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# The Influence of Pairing on the Distribution of Independent Yield Strengths in Neutron-Induced Fission

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SUPPLEMENT

Table VI, calculated proton and neutron pairing enhancements, has been extended to six additional proposed ENDF/B-V fissionable nuclides: $^{229}Th$, $^{237}U$, $^{244}Pu$, $^{242}Cm$, $^{244}Cm$, and $^{249}Cm$. Notice of the additional nuclides was received after LA-6430-MS had gone to press.

Fission barriers for the calculations were taken from Ref. 19 of LA-6430-MS and A. Gavron, H. C. Britt, E. Konecny, J. Weber, and J. B. Wilhelmy, "$\Gamma_n/\Gamma_f$ for Actinide Nuclei using ($^3He$, df) and ($^3He$, tf) Reactions," Phys. Rev. C13, 2374 (1976).

TABLE VI (continued)

CALCULATED X AND Y VALUES

| | $E_n$ (MeV) | $X \pm \Delta X$ | $Y \pm \Delta Y$ |
|---|---|---|---|
| (L) $^{229}Th + n$ | | | |
| | 0.0 | $0.274^{+0.416}_{-0.274}$ | $0.053^{+0.090}_{-0.053}$ |
| | 0.5 | $0.170^{+0.222}_{-0.170}$ | $0.033^{+0.050}_{-0.033}$ |
| | 1.0 | $0.124^{+0.152}_{-0.124}$ | $0.024^{+0.035}_{-0.024}$ |
| | 2.0 | $0.080^{+0.095}_{-0.080}$ | $0.015^{+0.022}_{-0.015}$ |
| | 3.0 | $0.059^{+0.069}_{-0.059}$ | $0.011^{+0.016}_{-0.011}$ |
| | 8.0 | $0.026^{+0.029}_{-0.026}$ | $0.005^{+0.007}_{-0.005}$ |
| | 14.0 | $0.015^{+0.018}_{-0.015}$ | $0.003^{+0.004}_{-0.003}$ |

TABLE VI (continued)

(M) $^{237}U + n$

| | | |
|---|---|---|
| 0.0 | $1.093^{+4.493}_{-1.093}$ [a] | $0.211^{+0.883}_{-0.211}$ [a] |
| 0.5 | $0.319^{+0.519}_{-0.319}$ | $0.062^{+0.111}_{-0.062}$ |
| 1.0 | $0.187^{+0.249}_{-0.187}$ | $0.036^{+0.056}_{-0.036}$ |
| 2.0 | $0.102^{+0.123}_{-0.102}$ | $0.020^{+0.028}_{-0.020}$ |
| 3.0 | $0.070^{+0.083}_{-0.070}$ | $0.014^{+0.019}_{-0.014}$ |
| 8.0 | $0.027^{+0.032}_{-0.027}$ | $0.005^{+0.007}_{-0.005}$ |
| 14.0 | $0.016^{+0.018}_{-0.016}$ | $0.003^{+0.004}_{-0.003}$ |

[a] May be unrealistically large.

(N) $^{244}Pu + n$

| | | |
|---|---|---|
| 0.0 | --- | --- |
| 0.5 | --- | --- |
| 1.0 | $0.498^{+1.093}_{-0.498}$ | $0.096^{+0.224}_{-0.096}$ |
| 2.0 | $0.155^{+0.200}_{-0.155}$ | $0.030^{+0.045}_{-0.030}$ |
| 3.0 | $0.092^{+0.110}_{-0.092}$ | $0.018^{+0.025}_{-0.018}$ |
| 8.0 | $0.030^{+0.035}_{-0.030}$ | $0.006^{+0.008}_{-0.006}$ |
| 14.0 | $0.017^{+0.019}_{-0.017}$ | $0.003^{+0.004}_{-0.003}$ |

$E_n(X \to \infty) = 0.548$ MeV

$E_n$(one-half plateau) $\simeq 0.750$ MeV

TABLE VI (continued)

(O) $^{242}$Cm + n

| | | |
|---|---|---|
| 0.0 | $0.586^{+1.454}_{-0.586}$ [a] | $0.113^{+0.295}_{-0.113}$ [a] |
| 0.5 | $0.255^{+0.381}_{-0.255}$ | $0.049^{+0.083}_{-0.049}$ |
| 1.0 | $0.163^{+0.212}_{-0.163}$ | $0.031^{+0.048}_{-0.031}$ |
| 2.0 | $0.094^{+0.114}_{-0.094}$ | $0.018^{+0.026}_{-0.018}$ |
| 3.0 | $0.067^{+0.078}_{-0.067}$ | $0.013^{+0.018}_{-0.013}$ |
| 8.0 | $0.027^{+0.031}_{-0.027}$ | $0.005^{+0.007}_{-0.005}$ |
| 14.0 | $0.016^{+0.018}_{-0.016}$ | $0.003^{+0.004}_{-0.003}$ |

[a] May be unrealistically large.

(P) $^{244}$Cm + n

| | | |
|---|---|---|
| 0.0 | $0.150^{+0.192}_{-0.150}$ | $0.029^{+0.043}_{-0.029}$ |
| 0.5 | $0.112^{+0.138}_{-0.112}$ | $0.022^{+0.032}_{-0.022}$ |
| 1.0 | $0.090^{+0.108}_{-0.090}$ | $0.017^{+0.025}_{-0.017}$ |
| 2.0 | $0.064^{+0.076}_{-0.064}$ | $0.012^{+0.018}_{-0.012}$ |
| 3.0 | $0.050^{+0.058}_{-0.050}$ | $0.010^{+0.014}_{-0.010}$ |
| 8.0 | $0.024^{+0.027}_{-0.024}$ | $0.005^{+0.006}_{-0.005}$ |
| 14.0 | $0.012^{+0.014}_{-0.012}$ | $0.002^{+0.003}_{-0.002}$ |

TABLE VI (continued)

(Q) $^{249}$Cm + n

| | | |
|---|---|---|
| 0.0 | $0.109^{+0.133}_{-0.109}$ | $0.021^{+0.031}_{-0.021}$ |
| 0.5 | $0.088^{+0.105}_{-0.088}$ | $0.017^{+0.024}_{-0.017}$ |
| 1.0 | $0.074^{+0.087}_{-0.074}$ | $0.014^{+0.020}_{-0.014}$ |
| 2.0 | $0.055^{+0.065}_{-0.055}$ | $0.011^{+0.015}_{-0.011}$ |
| 3.0 | $0.044^{+0.052}_{-0.044}$ | $0.009^{+0.012}_{-0.009}$ |
| 8.0 | $0.022^{+0.026}_{-0.022}$ | $0.004^{+0.006}_{-0.004}$ |
| 14.0 | $0.014^{+0.016}_{-0.014}$ | $0.003^{+0.004}_{-0.003}$ |

# THE INFLUENCE OF PAIRING ON THE DISTRIBUTION OF INDEPENDENT YIELD STRENGTHS IN NEUTRON-INDUCED FISSION

by

David G. Madland and Talmadge R. England

ABSTRACT

This report is a summary of the current status of an ongoing investigation of the influence of the pairing force in the distribution of independent yields. At this time pairing systematics have been obtained for $^{235}U$ thermal and fast fission yields by a comparison of experimental data to the normal yield curves predicted by the phenomenological model. A semiempirical formalism has been developed and tested (in so far as the available data permits) by which estimates for the magnitudes of the pairing effects can be easily incorporated into the version of the phenomenological model to be used in the ENDF/B-V Fission Product Evaluated Data File. The formalism is based upon the $^{235}U$ thermal and fast fission data analysis and has been extended to other proposed ENDF/B-V fissionable nuclides. Neutron energy dependence has been incorporated (in a simple fashion) in terms of excitation energies of the compound system and measured fission barriers. Again, the energy dependence has been expressed in a manner which is easily assimilated by the ENDF/B-V yield model.

The initial efforts in this study have been governed by the need to include some quantitative description of the pairing effects within the framework of the existing phenomenological treatment. The results reported herein are thus empirically based and it is to be expected that they will change with the accumulation of more (and better) data. Hopefully this work will provide important clues to a more detailed calculation of the pairing influence as well as other theoretical work in progress.

The discussion is given in two parts. Part I summarizes the $^{235}U$ thermal and fast fission studies. Part II extends the results to 11 ENDF/B-V fissionable nuclides for neutron energies ranging from thermal to 14 MeV.

