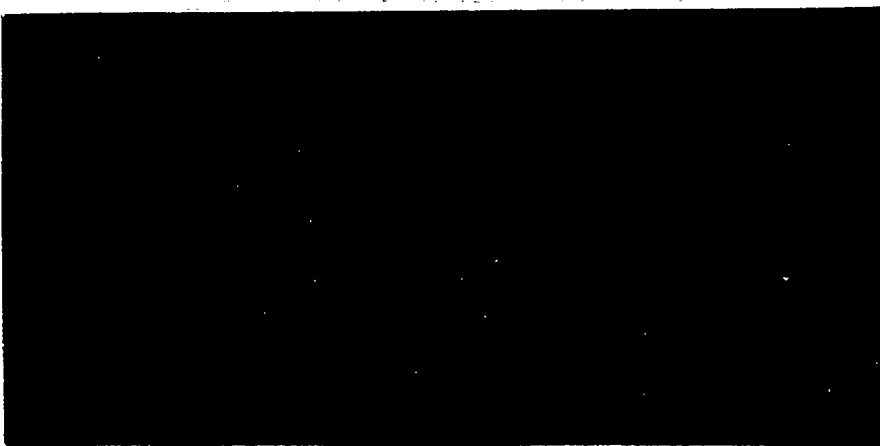


34
C2

LOS ALAMOS SCIENTIFIC LABORATORY

OF THE UNIVERSITY OF CALIFORNIA
LOS ALAMOS, NEW MEXICO



LOS ALAMOS NATIONAL LABORATORY
3 9338 00356 2302

CONTRACT W-7405-ENG.36 WITH THE
U.S. ATOMIC ENERGY COMMISSION

FOR REFERENCE
NOT TO BE TAKEN FROM THIS ROOM
LIBRARY BUREAU
EX. NO. 1935

LOS ALAMOS SCIENTIFIC LABORATORY
of the
UNIVERSITY OF CALIFORNIA

Report written:
August, 1954

Report distributed: DEC 6 1954

LA-1740

ON THE INTERACTION OF A SHOCK
WITH A CONSTRICTION



Report written by:
Otto Laporte

Work done by:
Otto Laporte

PHYSICS



ABSTRACT

A detailed theoretical treatment of the passage of a shockwave along a channel possessing a constriction or area reduction at one point is given. The purpose is to furnish a theory of the diffraction of a shock by a flat plate which has upon it a regular array of perpendicular spikes or wedges. This type of diffraction has been studied experimentally by the shock tube laboratory of the University of Michigan and a report on this work is being issued at the present time.

The pressures, densities and temperatures encountered are calculated assuming a monatomic gas and ideal gas law. Two entirely different treatments have to be employed according as the Mach number at the constriction is less or greater than unity. Numerical results are presented for constriction ratios 2:1, 5:1, and ∞ :1. Of interest to experimental investigators are quantitative predictions on the possibility of attaining greater shock strengths in this way and of, therefore, exceeding the inherent limitations of a shock tube.

I. INTRODUCTION

About a year ago, the shock tube laboratory of the Physics Department of the University of Michigan began the study of the reflection of shock waves by stacks of plates or prongs arranged upon a base plate so as to form a regular array. The ratio of plate thickness to the width of the interstices as well as the wedge angle of the plates was varied. A report containing many schlieren pictures and a few interferograms is being put out by the University of Michigan¹⁾. In brief, the events

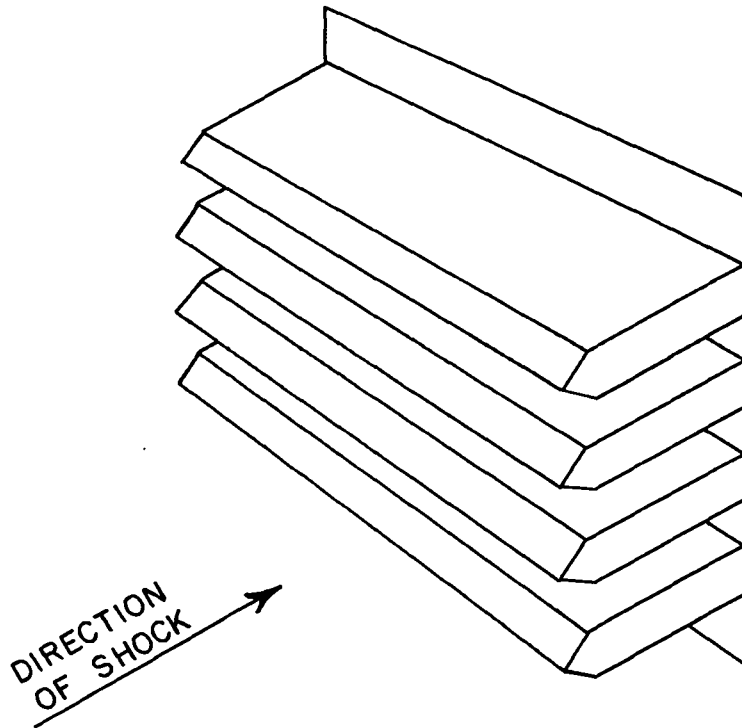


Fig. 1

¹⁾ Report on the Reflection of Shock Waves from Stacks of Wedges, by Alfred Hunting, Otto Laporte, and Eugene Turner. Engineering Research Institute of the University of Michigan, Report No. 2183-1-F, July, 1954.

