/2} . \quad (8)$$

Thus, S(E) is a normalized Gaussian distribution. The full width at half maximum FW is related to W by

$$FW = 2W\sqrt{\ln 2} . \quad (9)$$

For the D-T reaction $(m_1 + m_2)/m \simeq 5$ and $E_0 = 14.07$ MeV. From Eqs. (8) and (9), then

$$FW_{DT}(\text{MeV}) = 1.665\, W_{DT}(\text{MeV}) = 5.59\sqrt{kT(\text{MeV})} . \quad (10)$$

The width of this distribution is much larger than kT for most cases of interest. For example, if $kT = 20$ keV, the width of the 14-MeV neutron peak is 0.79 MeV, or a factor of forty greater than kT.

III. THRESHOLD CROSS SECTIONS WITH LINEAR ENERGY DEPENDENCE

The simple analytic nature of the function S(E) simplifies the task of cross-section averaging in certain cases. For example, we consider the case of a cross section which rises linearly above a threshold energy ε

$$\sigma(E) = \begin{cases} C(E - \varepsilon), & E > \varepsilon \\ 0, & E \leq \varepsilon \end{cases} .$$

This shape fits the gross features of many (n,2n) and (n,3n) reactions, two examples of which are discussed in the following section. The average cross section for fusion-born neutrons can be written as follows

$$\sigma_{av} \equiv \int_{\varepsilon}^{\infty} \sigma(E)\, S(E)\, dE = C \int_{\varepsilon}^{\infty} (E - \varepsilon)\, S(E)\, dE \quad .$$

From Eq. (7), then

$$\sigma_{av} = \frac{1}{W\sqrt{\pi}} \int_{\varepsilon}^{\infty} C(E - \varepsilon) \exp\left[-\left(\frac{E - E_0}{W}\right)^2\right] dE \quad . \qquad (12)$$

For convenience we introduce the parameter

$$t = \frac{\sqrt{2}}{W}(\varepsilon - E_0) \quad . \qquad (13)$$

We also introduce the tabulated function G(t), which is the area under the normal curve of error.[6]

$$G(t) \equiv \frac{1}{\sqrt{2\pi}} \int_0^t e^{-x^2/2}\, dx \qquad (14)$$

Integration of Eq. (12) then gives

$$\sigma_{av} = \frac{WC}{\sqrt{4\pi}} \left\{ e^{-t^2/2} - \sqrt{2\pi}\, t \left[1/2 - G(t)\right] \right\} . \qquad (15)$$

IV. NUMERICAL EXAMPLES

Consider the nuclear reaction $^{27}Al(n,2n)^{26m}Al$. The isomeric state of ^{26}Al decays by positron emission with a half-life of 6.4 sec. The average energy of the positron is about 1.4 MeV, and two 0.51 MeV annihilation photons are produced as well. Although the cross section for this reaction is small ($\lesssim$ 1 mb), the short range of the reaction products may lead to a degradation of the performance of the common electrical insulator, Al_2O_3. The available data for the ^{26m}Al production cross section[7] is adequately described below 16 MeV by the linear relation $\sigma(E) = C(E - \varepsilon)$, where $C = 0.276$ mb/MeV and $\varepsilon = 13.77$ MeV, and E is the neutron energy in MeV. Thus the cross section averaging formula, Eq. (15), can be used for this reaction.

Another reaction of interest in fusion reactor technology is $^{181}Ta(n,3n)^{179}Ta$. Tantalum has been considered as a possible material for a fusion reactor vacuum "first-wall," because it has a very small (n,α) cross section. This property should make tantalum very resistant to helium-induced embrittlement. ^{179}Ta production is of interest in that the half-life (600 d) is much longer than that of other activities which might be produced in tantalum structures. The cross section for the (n,3n) reaction is represented in a recent evaluation[8] as rising linearly from a threshold ε at 14.5 MeV. The slope C is 584 mb/MeV.

In order to illustrate the sensitivity of the average cross section to the plasma temperature and to the position of the threshold, we now calculate σ_{av} for the two reactions described above. The approximate average plasma temperatures for a variety of fusion reactor concepts are given in Table I (first column). In the second column are listed the widths of the 14-MeV peak, calculated from Eq. (10). The third column lists spectrum-averaged cross sections for the $^{27}Al(n,2n)^{26m}Al$ reaction ($\varepsilon = 13.77$ MeV), and the fourth column contains similar information for the $^{181}Ta(n,3n)^{179}Ta$ reaction ($\varepsilon \doteq 14.50$ MeV). All cross sections shown were calculated using the linear approximation to $\sigma(E)$ given above and the averaging formula, Eq. (15).

Summarizing the results in Table I, we see that for the ^{27}Al case, where the threshold is 300 keV below E_0 (14.07 MeV), the average cross section varies by less than 20% for changes in kT of a factor of four. For the ^{181}Ta case, where the threshold is 430 keV above E_0, the picture is quite different. Here a factor of ten change in σ_{av} is seen for this factor of four change in kT. It is expected that similar sensitivity will be observed for other high-threshold ($\varepsilon > 14$ MeV) reactions.

TABLE I

SPECTRUM-AVERAGED CROSS SECTIONS FOR DIFFERENT PLASMA TEMPERATURES

Reactor Concept	Approximate Average Plasma Temperature (keV)	Width of 14-MeV Peak (fwhm in MeV)	σ_{av} for $^{27}Al(n,2n)^{26m}Al$ (μb)	σ_{av} for $^{181}Ta(n,3n)^{179}Ta$ (mb)
Wisconsin Tokamak A[9]	11	0.59	87	2.5
Theta Pinch[10]	12	0.61	87	3.0
Princeton Tokamak[1]	15	0.68	89	5.2
Oak Ridge Tokamak[11]	20	0.79	92	9.3
Wisconsin Tokamak B[9]	28	0.94	97	16.5
Laser-Pellet[12]	40	1.12	104	27.8
D-T Mirror[1]	400	3.53	210	238.2

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