## I. PAIRING SYSTEMATICS IN $^{235}U$ THERMAL AND FAST FISSION INDEPENDENT YIELDS

Several investigators have reported experiments that show an "even-odd Z effect" in the magnitudes of the independent yields for a given mass chain when plotted against the Gaussian distribution of the phenomenological model* for that mass chain.[6-9] This fine structure effect has been observed for several mass chains in both the light and heavy peaks of the mass distribution for $^{235}U$ thermal fission. Independent yields with even-Z are 20-25% larger than the Gaussian fit while those with odd-Z are 20-25% smaller.[8] Thus, two separate Gaussians per mass chain could perhaps be used differing only in their amplitudes (-1.2 for even-Z and -0.8 for odd-Z). In fact, the ENDF/B-IV Fission Product File has, in effect, used (1 ± 0.20) Gaussian amplitudes for all of the fissionable nuclides included.[3]

Recently, fairly exhaustive studies by Amiel and Feldstein have shown the existence of even-odd Z amplitudes of (1 ± 0.22) for $^{233}U$ and $^{235}U$ thermal fission and (1 ± 0.08) for $^{235}U$ fast fission (-1.9 MeV).[8,9] They also report even-odd N amplitudes of (1 ± 0.08) and (1 ± <0.03) for, respectively, the heavy and light mass peaks in both $^{233}U$ and $^{235}U$ thermal fission. These results are based on comparisons of data compiled by Wolfsberg,[4] Amiel,[8] and others[10] to the phenomenological Gaussian model fit by Wahl[1] which uses an empirical relation for the

*The Gaussian form of the phenomenological model was first proposed by Wahl.[1] Refinements include work by England and Schenter,[2] Rider,[3] and Wolfsberg.[4] A review on the topic of yield distributions in general has been given by Pappas.[5]

most probable change, $Z_p$, and a width $\sigma$ = 0.56 ± 0.06 (yields from the Gaussian model are henceforth referred to as "normal yields"). Essentially these same data and identically the same Gaussian model have been used in the work described here.

It is well known that because of the proton and neutron pairing interactions it generally costs less energy to make even-even nuclei than odd-odd nuclei, all other factors being fixed. Even-odd nuclei and odd-even nuclei fall in between. Applying this fact to the fission process, without incorporating any explicit detail of the various saddle-to-scission models, readily provides a simple form to represent the pairing force modulation to the normal independent yields.

Let X = Fractional enhancement relative to the normal yield due to proton pairing.

Y = Fractional enhancement relative to the normal yield due to neutron pairing.

IY = Independent yield.

FIY = Fractional independent yield.

NIY = Normal independent yield.

NFIY = Normal fractional independent yield.

Then, the basic modulation equations to the normal yield model are, for each mass chain,

$$\left.\begin{aligned}(IY) &= F_i\,(NIY) \text{ or}\\ (FIY) &= F_i\,(NFIY)\quad ,\end{aligned}\right\} \quad (1)$$

where the $F_i$ are as follows:

| Z | N | $F_i$ |
|---|---|---|
| E | E | $F_1 = [1 + (X + Y)]$ |
| E | 0 | $F_2 = [1 + (X - Y)]$ |
| 0 | E | $F_3 = [1 - (X - Y)]$ |
| 0 | 0 | $F_4 = [1 - (X + Y)]$ . |

(2)

If the independent yields were measures of primary fission fragment distributions (before prompt neutron emission) then one might expect $X \approx Y$ and (for example) if $X = Y = 0.2$ then $F_1 = 1.40$, $F_2 = F_3 = 1.0$, and $F_4 = 0.60$. However, the independent yields are in fact measures of primary fission product distributions (after prompt neutron emission but before β decay). Thus, the neutron pairing effect can not be expected to be as sharp as the proton pairing because of the effects of folding in the prompt neutron distribution. We therefore maintain X and Y as distinct quantities and claim that four distinct F factors are required for each mass chain.

A second point to be made is that a calculation of the pairing influence would, in principle, determine X and Y from detailed nuclear structure considerations along the entire fission path up through the formation of the primary fission products. For a given fissionable nuclide and a fixed excitation energy of the compound system, X and Y would be functions of A, Z, N, and excitation energy (thus, shell structure) of each primary fission fragment as well as functions of A', Z, N', and excitation energy (shell structure) of each primary fission product. A plot of the deduced X and Y values against primary product mass would then presumably show variations indicative of the relative influence of the pairing force per mass channel. Studies to date,[3,4,8,9] however, assume that X is roughly constant (within the experimental uncertainties) for a given fissionable system at a fixed excitation energy (~ constant neutron energy). Accordingly, we assume simply that (X, Y) can vary with fissionable nuclide, neutron energy, and (perhaps) light or heavy mass peak of the fission product mass distribution.*

Before proceeding to the discussion of the data analysis, it is worthwhile to point out some properties related to the F factors given in Eq. (2). These are, Even/Odd Combinations:

$$\left.\begin{aligned}F_e^z &= \tfrac{1}{2}(F_1 + F_2) = 1 + X\\ F_o^z &= \tfrac{1}{2}(F_3 + F_4) = 1 - X\\ F_e^n &= \tfrac{1}{2}(F_1 + F_3) = 1 + Y\\ F_o^n &= \tfrac{1}{2}(F_2 + F_4) = 1 - Y\quad ,\end{aligned}\right\} \quad (3)$$

Normalization:

$$\tfrac{1}{4}(F_1 + F_2 + F_3 + F_4) = 1\quad ,$$

*Recent work by H. -G. Clerc and collaborators at Darmstadt and Grenoble[11] has provided a measurement of the variation of X and Y with primary product mass in the light mass peak, but only for the "most probable fragment kinetic energies." Since the present work utilizes data measured over all kinetic energies, we do not at this time include their results.

X and Y values:

$$\left.\begin{aligned} X &= \tfrac{1}{4}(F_1 + F_2 - F_3 - F_4) \\ Y &= \tfrac{1}{4}(F_1 + F_3 - F_2 - F_4) \quad , \end{aligned}\right\} \quad (4)$$

Estimated uncertainties in X and Y values:

$$\Delta X = \Delta Y = \frac{1}{4}\sqrt{\sum_{i=1}^{4} (\Delta F_i)^2} \quad . \qquad (5)$$

In Eq. (3), $F_e^z$ and $F_o^z$ are equivalent to the "even-odd Z effect" factors previously reported (see Refs. 3,4,8,9) which lump even-even and even-odd species together and odd-even and odd-odd species together, respectively.

A summary of the number of measured independent yields studied in $^{235}U$ thermal and fast fission appears in Table I. As previously stated the data set is that used by Amiel and Feldstein[8,9] plus a few additions.[3,*] The data analysis has been performed in two ways.

A. Method of Average Deviations of the Fractional Independent Yield from the Normal Fractional Independent Yield

The first method is to compute average deviations of the fractional independent yields (FIY) from the normal fractional independent yields (NFIY) for each of the four types of nuclides and for light, heavy, and total components of the mass distribution. This amounts to calculating 12 numbers together with uncertainties for both $^{235}U$ thermal and $^{235}U$ fast fission. A typical calculation of an F value is as follows [heavy mass peak, $F_1$ (even-even), $^{235}U$ thermal fission]:

$$F_1 - 1 = \frac{1}{13}\sum_{i=1}^{13}\left\{\frac{\text{FIY(E-E)} - \text{NFIY(E-E)}}{\text{NFIY(E-E)}}\right\} ,$$

(sum over heavy mass peak)

$\Delta F_1$ = ± (one standard deviation from the mean).

In this case the result is $F_1 = 1 + 0.370$ and $\Delta F_1 = \pm\ 0.192$. The other three F factors are calculated in a similar manner and (X, Y) values for the heavy mass peak are extracted using Eq. (4). Figures 1 and 2 illustrate the fractional deviations (argument of the above expression) for $^{235}U$ thermal fission lumped according to the four nuclide types and split into light and heavy mass peaks. The uncertainties range from ± 2 to 90% of the magnitudes of the individual points plotted and are not illustrated for purposes of clarity. While a clear cut (although somewhat scattered) case exists for the even-odd Z pairing influence over both mass peaks, the even-odd N pairing effect is strongly evident only in the heavy mass peak.

Results of this first method of calculation for thermal fission of $^{235}U$ are tabulated in Table II (A) and illustrated in Figs. 3 and 5. Figure 3 shows the four values of $(F_i - 1)$ for light mass peak, heavy mass peak, and total. Although the uncertainties are quite large (~50% of the magnitude of $F_i$ - 1) tendencies do exist for even-Z even-N yields to be larger than even-Z odd-N yields and odd-Z odd-N yields to be smaller than odd-Z even-N yields. Thus, the data support the basic assumptions contained in Eq. (2). The data for the light mass peak, however, do not demonstrate the fine differences, but only the even-odd Z pairing influence. These results are more clearly shown in Fig. 5 wherein the extracted X and Y values are plotted (open symbols for present discussion). While the X values (Z pairing) are in reasonable agreement for the three mass ranges, there does appear to be evidence that proton pairing may be stronger in the heavy mass peak. The Y values (N pairing) indicate significant neutron pairing only in the heavy mass peak and total mass range calculations. Note, however, that the error bars do overlap for both X and Y over the three mass ranges calculated. For the total mass range, X = 0.264 ± 0.145 and Y = 0.041 ± 0.145.

The results for $^{235}U$ fast fission (~1.9 MeV) are tabulated in Table III (A) and illustrated in Figs. 6 and 8. Here the data are far more sparse (see Table I) and less well determined. In the light mass peak the data are insufficient to calculate $F_4$ and there is only a single datum for $F_1$

*Error estimates for $^{235}U$ fast fission chain yields were taken from the review article by J. G. Cuninghame.[12]

($^{92}$Kr, with a large assigned uncertainty of ± 92%) which, incidentally, predicts the wrong sign for X. Thus, F factors, and X and Y values can only be calculated for the heavy mass peak and the (sparse) total mass range. Figure 8 indicates that $X \approx 0.075 \rightarrow 0.100$ and $Y \approx 0 \rightarrow 0.025$ or about 1/3 to 1/2 of the thermal fission values. Clearly, more fast fission independent yield data are needed. The present data, over the whole mass range, give X = 0.078 ± 0.077 and Y = -0.004 ± 0.077.