studied can be described as follows: As the shock strikes the tips of the wedges, a complicated interaction takes place which, however, after some time results in the formation of a reflected shock. However, a shock is transmitted down the interstices as well, is reflected at the end and upon emergence from the flaring ends forms a second shock following the first.

The quantitative account of this process can, therefore, be divided into two parts of which the second, the amalgamation of two shocks is, thanks to the work of von Neumann, Courant and Friedrichs, well understood. The first earlier part of the process, i. e., the formation of the two reflected shocks, the prediction of their strengths, of all pressures and densities outside as well as inside the interstices is a rather complex problem of fluid dynamics. This is the problem which is treated in the following pages.

This work was carried out during the author's stay at Los Alamos in the summer of 1954. For generous and understanding help with the lengthy and often involved numerical work the author is glad to express his indebtedness to Max Goldstein and Josephine E. Powers.

II. DESCRIPTION OF EVENTS

If two planes are laid, one through the median lines of the wedges, the other through the median lines of the interstices, then the problem is reduced to the passage of a shock through a channel with a constriction, followed by reflection and ultimate emergence. Let us first consider the channel extended toward infinity in both directions.

The problem of a channel possessing a constriction through which a shock passes has received some attention, especially by F. Schultz-Grunow²⁾, 3), 4), 5); and by Kahane, Warren, Griffith, and Marino⁶⁾. All of these authors consider only the case of such weak shocks that all waves, although of finite amplitude, may still be regarded as isentropic. It is intended to treat the problem for incident and reflected shocks of arbitrary strengths.

Let us consider what happens as the incident shock, defined by the pressure ratio

$$y_0 = \frac{p_1}{p_0} \quad (1)$$

passes through the constriction (Figs. 1 and 2).

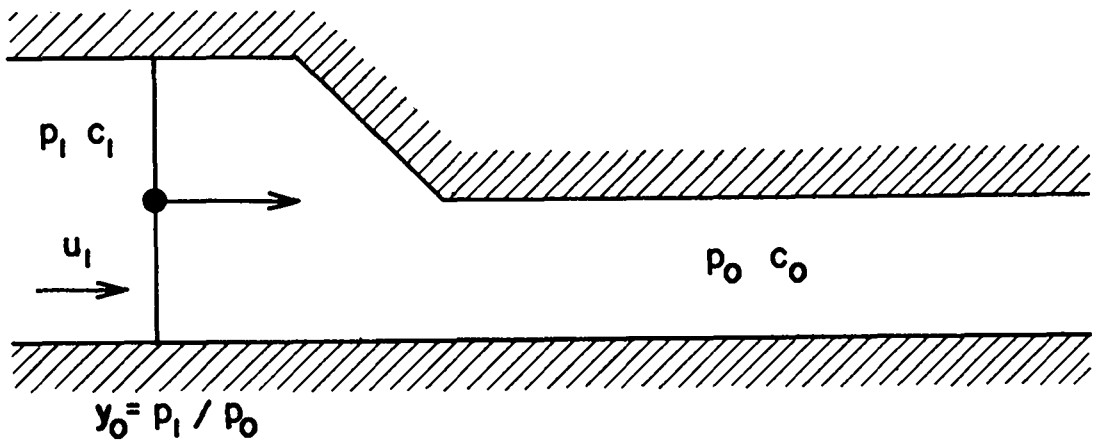
2) Forschung Ingenieur Wesen, Vol. 13, p. 125, 1942.

3) Ingenieur - Archiv., Vol. 16, p. 21, 1943.

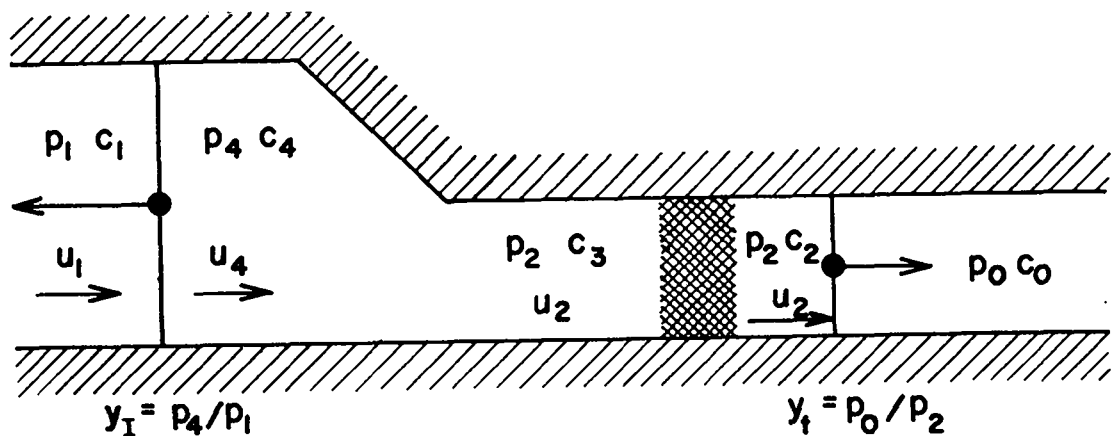
4) Zeitschrift f. Angewandte Math. and Mech., Vol. 24, p. 284, 1944.

5) ibidem, Vol. 29, p. 257, 1949.

6) Rpt. Princeton University 1954. See also: Journ. Aeronaut. Sciences 21, p. 505, 1954.



BEFORE



AFTER

Fig. 2

As the photographs of the Michigan report show, after some time a reflected shock y_I travels to the left while a transmitted shock y_t continues down the narrowed channel. But the interaction of the primary shock with the constriction will give rise to a more or less extended region of changing entropy, an interface region as it were. It is for this reason that in the above diagram the flow variables on the two sides of the cross-hatched interface region possess different

sound velocities (and, of course, also different densities and temperatures) c_3 and c_2 while the pressure p_2 and the flow velocity u_2 are continuous.

As for the transition from the flow field $(p_4 c_4 u_4)$ to the flow field $(p_3 c_3 u_3)$, it is assumed to be adiabatic which we believe to be correct as long as (a) the wedge angle is not too near to 90° , and (b) as long as the calculated Mach number u_3/c_3 is less than unity. Concerning (a) it is clear that if the transition from the wide to the narrow part of the channel is too abrupt, the entire flow in the narrow part will be turbulent. And concerning (b), it should be remembered that the flow from region $(p_4 c_4 u_4)$ to $(p_3 c_3 u_3)$ can at best be accelerated to sonic conditions, and that for production of supersonic flow in the narrow channel, a throat (Laval nozzle) would be necessary. But this we do not have with the present type of wedge system.

Assuming, for the present, a subsonic region at both sides of the constriction, we have a gradually lengthening region $p_3 c_3 u_3$ since the (cross-hatched) interface region moves with the fluid, and also a lengthening region $p_2 c_2 u_2$, since the speed of the transmitted shock is, of course, greater than u_2 . When this shock of strength y_t strikes the end of the interstice, it is reflected as a shock of strength y'_t given by

$$y'_t = \frac{6y_t - 1}{y_t + 4} \quad (2)$$

which formula shows that the strength of a reflected shock can, for argon, never rise beyond 6. Before emerging from the model as a second plane

shock, the reflected interstitial shock y_t^i encounters two hurdles: (1) it has to pass through the interface region, and (2) it has to emerge past the wedge tips, into the wider region at very left of Fig. 2.

At the interaction of the shock with the interface, a slightly modified shock is transmitted and at the same time a shock is reflected. The slightly strengthened transmitted shock then passes through the divergent part of the channel. This second process is more important than the former one; it may be regarded as the converse of the process the primary shock undergoes as it first meets the wedge tips.

What happens is that the emerging shock becomes, with the aid of Huygens wavelets, and by Mach stem formation, a transmitted shock y_{II} , while at the same time a rarefaction is sent back into the interstices. But also a more or less extended interface region (which owes its existence to the temporary presence of curved shocks) is created which, because of the almost vanishing fluid speed behind y_{II} , continues to hover about the tips of the wedges.

Fig. 3 is an $x-t$ plot of the interactions dealt with in this and the next section.