B. Method of the Deviation of the Summed Independent Yield from the Summed Normal Independent Yield

The results presented thus far have utilized a calculational approach which places equal weight on all data (and data uncertainties) by the use of deviations between fractional independent yields and normal fractional independent yields. That is, equal weight has been given to points on the tail and the peak of the isobaric yield distribution for each mass value. The second method of analysis amounts to weighting the fractional independent and normal fractional independent yield by the experimental mass chain yield, for each mass value. In other words, the deviation between the independent yield (IY) and the normal independent yield (NIY) is systematically investigated. The major difference between this method and that already discussed is that the weighting of a given primary fission product is proportional to its frequency of occurrence in the fission process. A direct consequence of this method is that the errors in the computed F factors and extracted X and Y values will be smaller than the previous method. This is, of course, due to the experimental fact that chain yields and independent yields are generally most well determined when they are large. A typical calculation of an F value is as follows [heavy mass peak, $F_1$ (even-even), $^{235}$U thermal fission]:

$$F_1 - 1 = \left\{ \frac{\sum_{i=1}^{13} IY(E\text{-}E) - \sum_{i=1}^{13} NIY(E\text{-}E)}{\sum_{i=1}^{13} NIY(E\text{-}E)} \right\}$$

(sum over heavy mass peak)

$$\Delta F_1 = \pm \left\{ \frac{\sqrt{\sum_{i=1}^{13} [\Delta IY(E\text{-}E)]^2}}{\sum_{i=1}^{13} NIY(E\text{-}E)} \right\} .$$

(sum over heavy mass peak)

In this case the result is $F_1$ = 1 + 0.303 and $\Delta F_1$ = ± 0.042 whereas in the previous method we found $F_1$ = 0.370 and $\Delta F_1$ = ± 0.192. The other three F factors are calculated in a similar manner and (X, Y) values for the heavy mass peak are extracted using Eq. (4).

Results of this second method of calculation for thermal fission of $^{235}$U are tabulated in Table II (B) and illustrated in Figs. 4 and 5. Figure 4 shows the four values of ($F_i$ - 1) for light mass peak, heavy mass peak, and total. The uncertainties are typically ~20% of the magnitude of ($F_i$ - 1) whereas they are ~2½ times larger using the previous method.* The conclusions reached with the previous method (see Fig. 3 and discussion), however, are more firmly verified. Namely, the formalism of Eq. (2) is consistent with the data, proton pairing effects (X) dominate over neutron pairing effects (Y), proton pairing effects are somewhat stronger (~10-15%) in the heavy mass peak compared to the light mass peak, and neutron pairing effects are much stronger in the heavy mass peak. The extracted X and Y values are plotted in Fig. 5 (closed symbols). For the total mass range, X = 0.228 ± 0.034 and Y = 0.044 ± 0.034 to be compared with 0.264 ± 0.145 and 0.041 ± 0.145, respectively, by the previous method.

The results for $^{235}$U fast fission are tabulated in Table III (B) and illustrated in Figs. 7 and 8. Again, the fast fission analysis suffers from both data quantity and data quality. In contrast to the first method of calculation [Fig. 6 and Table III (A)] however, this method indicates a small positive,

*Uncertainties in $F_i$ for the first method of calculation (average deviation of the FIY from the NFIY) were equated to ± (one standard deviation from the mean), i.e., square root of the variance. Uncertainties for $F_i$ in the second method of calculation (deviation of summed IY from summed NIY) were obtained by combining in quadrature the error assignments for the IYs in the sum.

neutron pairing enhancement. For the total mass range, X = 0.078 ± 0.063 and Y = 0.015 ± 0.063 where as previously, X = 0.078 ± 0.077 and Y = - 0.004 ± 0.077.

### C. Conclusions and Recommendations for $^{235}U$

By inspection of Tables II and III and Figs. 5 and 8 it is evident that both methods of analysis give the same results, within the estimated uncertainties, for the extracted (X, Y) relative pairing enhancements. This implies (although somewhat weakly) that the pairing effects are of equal strength throughout the isobaric yield distribution, for any given mass. On the other hand, systematic differences appear to exist between light and heavy mass peak results, namely, proton pairing effects are slightly stronger in the heavy mass peak and neutron pairing effects are much weaker in the light mass peak. Because there is (a) no obvious reason to expect the neutron pairing effect to be selective according to light or heavy mass region* and (b) because the proton pairing effect is almost equal (to within ~15%) in both mass regions, it is reasonable to adopt (X, Y) sets deduced from the entire mass range, that is, the last column in Tables II and III. It is obvious that the second method of analysis (deviation of summed independent yield from summed normal independent yield) gives the most precise results because the largest yields, with generally the smallest uncertainties, are given the most weight. *Thus, it is recommended that (X, Y) sets be used from the last column of Table II (B) and Table III (B).*

In closing this section we note that $F_e^z$ and $F_o^z$ values [Eq. (3)] have been separately calculated from the data and tabulated in Tables II and III for comparison. Their averaged difference, $\langle F_z \rangle - 1$, is the quantity which has most often been quoted in the literature as a measure of the pairing influence (see Refs. 3,4,8,9).

*In fact, a neutron pairing enhancement has been observed in the light mass peak for $^{235}U$ thermal fission when the measurements are confined to fission products of the most probable energy. This observation has been made by the Darmstadt-Grenoble collaboration.[11]

## II. EXTENSION TO OTHER FISSIONABLE NUCLIDES

In this section proton and neutron pairing effects are estimated for 11 fissionable nuclides as a function of incident neutron energy. As in Sec. I these effects are represented by fractional quantities, X (proton pairing) and Y (neutron pairing), which modulate the unit amplitude of the normal (Gaussian) yield model.

### A. Development of Semiempirical Relationship to Estimate Pairing Effects in Independent Yield Strengths

The first step in deciding possible alternatives for the calculational approach is to survey existing data evaluations which have led to a measure of the pairing influence. A summary of several pertinent studies is contained in Table IV. The following observations can be made from the table.

1. Comparable sets of X and Y values from different sources are in agreement, that is, the tabulated data are reasonably consistent within the quoted uncertainties.

2. Both X and Y decrease rapidly with increasing excitation energy of the compound system.*

3. In the four reported cases of (X, Y) pairs (three at thermal energy and one at fast energy) the ratio Y/X is constant within the quoted uncertainties.

4. For thermal fission, X($^{239}Pu$, $^{241}Pu$) < X($^{233}U$, $^{235}U$).

5. The X value for fast fission of $^{232}Th$ is a factor ~4 larger than any other tabulated fast fission X value.

It seems clear that the decreasing magnitude of the pairing effect is strongly correlated with the increasing excitation energy of the compound system, and indeed, similar behavior occurs in realistic single-particle calculations of level densities and fission probabilities.[16] For a given excitation energy, however, the compound system can generally decay by several different channels (neutron emission, alpha decay, or fission, for example)

*This statement is reinforced by the fact that Nethaway[15] in an analysis of combined 14-MeV data from several even-Z actinides found $F_e^z - 1 = -0.02 \pm 0.03$ and $F_o^z - 1 = -0.04 \pm 0.08$.

each having the same initial condition of the energy. To distinguish channels we note that in this work the fission channel is already presumed open. *Hence, the pertinent excitation energy is really the available excitation energy with respect to the fission barrier.*

Let $E^*$ = Excitation energy of the compound system due to absorption of a zero-energy neutron,

$E_n$ = Incident neutron energy, and

$E_a$, $E_b$ = Inner, outer fission barrier heights in the double-humped barrier model.[17]

Then, the available excitation energy† with respect to either the inner barrier ($\varepsilon_a$) or the outer barrier ($\varepsilon_b$) is just

$$\varepsilon_{a,b} = (E^* + E_n) - E_{a,b} \quad . \qquad (6)$$

Values of $E^*$, $\langle E_a \rangle$, and $\langle E_b \rangle$ are listed in Table V for 13 fissionable nuclei to be included in ENDF/B-V. The $E^*$ values were calculated from experimental masses tabulated by Wapstra[18] and the barrier heights are averages of experimental values determined by Britt et al.[19]

Assuming the existence of a correlation between excitation energy, $\varepsilon_{a,b}$, and the pairing effect, X, it remains (a) to determine whether the use of the inner barrier or the outer barrier is more appropriate for our purpose, (b) to determine a suitable functional relationship between $\varepsilon_{a,b}$ and X, and (c) having determined X to then determine Y. Guidelines in accomplishing the above will be the previously mentioned observations from the data of Table IV together with the data of Table V.

We assume that the relationship between $\varepsilon_{a,b}$ and X can be approximated by expressions which are either linear ($\varepsilon_{a,b} = -a_1X + a_2$), exponential [$\varepsilon_{a,b} = a_3 \exp(-a_4X)$ or $\varepsilon_{a,b} = a_5 \exp(-a_6\sqrt{X})$], or hyperbolic ($\varepsilon_{a,b} + a_7 = a_8/X$). Features common to the four forms are that there are two adjustable parameters in each and that as $\varepsilon_{a,b}$ increases, X decreases. The distinguishing feature between them is the rate at which X decreases.

The decision as to which barrier should be used in Eq. (6) can be made by invoking observation (4) as a constraint (see above discussion on Table IV): for thermal fission, $X(^{239}Pu, ^{241}Pu) < X(^{233}U, ^{235}U)$. If the inner barrier, $E_a$, is used in Eq. (6) and values of $X(^{235}U$ thermal$) = 0.22$ and $X(^{235}U$ fast, 1.9 MeV$) = 0.08$ are used to determine the parameters, $a_i$, the following results are obtained for thermal fission of plutonium:

| Form | $X(^{239}Pu)$ | $X(^{241}Pu)$ |
|---|---|---|
| Linear | 0.22 | 0.23 |
| Exponential (X) | 0.23 | 0.24 |
| Exponential ($\sqrt{X}$) | 0.24 | 0.26 |
| Hyperbolic | 0.23 | 0.25 |

Thus, by defining the available excitation energy, $\varepsilon_a$, with respect to the inner barrier, $E_a$, one finds $X(^{239}Pu, ^{241}Pu) \geq X(^{233}U, ^{235}U)$ which violates the trend in the data or, at best, is marginal. If the calculations are repeated using the outer barrier, $E_b$, in the expression for $\varepsilon_b$ the results are essentially reversed and the inequality becomes $X(^{239}Pu, ^{241}Pu) \leq X(^{233}U, ^{235}U)$ which agrees with the trend in the data or, at worst, is marginal. The outer barrier, $E_b$, is therefore preferred. This result, although based upon sparse data, is encouraging because it is generally believed that asymmetric mass division is strongly coupled to the reflection asymmetric shape in the potential energy surface at (and beyond) the outer barrier.[20,21] The inference, of course, is that the mass split is decided at the outer barrier (or slightly beyond). If this is the case, it is not unreasonable to suspect that the pairing forces exert their influence upon the even-odd character of the nascent fragments when the system has evolved to the outer barrier in the energy surface.

Using the outer barrier in Eq. (6), calculations of X values for the ENDF/B-V nuclides listed in Table V were attempted for each of the proposed functions. The linear function was discarded because it yields negative X values for $E_n > -4$ MeV when thermal and fast fission $^{235}U$ values from Table IV* are used to determine the parameters $a_1$ and $a_2$. This problem

†Neglecting the recoil energy correction which is typically -100 keV for 14 MeV neutrons on uranium.

*Note: The most trustworthy data of Table IV are probably those for thermal and fast fission of $^{233}U$ and $^{235}U$.

could be circumvented by calculating only thermal X values and scaling the results according to the energy dependence of the peak-to-valley ratio of the mass distribution. The difficulty in this approach is that this energy dependence is not well known for most of the ENDF/B-V nuclides and even if it were, it is not clear whether the results would be more (or less) realistic than the approach being used. The exponential (X) function was rejected for the same reason, that is, negative X values occur for $E_n > -5$ MeV. However, neither the exponential ($\sqrt{X}$) function nor the hyperbolic function display this property for higher neutron energies so that either of these forma could be adopted. Of the two, the hyperbolic function was chosen for the calculations because it gives slightly better agreement with the fast fission data point for $^{232}$Th (see Table IV).

Having determined how the X (proton pairing) values are to be calculated, the Y (neutron pairing) values are extracted by a simple argument. As previously noted, the ratio Y/X is constant, within quoted uncertainties, for the thermal and fast fission (X, Y) pairs reported in Table IV. It is therefore assumed that Y is proportional to X, that is, $Y = \alpha X$. Using the recommended (X, Y) values for $^{235}$U fission from Sec. I.C, (also in Table IV) one finds $\alpha = 0.193 \pm 0.152$, ($^{235}$U thermal fission), and $\alpha = 0.192 \pm 0.821$, ($^{235}$U fast fission).† The close agreement for the two energies is perhaps fortuitous; nevertheless, until more data become available it will be assumed that α is independent of fissionable species as well as neutron energy and that its value is 0.193 for all isobaric chains. It is acknowledged that detail such as the onset of second and third chance fission, or the energy dependence of the prompt neutron distribution, may well produce an energy and mass dependent α.

B. Calculation of Pairing Effects

Applying the conclusions of Sec. II.A, the X values together with their uncertainties, ΔX, are calculated by assuming a hyperbolic relationship between X and the excitation energy of the compound system relative to the outer fission barrier:

$$(\varepsilon_b + c) = k/X \quad , \tag{7}$$

where $\varepsilon_b$ is calculated using Table V values for $E^*$ and $\langle E_b \rangle$ in Eq. (6), and ($k \pm \Delta k$), ($c \pm \Delta c$) are determined by using the recommended X values obtained in the $^{235}$U thermal and fast fission data analysis [last column of Table II (B) and Table III (B)], namely,

X (thermal) = 0.228 ± 0.034, and
X (fast, 1.9 MeV) = 0.078 ± 0.063, resulting in

$k = 0.225 \pm 0.259$, and
$c = 0.182 \mp 0.789$ .

The Y values are given by

$$Y = \alpha X = (0.193 \pm 0.152)X \quad . \tag{8}$$

The uncertainties in X and Y are given by

$$\frac{\Delta X}{X} = \sqrt{\left(\frac{\Delta k}{k}\right)^2 + X^2\left(\frac{\Delta\varepsilon_b^2 + \Delta c^2}{k^2}\right)} \quad \text{, and} \tag{9}$$

$$\frac{\Delta Y}{Y} = \sqrt{\left(\frac{\Delta\alpha}{\alpha}\right)^2 + \left(\frac{\Delta X}{X}\right)^2} \quad . \tag{10}$$

Calculations were performed for all even-Z nuclides listed in Table V for neutron energies of 0.0, 0.5, 1.0, 2.0, 3.0, 8.0, and 14.0 MeV. These are $^{232}$Th, $^{233,234,235,236,238}$U, and $^{238-242}$Pu. The resulting X and Y values, together with estimated uncertainties, are tabulated in Table VI.

In Table VI, uncertainties in X and Y with the positive sign were calculated with Eqs. (9) and (10), respectively. Uncertainties with the negative sign were calculated similarly or are the maximum possible uncertainties such that the ranges in X and Y never include negative values. This is consistent with the definitions of X and Y as pairing enhancements (see discussion preceeding Eq. 1) and with the defining equations for the $F_i$ in which X and Y are positive quantities [see Eq. (2)].

There are six nuclides in Table VI for which X and Y values have not been tabulated at lower values

†The value of α would be larger if the results of Amiel and Feldstein[9] were used (Table IV), but it must be remembered that their quoted Y value (0.08) refers to an analysis of the heavy mass peak alone.

of the incident neutron energy ($^{232}Th$, $^{234,236,238}U$, and $^{240,242}Pu$). This comes about in the following manner. Inspection of Eqs. (6) and (7) indicate that it is possible for X to approach an infinite value wherever

$$(E^* - E_b) + E_n + c \rightarrow 0, \text{ where} \qquad (11)$$

all quantities have been previously defined and c = (0.182 ± 0.789) MeV. This occurs for a positive value of the neutron energy, denoted by $E_n(X \rightarrow \infty)$, provided a negative value exists for $(E^* - E_b)$. As Table V illustrates, $(E^* - \langle E_b \rangle)$ is negative for the six nuclides listed above. For these cases, the value of $E_n(X \rightarrow \infty)$ is listed in Table VI. It is clear that X is negative for $E_n < E_n(X \rightarrow \infty)$. These X values are unphysical and are therefore not tabulated in Table VI.

Large values of X, which are also unphysical, occur because Eq. (7) is singular in the quantity $(\varepsilon_b + c)$. In the spirit of the model embodied by Eqs. (6) and (7), a large value of X corresponds to the situation in which the pairing influence is dominant, that is, the energy associated with the even-odd character of the mass split (or some function of that energy) is large with respect to $(\varepsilon_b + c)$. It is therefore of interest to compare the $E_n(X \rightarrow \infty)$ values with the neutron fission thresholds, $E_n$ (threshold), because $(\varepsilon_b + c)$ is smallest at threshold. One finds that $E_n$ (threshold) = $E_n(X \rightarrow \infty) \pm \Delta c$ for five of the six cases (the threshold energies[22] are also listed in Table VI). Thus, in this idealized model one would expect the largest pairing effects in independent yields to appear at, or slightly above, threshold.

Finally, for comparison to $E_n$ (threshold) and $E_n(X \rightarrow \infty)$, the neutron energy at which the fission cross section reaches one-half of the plateau value for first chance fission[22] is also listed in Table VI, and is denoted by $E_n$ (one-half plateau).

C. Conclusions and Recommendations

A comparison of the pairing effects reported in Table IV to the calculated values in Table VI follows (the $^{235}U$ thermal and fast fission data have not been included because of their use in the calculation).

1. $^{232}Th$ (fast). The reported value is X - 0.35 and the calculated value is X = 0.327.

2. $^{233}U$ (thermal). The average of the measured X values is 0.198 and the calculated value is 0.210. The measured value of Y is 0.08 and the calculated value is 0.041.†

3. $^{233}U$ (fast). The measured value is X - 0.08 and the calculated value, at 2 MeV, is X = 0.073.