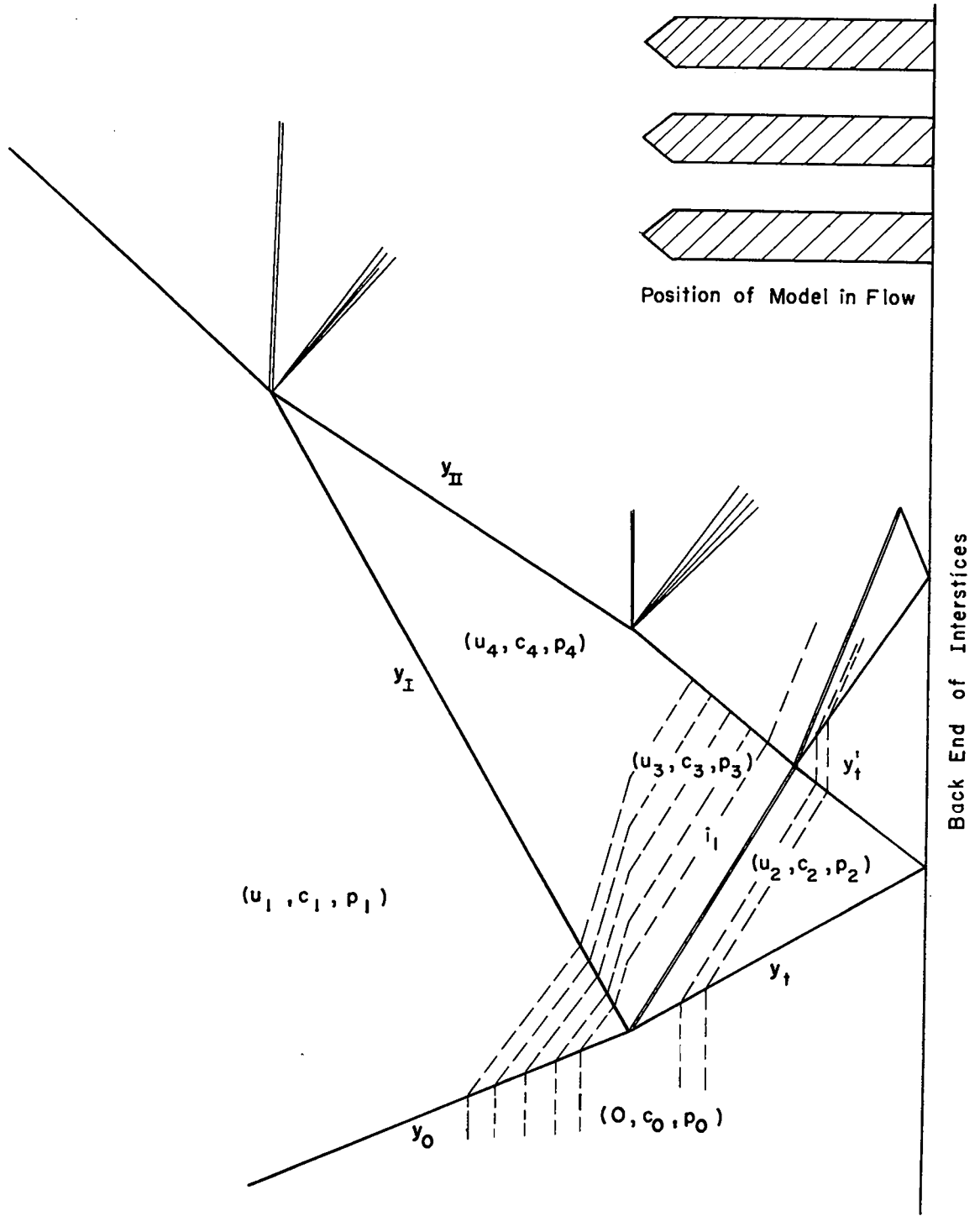


FIG. 3

III. CALCULATION OF y_t , y_I FOR WEAK SHOCKS

The primary shock is defined by its pressure ratio

$$y_o = \frac{p_1}{p_o} . \quad (1)$$

The fluid velocity and the sound speed behind y_o are given by:

$$\frac{u_1}{c_o} = \frac{3}{\sqrt{5}} \frac{y_o - 1}{\sqrt{4y_o + 1}} \quad (3)$$

$$\frac{c_1}{c_o} = \sqrt{\frac{y_o(y_o + 4)}{4y_o + 1}}$$

where c_o and p_o are the ambient quantities ahead of y_o . Some time after y_o has been diffracted by the wedge tips, the reflected shock of as yet unknown pressure ratio y_I recedes, the pressure behind which is

$$p_4 = y_I p_1 = y_o y_I p_o . \quad (4)$$

The fluid and sound speeds u_4 and c_4 are given by

$$\frac{u_4}{c_o} = \frac{3}{\sqrt{5}} \frac{1}{\sqrt{4y_o + 1}} \left\{ y_o - 1 - (y_I - 1) \sqrt{\frac{y_o(y_o + 4)}{4y_o + 1}} \right\} \quad (5)$$

$$\frac{c_4}{c_o} = \sqrt{\frac{y_o(y_o + 4)}{4y_o + 1}} \sqrt{\frac{y_I(y_I + 4)}{4y_I + 1}} .$$

Densities, entropies, etc., can, of course, also be written down, but are not necessary just now.