4. $^{238}U$ (fast). The reported value is X - 0.08 and the calculated value, at 2 MeV, is X = 0.329.

5. $^{239}Pu$ (thermal). The average of the measured X values is 0.121 and the calculated value is 0.171.

6. $^{241}Pu$ (thermal). The reported value is X - 0.09 and the calculated value is X = 0.206.

7. 14-MeV Data. The two reported values are X - 0 and the two calculated values are X = 0.015.*

Although the agreement between the data and the calculation is quite acceptable, a few remarks are in order. First, a "measured" value from Table IV is really an indirect measurement based upon an assessment of sets of independent yield data points or, in some cases, is a reported estimate. Second, the phrase "average of measured values" from Table IV means the average of the absolute values. This type of average has been used because of the various ways in which X is defined (see footnotes to Table IV). Third, although the uncertainties in the calculated X and Y values have not been transferred from Table VI to the above summary, they overlap the "measured" Table IV value in every case. It should be noted, however, that the magnitude of these uncertainties is large due to the fact that only two data points were used to determine the constants in Eq. (7).

Calculations have not been done for the $^{241}Am$ + n and $^{243}Am$ + n systems listed in Table V. These are both odd-Z odd-N compound systems, hence, if one considers formation of complementary primary fragment pairs, Z must be odd in one fragment and even

†The measured value of 0.08 is due to an analysis of the heavy mass peak alone[9] while the calculated value of 0.041 is based upon the entire mass range in the $^{235}U$ data analysis of Sec. I.

*A realistic calculation of pairing effects at 14 MeV would account for second and third chance fission processes. In the present 14-MeV calculation the agreement is due to the fact that the model demands that pairing effects "wash out" with increasing excitation energy.

in the other. This is also true for the subsequent complementary primary product pairs. The proton pairing effect should therefore vanish as there exists only a single option in the even-odd character of the charge division. This same argument could be equally applied to the neutron pairing effect for these two cases were it not for the effects of prompt neutron emission at or before scission.

For compound systems of an even-Z even-N character four even-odd options exist in the formation of complementary primary fragment pairs. The four options constitute two sets of equal and symmetric options for protons and neutrons, separately. This is the system which is most appropriate to the formalism of Eq. (2), hence, we place the most confidence in Table VI values for the following four cases: $^{233}U + n$, $^{235}U + n$, $^{239}Pu + n$, and $^{241}Pu + n$.

The remaining seven cases in Table VI have even-Z odd-N compound systems, thus two even-odd options exist in the formation of complementary primary fragment pairs. These are two odd-Z fragments or two even-Z fragments. Since the choices are with respect to Z, the preceding comment holds for the primary products as well. Thus, we have no reason to suspect the X (proton pairing) values for these seven cases. However, the Y (neutron pairing) values should probably be considered as upper limits because an odd-N compound system allows only a single option in the even-odd character of the division of N for complementary fragment pairs. How the magnitude of Y might be modified by the time that the fragments become products is not known for even-Z odd-N compound systems. Note, for example, that an even-Z odd-N compound system becomes an even-Z even-N compound system by the emission of a neutron at the scission point.

In conclusion, a sample calculation of F factors is presented, using X and Y values from Table VI. Consider fast fission of $^{232}Th$ at 2 MeV. From Table VI one finds X = 0.327 (+ 0.540, - 0.327) and Y = 0.063 (+ 0.116, - 0.063). Thus, with the use of Eq. (2), one obtains

$F_1$ (even-even) = 1 + 0.390
$F_2$ (even-odd) = 1 + 0.264
$F_3$ (odd-even) = 1 - 0.264
$F_4$ (odd-odd) = 1 - 0.390

The estimated uncertainty in $F_i$ is given by

$$\Delta F_i = \pm \sqrt{\Delta X_{max}^2 + \Delta Y_{max}^2} \quad , \text{ for all i.}$$

In this example, $\Delta F_i = \pm 0.552$.

Use of these four F factors in the normal yield model then requires renormalization to the mass chain yield.

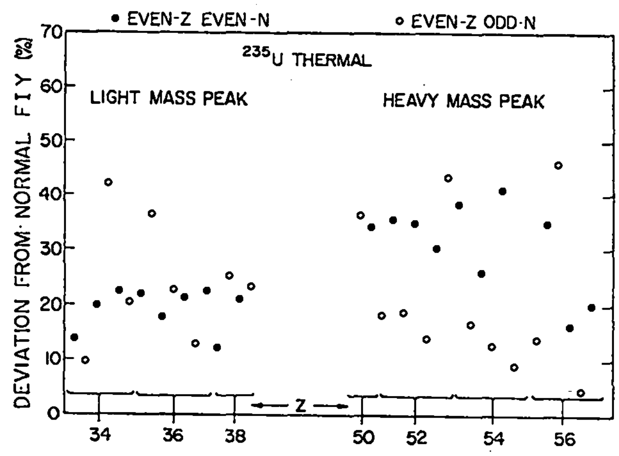


Fig. 1. Deviation of the measured FIY from the NFIY for individual even-even and even-odd fission products in $^{235}U$ thermal fission. Experimental uncertainties on the individual points range from ± 2 to 90% and have not been plotted.

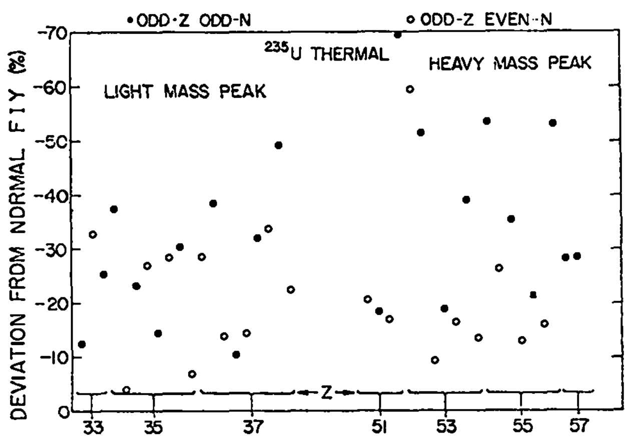


Fig. 2. Deviation of the measured FIY from the NFIY for individual odd-even and odd-odd fission products in $^{235}U$ thermal fission. Experimental uncertainties on the individual points range from ± 2 to 90% and have not been plotted.

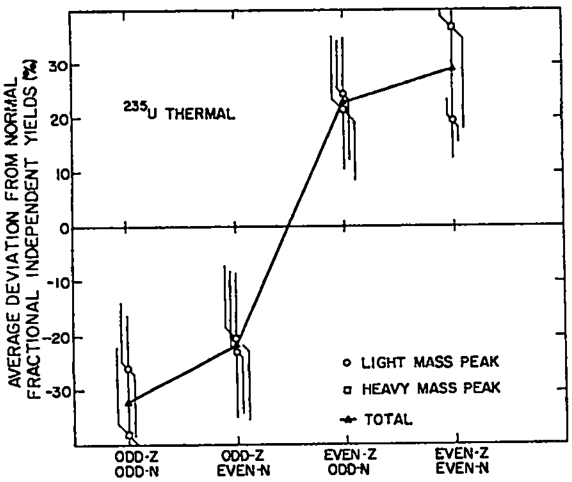


Fig. 3. Average of the deviations of measured FIYs from NFIYs for odd-odd, odd-even, even-odd, and even-even fission products, and for the light mass peak, heavy mass peak, and the total mass distribution, in $^{235}U$ thermal fission.

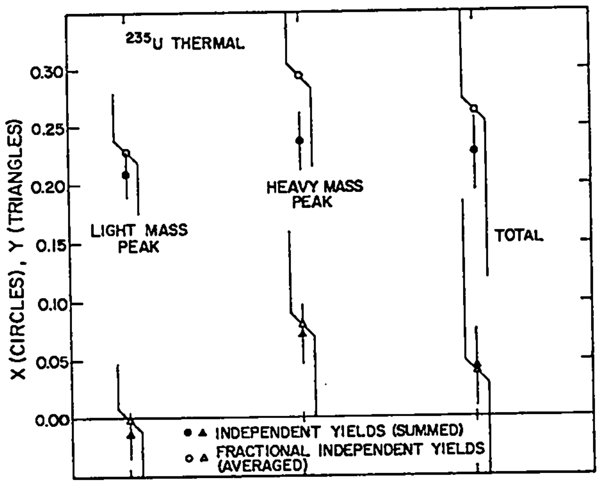


Fig. 5. Extracted pairing effects, X (protons) and Y (neutrons), for the light mass peak, heavy mass peak, and total mass distribution, in $^{235}U$ thermal fission. Results are shown in open symbols for the first method of analysis and closed symbols for the second method of analysis.

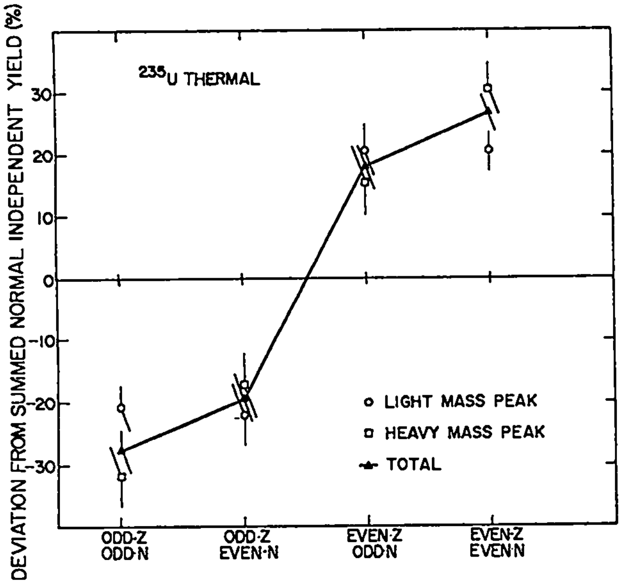


Fig. 4. Deviation of the summed IY from the summed NIY for odd-odd, odd-even, even-odd, and even-even fission products, and for the light mass peak, heavy mass peak, and the total mass distribution, in $^{235}U$ thermal fission.

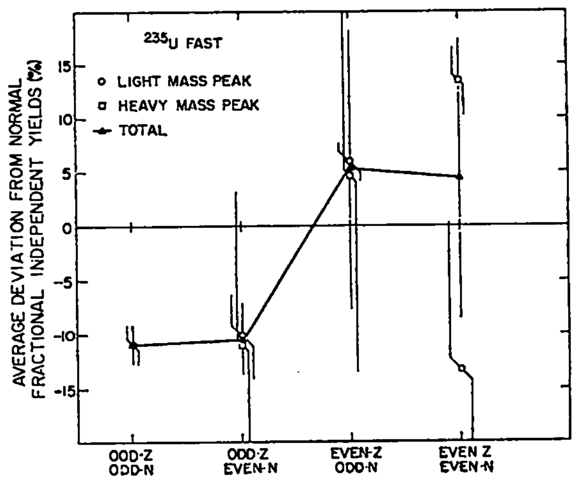


Fig. 6. Average of the deviations of measured FIYs from NFIYs for odd-odd, odd-even, even-odd, and even-even fission products, and for the light mass peak, heavy mass peak, and the total mass distribution, in $^{235}U$ fast fission.

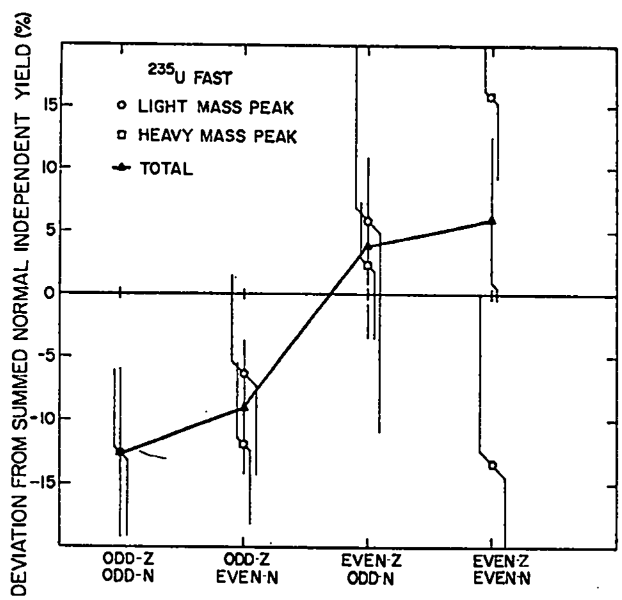


Fig. 7. Deviation of the summed IY from the summed NIY for odd-odd, odd-even, even-odd, and even-even fission products, and for the light mass peak, heavy mass peak, and the total mass distribution, in $^{235}U$ fast fission.

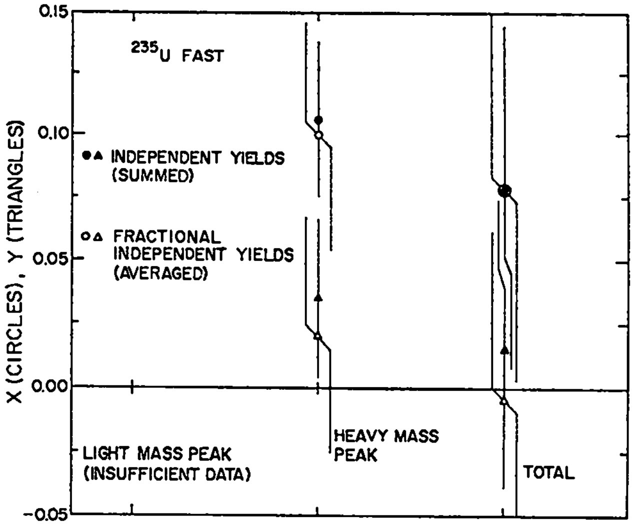


Fig. 8. Extracted pairing effect, X (protons) and Y (neutrons), for the light mass peak, heavy mass peak, and total mass distribution, in $^{235}U$ fast fission. Results are shown in open symbols for the first method of analysis and closed symbols for the second method of analysis.

TABLE I

INDEPENDENT YIELD DATA SUMMARY

| Quantity | Light Mass Peak | Heavy Mass Peak | Total |
|---|---|---|---|
| (A) Number and mass range of $^{235}U$ thermal fission independent yields. | | | |
| #E-E | 10 | 13 | 23 |
| (range) | (84-94) | (132-144) | |
| #E-0 | 10 | 14 | 24 |
| (range) | (85-95) | (131-143) | |
| #0-E | 13 | 11 | 24 |
| (range) | (85-97) | (131-143) | |
| #0-0 | 12 | 12 | 24 |
| (range) | (84-96) | (132-144) | |
| (B) Number and mass range of $^{235}U$ fast fission independent yields. | | | |
| #E-E | 1 | 2 | 3 |
| (range) | ( 92 ) | ( 140 ) | |
| #E-0 | 3 | 3 | 6 |
| (range) | (87-91) | (139-141) | |
| #0-E | 3 | 2 | 5 |
| (range) | (89-91) | ( 139 ) | |
| #0-0 | 0 | 2 | 2 |
| (range) | ( 0 ) | ( 140 ) | |

TABLE II

F FACTORS, X AND Y VALUES, FOR $^{235}U$ THERMAL FISSION

| Quantity | Light Mass Peak | Heavy Mass Peak | Total[a] |
|---|---|---|---|
| (A) As determined from average deviations of the fractional independent yield from the normal fractional independent yield. | | | |
| $F_1$ | 1 + (.193 ± .036) | 1 + (.370 ± .192) | 1 + (.290 ± .169) |
| $F_2$ | 1 + (.243 ± .101) | 1 + (.216 ± .133) | 1 + (.228 ± .122) |
| $F_3$ | 1 - (.219 ± .125) | 1 - (.214 ± .142) | 1 - (.217 ± .133) |
| $F_4$ | 1 - (.258 ± .124) | 1 - (.380 ± .164) | 1 - (.319 ± .157) |
| X | 0.228 ± .052 | 0.295 ± .080 | 0.264 ± .145 |
| Y | - 0.003 ± .052 | 0.080 ± .080 | 0.041 ± .145 |
| $F_e^z$ | 1 + (.217 ± .078) | 1 + (.293 ± .182) | 1 + (.260 ± .151) |
| $F_o^z$ | 1 - (.237 ± .126) | 1 - (.305 ± .161) | 1 - (.269 ± .155) |
| $\langle F^z \rangle$ | 1 ± (.227 ± .102) | 1 ± (.299 ± .171) | 1 ± (.264 ± .153) |
| (B) As determined from deviations of the summed independent yield from the summed normal independent yield. | | | |
| $F_1$ | 1 + (.202 ± .032) | 1 + (.303 ± .042) | 1 + (.266 ± .029) |
| $F_2$ | 1 + (.206 ± .040) | 1 + (.156 ± .054) | 1 + (.176 ± .036) |
| $F_3$ | 1 - (.221 ± .050) | 1 - (.172 ± .052) | 1 - (.192 ± .037) |
| $F_4$ | 1 - (.209 ± .032) | 1 - (.318 ± .048) | 1 - (.277 ± .032) |
| X | 0.209 ± .020 | 0.237 ± .025 | 0.228 ± .034 |
| Y | - 0.016 ± .020 | 0.073 ± .025 | 0.044 ± .034 |
| $F_e^z$ | 1 + (.204 ± .026) | 1 + (.232 ± .034) | 1 + (.221 ± .032) |
| $F_o^z$ | 1 - (.215 ± .030) | 1 - (.247 ± .035) | 1 - (.234 ± .035) |
| $\langle F^z \rangle$ | 1 ± (.209 ± .028) | 1 ± (.239 ± .034) | 1 ± (.228 ± .033) |

[a]Uncertainties for X and Y values in this column are given by $\Delta X = \Delta Y = \frac{1}{4}\sum_{i=1}^{4} |\Delta F_i|$ instead of using Eq. (5). This is because of averaging effects over light and heavy mass peaks where the X and Y values are noticeably different from one another.