Now the essential hypothesis of the adiabatic connection between the regions $(u_4 c_4 p_4)$ and $(u_3 c_3 p_3)$ is used. Let the area in the wider part of the duct be A_4 , that in the narrower part A_3 ⁷⁾. Then the continuity equation together with the adiabatic law states that

$$\left. \begin{aligned} A_4 u_4 c_4^3 &= A_3 u_3 c_3^3 \\ \text{and the energy law} \\ u_4^2 + 3c_4^2 &= u_3^2 + 3c_3^2. \end{aligned} \right\} \quad (6)$$

Introducing Mach numbers in regions 3 and 4 a single equation connecting M_3 and M_4 may be found, which is often convenient:

$$\frac{(M_4^2 + 3)^2}{A_4 M_4} = \frac{(M_3^2 + 3)^2}{A_3 M_3}. \quad (7)$$

For a given area ratio this or the previous pair of equations relate the variables $M_3, u_3, c_3, p_3, \rho_3$ to the flow variables in the sub-four region.

Across the interface i , velocity and pressure are continuous

$$u_2 = u_3, \quad p_2 = p_3 \quad (8)$$

and these latter quantities in turn are related to the ambient state $p_0, c_0, u_0 = 0$ via the shock y_t with equations similar to (3).

Equations (4) to (8) make it possible to determine the shock strengths y_t and y_I for a given y_0 .

The following essential point was brought out by George Carrier with whom the author had several stimulating discussions. While equating the u and p according to (8), to require the same continuity of ρ would mean to overdetermine the situation. The ρ 's in the two

⁷⁾ Consequently one has for the ratio

$$\frac{\text{wedge thickness}}{\text{width of interstice}} = \frac{A_4 - A_3}{A_3}.$$

regions, ρ_2 and ρ_3 , are not equal, a fact which proves the existence of the interface region.

Equations (4) and (8) can be interpreted graphically in a u, p plane, using a procedure generalizing the almost classical procedure for shock and rarefaction interaction. However, a direct numerical evaluation was carried out, using the facilities of the hand calculation group of T Division. The area ratio chosen was 2:1 which corresponds to the models that were investigated most thoroughly.

IV. NUMERICAL RESULTS FOR WEAK SHOCKS AND AREA RATIO 1:2

The following table gives the values of the flow variables in the various regions as functions of the independent variable y_0 , the strength of the incident shock. The first five columns contain the variables for region 4 (see Fig. 2), i. e., the strength of the first reflected shock, the speeds and Mach number and the pressure in terms of ambient pressure p_0 .

The next four columns display the same information for region 3. They show in particular that the local Mach numbers M_3 are always much greater than the corresponding M_4 's and that somewhere between $y_0 = 6$ and $y_0 = 10$, Mach unity is reached. This all important fact is reflected by the behavior of M_4 which reaches a maximum for $M_3 = 1$. As is well known to the aerodynamicist, this heralds the beginning of a condition known as "choking" of the duct⁸⁾. It is a proven fact that the steady flow in a duct with narrowing cross section is only able to reach supersonic Mach numbers if there is a throat, i. e., a non-monotonic change in cross section. Of course, it is problematic whether our problem still permits a steady state treatment, but the experiments would seem to indicate that it does.

In view of this circumstance it is proposed to employ the theory contained in Eqs. (3) to (8) and, therefore, the data in the foregoing

⁸⁾ See, for instance, the presentation in A. H. Shapiro's Compressible Flow, p. 89 (Ronald Press, New York, 1953).

TABLE I.
FLOW VARIABLES FOR WEAK SHOCKS, PRIOR TO CHOKING.

y_0	$y_I = p_4/p_1$	u_4/c_0	c_4/c_0	$M_4 = u_4/c_4$	$y_0 y_I = p_4/p_0$
3.0	1.59429	.37110	1.39749	.2655	4.7829
4.0	1.83212	.44544	1.55403	.2866	7.3288
5.0	2.03269	.49988	1.69643	.2946	10.1635
6.0	2.20200	.54391	1.82806	.2975	13.2120
10.0	2.66828	.67521	2.28140	.2960	26.6828

-14-

y_0	u_3/c_0	c_3/c_0	$M_3 = u_3/c_3$	ρ_3/ρ_0	ρ_2/ρ_0	ρ_3/ρ_2	$y_t = p_2/p_0$	$y_t' = p_2'/p_2$	p_2'/p_0
3.0	.88358	1.31858	.6701	2.0571	2.0202	1.016	3.57668	2.7004	9.6585
4.0	1.14235	1.43044	.7986	2.3666	2.3036	1.027	4.84245	3.1727	15.3636
5.0	1.35790	1.53184	.8865	2.6002	2.5151	1.034	6.10138	3.5251	21.5079
6.0	1.54593	1.62597	.9508	2.7820	2.6790	1.038	7.35494	3.7983	27.9363
10.0	2.14446	1.95547	1.0966	3.2283	3.0823	1.047	12.3447	4.4705	55.1870

only as long as $M_3 \ll 1$ which will be as long as

$$y_0 < \sim 7.$$

After that a different approach, to be presented in the next section, will have to be used.