TABLE III

F FACTORS, X AND Y VALUES, FOR $^{235}$U FAST FISSION (~1.9 MeV)

| Quantity | Light Mass Peak | Heavy Mass Peak | Total[a] |
|---|---|---|---|
| (A) As determined from average deviations of the fractional independent yield from the normal fractional independent yield. | | | |
| $F_1$ | 1 - (.134 ± .134) | 1 + (.133 ± .032) | 1 + (.044 ± .129) |
| $F_2$ | 1 + (.059 ± .017) | 1 + (.046 ± .181) | 1 + (.052 ± .129) |
| $F_3$ | 1 - (.101 ± .039) | 1 - (.110 ± .141) | 1 - (.104 ± .033) |
| $F_4$ | insufficient data | 1 - (.109 ± .018) | 1 - (.109 ± .018) |
| X | | 0.100 ± .046 | 0.078 ± .077 |
| Y | | 0.021 ± .046 | -0.004 ± .077 |
| $F_e^z$ | | 1 + (.081 ± .148) | 1 + (.050 ± .102) |
| $F_o^z$ | | 1 - (.110 ± .017) | 1 - (.106 ± .029) |
| $\langle F^z \rangle$ | | 1 ± (.096 ± .082) | 1 ± (.078 ± .066) |
| (B) As determined from deviations of the summed independent yield from the summed normal independent yield. | | | |
| $F_1$ | 1 - (.134 ± .143) | 1 + (.157 ± .064) | 1 + (.060 ± .064) |
| $F_2$ | 1 + (.058 ± .166) | 1 + (.024 ± .050) | 1 + (.037 ± .071) |
| $F_3$ | 1 - (.064 ± .079) | 1 - (.118 ± .063) | 1 - (.089 ± .052) |
| $F_4$ | insufficient data | 1 - (.126 ± .066) | 1 - (.126 ± .066) |
| X | | 0.106 ± .031 | 0.078 ± .063 |
| Y | | 0.035 ± .031 | 0.015 ± .063 |
| $F_e^z$ | | 1 + (.071 ± .040) | 1 + (.045 ± .067) |
| $F_o^z$ | | 1 - (.123 ± .046) | 1 - (.103 ± .062) |
| $\langle F^z \rangle$ | | 1 ± (.097 ± .043) | 1 ± (.074 ± .064) |

[a]Uncertainties for X and Y values in this column are given by $\Delta X = \Delta Y = \frac{1}{4} \sum_{i=1}^{4} |\Delta F_i|$ instead of using Eq. (5).

## TABLE IV

SUMMARY OF EVALUATIONS OF THE PAIRING INFLUENCE UPON INDEPENDENT YIELD STRENGTHS

| System | X, Y | Thermal | Fast[a] | 14 MeV |
|---|---|---|---|---|
| $^{232}Th + n$ | X | --- | −± 0.35[b] | --- |
| $^{233}U + n$ | X | ± 0.21 ± 0.075[c] | −± 0.08[b] | --- |
| | Y | ± 0.08 ± 0.05[c] | --- | --- |
| | X | + 0.081 ± 0.097[d]<br>− 0.369 ± 0.098[d] | --- | ~0[d] |
| | X | ± 0.158[e] | --- | --- |
| $^{235}U + n$ | X | ± 0.22 ± 0.054[c] | ± 0.08 ± 0.04[c] | --- |
| | Y | ± 0.08 ± 0.05[c] | --- | --- |
| | X | + 0.185 ± 0.073[d]<br>− 0.209 ± 0.044[d] | --- | -0[d] |
| | X | ± 0.206[e] | | |
| | X | 0.228 ± 0.034[f] | 0.078 ± 0.063[f] | --- |
| | Y | 0.044 ± 0.034[f] | 0.015 ± 0.063[f] | --- |
| $^{238}U + n$ | X | --- | −± 0.08[b] | --- |
| $^{239}Pu + n$ | X | −± 0.09[b] | --- | --- |
| | X | + 0.030 ± 0.095[d]<br>− 0.341 ± 0.146[d] | --- | --- |
| | X | − 0.087[e]<br>+ 0.087[e] | --- | --- |
| $^{241}Pu + n$ | X | −± 0.09[b] | --- | --- |

[a]The magnitude of the average "fast" fission neutron energy depends upon the particular reactor used to make the yield measurements. It is manifestly evident that "fast" is an ill-defined quantity when one considers not only the measurement of yields, but also the use of yield models based upon these measurements. It is clear that as yield models and yield measurements become more microscopic the broad classification of "fast" neutrons will no longer be sufficient.

[b]S. Amiel, Soreq Nuclear Research Centre, Yavne, Israel, personal communication to T. R. England, October 1975. X values defined as in $\langle F^{z}\rangle - 1$ (see Tables II and III).

[c]S. Amiel and H. Feldstein.[9] X values defined as in $\langle F^{z}\rangle - 1$ (see Tables II and III). Y values similarly defined.

[d]E. A. C. Crouch.[13] X values defined as in $F^{z}_{e} - 1$ and $F^{z}_{o} - 1$ [see Eq. (3), Tables II and III]. This work assumed $\sigma = 0.60$ and confined the independent yield data set to within ± 2 charge units of $Z_p$.

[e]A. R. deL. Musgrove, J. L. Cook, and G. D. Trimble.[14] X values defined as in $\langle F^{z}\rangle - 1$ except for $^{239}Pu$ for which $F^{z}_{e} - 1$ and $F^{z}_{o} - 1$ are used [see Tables II and III, Eq. (3)]. In this work $\sigma(^{235}U) = 0.569$, $\sigma(^{233}U) = 0.582$, and $\sigma(^{239}Pu) = 0.67$.

[f]Present work [see last columns in Table II (B) and Table III (B)]. X and Y defined by Eq. (2).

TABLE V

THERMAL FISSION EXCITATION ENERGIES AND FISSION BARRIER HEIGHTS FOR SOME ACTINIDE NUCLEI[a]

| System | $E^*$ (MeV) | $\langle E_a \rangle$ (MeV) | $\langle E_b \rangle$ (MeV) | $E^*-\langle E_b \rangle$ (MeV) |
|---|---|---|---|---|
| $^{232}Th + n$ | 4.786 | 6.02 ± .25 | 6.28 ± .20 | -1.494 ± .20 |
| $^{233}U + n$ | 6.841 | 6.20 ± .25 | 5.95 ± .25 | +0.891 ± .25 |
| $^{234}U + n$ | 5.306 | 6.10 ± .30 | 5.65 ± .30 | -0.344 ± .30 |
| $^{235}U + n$ | 6.546 | 5.90 ± .20 | 5.74 ± .20 | +0.806 ± .20 |
| $^{236}U + n$ | 5.124 | 6.35 ± .30 | 5.95 ± .30 | -0.826 ± .30 |
| $^{238}U + n$ | 4.803 | 6.55 ± .20 | 6.30 ± .30 | -1.497 ± .30 |
| $^{238}Pu + n$ | 5.655 | 6.35 ± .25 | 5.35 ± .30 | +0.305 ± .30 |
| $^{239}Pu + n$ | 6.534 | 5.92 ± .20 | 5.40 ± .20 | +1.134 ± .20 |
| $^{240}Pu + n$ | 5.240 | 6.25 ± .20 | (5.50) | -0.260 ± -.3 |
| $^{241}Pu + n$ | 6.301 | 5.77 ± .20 | 5.39 ± .20 | +0.911 ± .20 |
| $^{242}Pu + n$ | 5.037 | 6.05 ± .20 | (5.60) | -0.563 ± -.3 |
| $^{241}Am + n$ | 5.528 | 6.39 ± .20 | 4.90 ± .20 | +0.628 ± .20 |
| $^{243}Am + n$ | 5.363 | 6.19 ± .20 | 4.80 ± .25 | +0.563 ± .25 |

[a]These represent a large fraction of the important fissionable nuclides to be included in Version V of ENDF/B; for brevity we refer to these as ENDF/B-V nuclides in the body of the text.

TABLE VI

CALCULATED X AND Y VALUES

| | $E_n$ (MeV) | X ± ΔX | Y ± ΔY |
|---|---|---|---|
| (A) $^{232}Th + n$ | | | |
| | 0.0 | --- | --- |
| | 0.5 | --- | --- |
| | 1.0 | --- | --- |
| | 2.0 | $0.327^{+0.540}_{-0.327}$ | $0.063^{+0.116}_{-0.063}$ |
| | 3.0 | $0.134^{+0.166}_{-0.134}$ | $0.026^{+0.038}_{-0.026}$ |
| | 8.0 | $0.034^{+0.039}_{-0.034}$ | $0.006^{+0.009}_{-0.006}$ |
| | 14.0 | $0.018^{+0.020}_{-0.018}$ | $0.003^{+0.005}_{-0.003}$ |

$E_n(X \rightarrow \infty) = 1.312$ MeV

$E_n(\text{threshold}) = 1.200$ MeV

$E_n(\text{one-half plateau}) \simeq 1.25$ MeV

TABLE VI (continued)

(B) $^{233}U + n$

| | | |
|---|---|---|
| 0.0 | $0.210^{+0.291}_{-0.210}$ | $0.041^{+0.064}_{-0.041}$ |
| 0.5 | $0.143^{+0.181}_{-0.143}$ | $0.028^{+0.041}_{-0.028}$ |
| 1.0 | $0.109^{+0.132}_{-0.109}$ | $0.021^{+0.030}_{-0.021}$ |
| 2.0 | $0.073^{+0.089}_{-0.073}$ | $0.014^{+0.020}_{-0.014}$ |
| 3.0 | $0.055^{+0.065}_{-0.055}$ | $0.011^{+0.015}_{-0.011}$ |
| 8.0 | $0.025^{+0.029}_{-0.025}$ | $0.005^{+0.007}_{-0.005}$ |
| 14.0 | $0.015^{+0.017}_{-0.015}$ | $0.003^{+0.004}_{-0.003}$ |

(C) $^{234}U + n$

| | | |
|---|---|---|
| 0.0 | --- | --- |
| 0.5 | $0.666^{+1.831}_{-0.666}$ a | $0.129^{+0.368}_{-0.129}$ a |
| 1.0 | $0.269^{+0.411}_{-0.269}$ | $0.052^{+0.089}_{-0.052}$ |
| 2.0 | $0.123^{+0.152}_{-0.123}$ | $0.024^{+0.035}_{-0.024}$ |
| 3.0 | $0.079^{+0.094}_{-0.079}$ | $0.015^{+0.022}_{-0.015}$ |
| 8.0 | $0.029^{+0.033}_{-0.029}$ | $0.006^{+0.008}_{-0.006}$ |
| 14.0 | $0.016^{+0.019}_{-0.016}$ | $0.003^{+0.004}_{-0.003}$ |

$E_n(X \to \infty) = 0.162$ MeV

$E_n$(threshold) = 0.0014 MeV

$E_n$(one-half plateau) ≃ 0.60 MeV

[a]May be unrealistically large.