Returning to the numerical table, it should be noticed that the interface region which travels with velocity u_3 does not represent a particularly violent change in density or temperature. In fact the column labelled ρ_3/ρ_2 shows the percentage change to 1.5% at $y_0 = 3$ rising to about 4% at $y_0 = 7$. Further the pressures to which the model is subjected at these comparatively weak incident shocks are worth noticing: the pressure p_4 behind the reflected tip shock rises from 4 to 13 times ambient values at $y_0 = 6$; the pressures p_2 behind the shock which runs down the interstices are but little higher than the p_1 's, ranging as they do, from 3.58 at $y_0 = 3$ to 7.35 at $y_0 = 6$. But upon return from the end of the interstice there is a real pressure increase, as the last column for p_2' shows, namely from $9.65p_0$ at $y_0 = 3$ to $27.93 p_0$ at $y_0 = 6$.

V. STRONGER INCIDENT SHOCK. CALCULATION FOR CHOKED FLOW

Assuming then $M_3 = 1$ as maximum Mach number near the entrance of the channel, the maximum Mach number in region 4 is obtained from Eq. (7):

$$(M_4^2 + 3)^2 = 16 \frac{A_4}{A_3} M_4. \quad (9)$$

Assuming the area ratio equal to 2, one calculates for this limiting Mach number:

$$M_* = .298168, \quad (9')$$

while for a constriction to one fifth the value

$$M_* = .11347 \quad (9'')$$

is obtained.

This number we now regard as fixed for a given area ratio, no matter how strong the incident shock may be. Division of the two relations (5) by one another leads to another expression for this Mach number, which after some elementary changes can be written:

$$M_* \frac{\sqrt{5}}{3} \frac{\sqrt{y_I(y_I + 4)} + y_I - 1}{\sqrt{4y_I + 1}} = \frac{y_0 - 1}{\sqrt{y_0(y_0 + 4)}}. \quad (10)$$

In this way y_I shows a functional dependence on y_0 which is different from that expressed by the earlier system of formulae, but which will, of course, connect with the earlier value $y_I \approx 2.3$ near $y_0 \approx 7$ where, for area ratio 2:1, the unchoked flow approaches sonic conditions in region 3.

Assuming then y_I to be this new function of y_o we reenter formulae (5) and calculate u_4 and c_4 . Since $M_3 = 1$, formulae (6) will immediately give the values for the speeds in region 3:

$$\frac{u_3}{c_o} = \frac{c_3}{c_o} = \frac{1}{2c_o} \sqrt{u_4^2 + 3c_4^2} = \frac{1}{2} \sqrt{M_*^2 + 3} \sqrt{\frac{y_o(y_o + 4)}{4y_o + 1} \cdot \frac{y_I(y_I + 4)}{4y_I + 1}}.$$

Also, using the adiabatic law:

$$\frac{p_3}{p_o} = y_o y_I \left(\frac{c_3}{c_4}\right)^5; \quad \frac{\rho_3}{\rho_o} = \frac{4y_o + 1}{y_o + 4} \frac{4y_I + 1}{y_I + 4} \left(\frac{c_3}{c_4}\right)^3 \quad (11)$$

and with (9)

$$\frac{p_3}{p_o} = \left(\frac{1}{2} \sqrt{M_*^2 + 3}\right)^5 y_o y_I; \quad \frac{\rho_3}{\rho_o} = \left(\frac{1}{2} \sqrt{M_*^2 + 3}\right)^3 \frac{4y_o + 1}{y_o + 4} \frac{4y_I + 1}{y_I + 4}.$$

To connect p_3 and u_3 with the ambient state p_o , $u_o = 0$ via a shock is generally not possible since region 3 is subject to the stringent condition of sonic flow. We are, therefore, forced to interpose a rarefaction wave whose high pressure head remains at the sonic region 3 and whose low pressure foot moves gradually into the interstice. From this low pressure side the fluid emerges with pressure p_2 and velocity u_2 , which two values will then be connectable to the zero region via a shock. To be sure, there is nothing to assure the continuity of ρ or c , so that one has to interpose an interface as well. But using solely u and p one can, of course, disregard its existence for the nonce.

Figs. 4 and 5 illustrate those rather involved wave interactions and indicate the notation used for the variables of the various regions.