TABLE VI (continued)

(D) $^{235}U + n$

| | | |
|---|---|---|
| 0.0 | $0.228 \pm 0.034$ [a] | $0.044 \pm 0.034$ [a] |
| 0.5 | $0.151^{+0.193}_{-0.151}$ | $0.029^{+0.044}_{-0.029}$ |
| 1.0 | $0.113^{+0.138}_{-0.113}$ | $0.022^{+0.032}_{-0.022}$ |
| 2.0 | $0.078 \pm 0.063$ [a] | $0.015 \pm 0.063$ [a] |
| 3.0 | $0.056^{+0.066}_{-0.056}$ | $0.011^{+0.015}_{-0.011}$ |
| 8.0 | $0.025^{+0.029}_{-0.025}$ | $0.005^{+0.007}_{-0.005}$ |
| 14.0 | $0.015^{+0.017}_{-0.015}$ | $0.003^{+0.004}_{-0.003}$ |

[a]Data used to determine the parameters k and c. See last column in Table II (B) and Table III (B).

(E) $^{236}U + n$

| | | |
|---|---|---|
| 0.0 | --- | --- |
| 0.5 | --- | --- |
| 1.0 | $0.632^{+1.666}_{-0.632}$ a | $0.122^{+0.336}_{-0.122}$ a |
| 2.0 | $0.166^{+0.217}_{-0.166}$ | $0.032^{+0.049}_{-0.032}$ |
| 3.0 | $0.096^{+0.115}_{-0.096}$ | $0.018^{+0.026}_{-0.018}$ |
| 8.0 | $0.031^{+0.035}_{-0.031}$ | $0.006^{+0.008}_{-0.006}$ |
| 14.0 | $0.017^{+0.020}_{-0.017}$ | $0.003^{+0.005}_{-0.003}$ |

$E_n(X \rightarrow \infty) = 0.644$ MeV

$E_n$(threshold) = 0.600 MeV

$E_n$(one-half plateau) $\simeq$ 1.05 MeV

[a]May be unrealistically large.

(F) $^{238}U + n$

| | | | |
|---|---|---|---|
| | 0.0 | --- | --- |
| | 0.5 | --- | --- |
| | 1.0 | --- | --- |
| | 2.0 | $0.329^{+0.554}_{-0.329}$ | $0.063^{+0.118}_{-0.063}$ |
| | 3.0 | $0.134^{+0.168}_{-0.134}$ | $0.026^{+0.038}_{-0.026}$ |
| | 8.0 | $0.034^{+0.039}_{-0.034}$ | $0.006^{+0.010}_{-0.006}$ |
| | 14.0 | $0.018^{+0.020}_{-0.018}$ | $0.003^{+0.005}_{-0.003}$ |

$E_n(X \to \infty) = 1.315$ MeV

$E_n$(threshold) = 0.050 MeV

$E_n$(one-half plateau) ≃ 1.50 MeV

(G) $^{238}Pu + n$

| | | | |
|---|---|---|---|
| | 0.0 | $0.462^{+0.962}_{-0.462}$ | $0.089^{+0.198}_{-0.089}$ |
| | 0.5 | $0.228^{+0.327}_{-0.228}$ | $0.044^{+0.072}_{-0.044}$ |
| | 1.0 | $0.152^{+0.194}_{-0.152}$ | $0.029^{+0.044}_{-0.029}$ |
| | 2.0 | $0.091^{+0.109}_{-0.091}$ | $0.018^{+0.025}_{-0.018}$ |
| | 3.0 | $0.065^{+0.076}_{-0.065}$ | $0.012^{+0.018}_{-0.012}$ |
| | 8.0 | $0.026^{+0.031}_{-0.026}$ | $0.005^{+0.007}_{-0.005}$ |
| | 14.0 | $0.016^{+0.018}_{-0.016}$ | $0.003^{+0.004}_{-0.003}$ |

(H) $^{239}Pu + n$

| | | | |
|---|---|---|---|
| | 0.0 | $0.171^{+0.223}_{-0.171}$ | $0.033^{+0.050}_{-0.033}$ |
| | 0.5 | $0.124^{+0.153}_{-0.124}$ | $0.024^{+0.035}_{-0.024}$ |
| | 1.0 | $0.097^{+0.117}_{-0.097}$ | $0.019^{+0.027}_{-0.019}$ |
| | 2.0 | $0.068^{+0.080}_{-0.068}$ | $0.013^{+0.018}_{-0.013}$ |
| | 3.0 | $0.052^{+0.061}_{-0.052}$ | $0.010^{+0.014}_{-0.010}$ |
| | 8.0 | $0.024^{+0.028}_{-0.024}$ | $0.005^{+0.007}_{-0.005}$ |
| | 14.0 | $0.015^{+0.017}_{-0.015}$ | $0.003^{+0.004}_{-0.003}$ |

TABLE VI (continued)

(I) $^{240}Pu + n$

| | | |
|---|---|---|
| 0.0 | --- | --- |
| 0.5 | $0.534^{+1.231}_{-0.534}$ [a] | $0.103^{+0.251}_{-0.103}$ [a] |
| 1.0 | $0.244^{+0.359}_{-0.244}$ | $0.047^{+0.078}_{-0.047}$ |
| 2.0 | $0.117^{+0.144}_{-0.117}$ | $0.023^{+0.033}_{-0.023}$ |
| 3.0 | $0.077^{+0.091}_{-0.077}$ | $0.015^{+0.021}_{-0.015}$ |
| 8.0 | $0.028^{+0.033}_{-0.028}$ | $0.006^{+0.008}_{-0.006}$ |
| 14.0 | $0.016^{+0.019}_{-0.016}$ | $0.003^{+0.004}_{-0.003}$ |

$E_n(X \to \infty) = 0.078$ MeV

$E_n$(threshold) = thermal

$E_n$(one-half plateau) $\simeq$ 0.68 MeV

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[a] May be unrealistically large.

(J) $^{241}Pu + n$

| | | |
|---|---|---|
| 0.0 | $0.206^{+0.282}_{-0.206}$ | $0.040^{+0.063}_{-0.040}$ |
| 0.5 | $0.141^{+0.178}_{-0.141}$ | $0.027^{+0.040}_{-0.027}$ |
| 1.0 | $0.108^{+0.131}_{-0.108}$ | $0.021^{+0.030}_{-0.021}$ |
| 2.0 | $0.073^{+0.086}_{-0.073}$ | $0.014^{+0.020}_{-0.014}$ |
| 3.0 | $0.055^{+0.064}_{-0.055}$ | $0.011^{+0.015}_{-0.011}$ |
| 8.0 | $0.025^{+0.029}_{-0.025}$ | $0.005^{+0.007}_{-0.005}$ |
| 14.0 | $0.015^{+0.017}_{-0.015}$ | $0.003^{+0.004}_{-0.003}$ |

TABLE VI (continued)

(K) $^{242}Pu + n$

| | | |
|---|---|---|
| 0.0 | --- | --- |
| 0.5 | $1.890^{+13.563}_{-1.890}$ [a] | $0.365^{+2.633}_{-0.365}$ [a] |
| 1.0 | $0.364^{+0.649}_{-0.364}$ | $0.070^{+0.137}_{-0.070}$ |
| 2.0 | $0.139^{+0.176}_{-0.139}$ | $0.027^{+0.040}_{-0.027}$ |
| 3.0 | $0.086^{+0.103}_{-0.086}$ | $0.017^{+0.024}_{-0.017}$ |
| 8.0 | $0.030^{+0.034}_{-0.030}$ | $0.006^{+0.008}_{-0.006}$ |
| 14.0 | $0.016^{+0.019}_{-0.016}$ | $0.003^{+0.004}_{-0.003}$ |

$E_n(X \rightarrow \infty) = 0.381$ MeV

$E_n$(threshold) = 0.010 MeV

$E_n$(one-half plateau) ≃ 0.75 MeV

[a] May be unrealistically large.

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