To work through a rarefaction, one remembers that the so-called Riemann invariant $u + 3c$ is constant. Using the adiabatic law connecting c and p , this gives rise to the following expression for the fluid velocity u_2 between the rarefaction and the transmitted shock:

$$u_2 = u_3 + 3c_3 \left\{ 1 - \left(\frac{p_2}{p_3} \right)^{1/5} \right\} \quad (12)$$

where p_3 , and $u_3 = c_3$ are known. To determine p_2 , region 2 is connected to region 0 via the transmitted shock y_t :

$$\left. \begin{aligned} p_2 &= y_t p_0 \\ \frac{u_2}{c_0} &= \frac{3}{\sqrt{5}} \frac{y_t - 1}{\sqrt{1 + 4y_t}} \end{aligned} \right\} \quad (13)$$

so that the equation for y_t becomes:

$$\frac{3}{\sqrt{5}} \frac{y_t - 1}{\sqrt{1 + 4y_t}} = \frac{u_3}{c_0} \left\{ 1 - 3y_t^{1/5} \left(\frac{p_0}{p_3} \right)^{1/5} \right\} \quad (14)$$

It is possible to eliminate all variables referring to region 3 by means of (11) and (5) with the result:

$$\frac{(y_t - 1)^2}{4y_t + 1} = \frac{20}{9} (M_*^2 + 3) \frac{y_0(y_0 + 4)}{4y_0 + 1} \frac{y_I(y_I + 4)}{4y_I + 1} \left\{ 1 - \frac{3}{4} \frac{2}{\sqrt{M_*^2 + 3}} \frac{y_t}{y_0 y_I} \right\}^{1/5} \quad (15)$$

In this relation, one regards y_I as a known function of y_0 , on account of having solved (10). Then the above is an equation for y_t . The fluid

speed u_2 may then be obtained from the second formula (13) and also the two densities, namely the one between the transmitted shock and the interface

$$\frac{\rho_2}{\rho_0} = \frac{4y_t + 1}{y_t + 4} \quad (16)$$

and the density between the interface and the rarefaction:

$$\frac{\rho_2^*}{\rho_0} = \frac{4y_0 + 1}{y_0 + 4} \frac{4y_I + 1}{y_I + 4} \left(\frac{y_t}{y_0 y_I} \right)^{3/5} \quad (16')$$

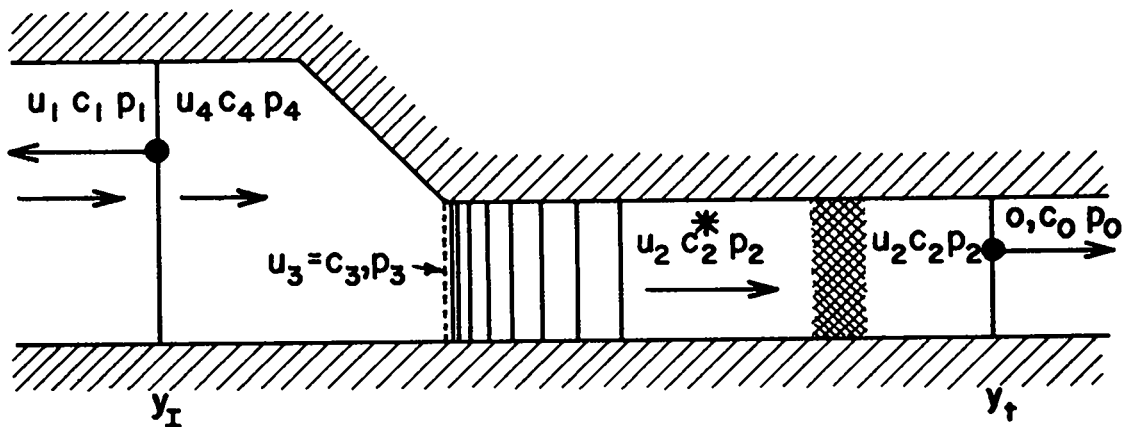


Fig. 4

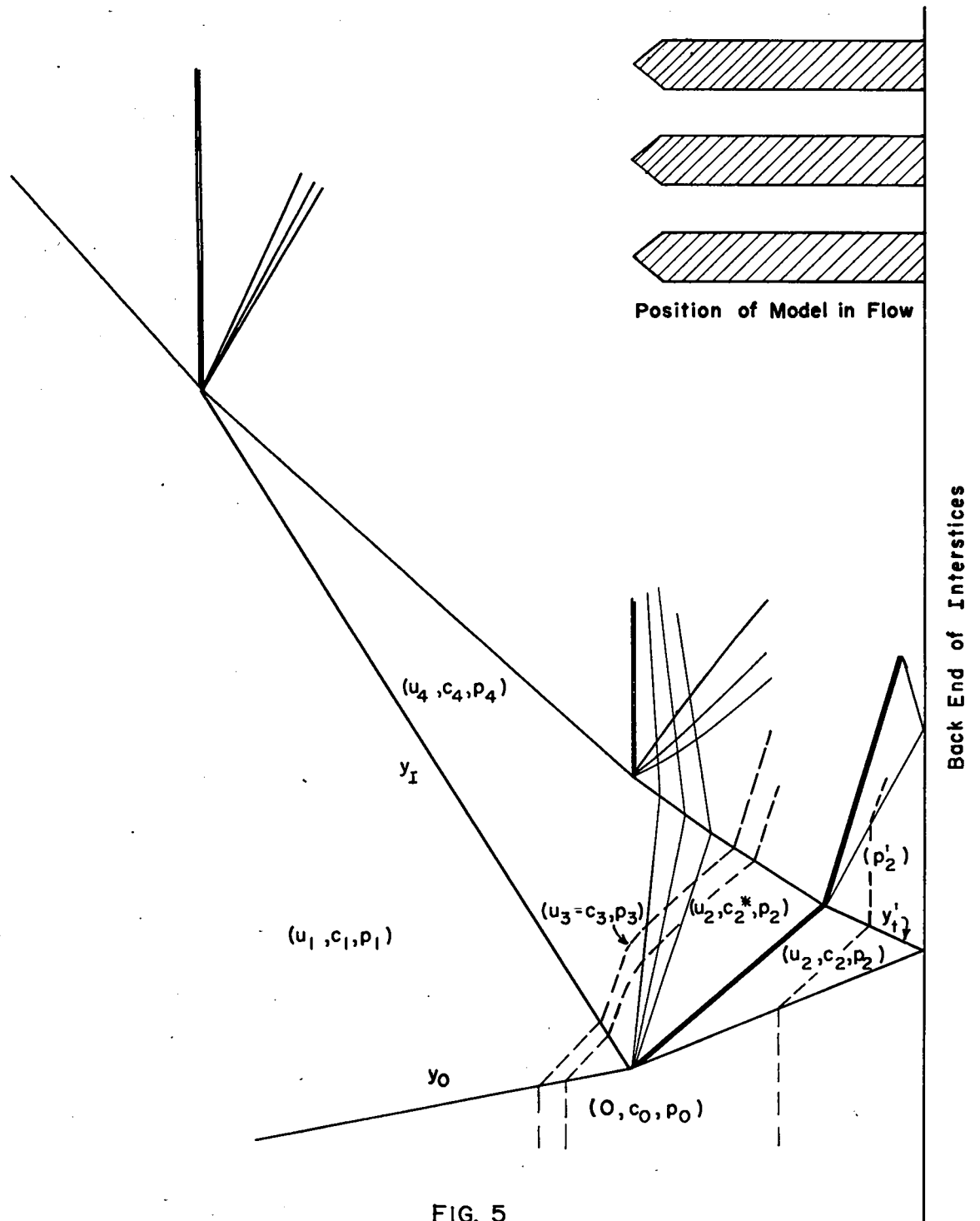


FIG. 5

VI. NUMERICAL RESULTS FOR CHOKED FLOW AND
AREA RATIOS 2:1 AND 5:1

Tables 2 and 3 exhibit, for choked flow, the values of the hydrodynamic variables as functions of the incident shock strength y_0 .

Just as in Table 1, for the unchoked flow, a row of values, viz., for $y_0 = 10$, was included in order to demonstrate a case where the method in question no longer applies, so do Tables 2 and 3 begin for y_0 which are too low for choking. This can be seen in Table 2 for $y_0 = 6$, because p_3 is less than p_2 , while for $y_0 = 7$, the proper pressure gradient is established, although the expansion fan is still weak. Similarly for the area ratio 5:1, Table 3 shows that for $y_0 = 4$ the flow is as yet unchoked, but already for $y_0 = 6$, $p_3 > p_2$ an upstream facing expansion fan arises. As the incident shock becomes stronger, the two quantities of principal interest, namely y_I and y_t , behave in quite a different manner. It is seen that y_I increases but slowly and approaches a finite value, while the transmitted shock strength y_t continues to increase. The behavior of y_I is understandable when one remembers that the strength of a shock reflected from a blank wall follows formula (2), according to which this quantity rises only slowly with 6 as ultimate limit. As for pressures, the tables show that the p_4 values, i. e., those in front of the model are appreciable although naturally not as high as they would be for a blank wall. Compared with p_4 the pressures p_3 and p_2 further down the narrow channel are not especially remarkable until one comes to consider

TABLE 2
 VARIABLES FOR CHOKED FLOW; AREA RATIO 2:1

y_0	$y_1 = p_1/p_0$	$y_0 y_1 = p_1/p_0$	$\frac{u_3}{c_0} = \frac{c_3}{c_0}$	p_3/p_0	ρ_3/ρ_0	$y_t = p_2/p_0$	u_2/c_0	ρ_2^*/ρ_0	ρ_2/ρ_0	ρ_2^*/ρ_2	$y_t' = p_2'/p_2$	$y_t' y_t = \frac{p_2'}{p_0}$
6	2.20000	13.2000	1.60609	6.91733	2.68156	7.3580	1.54629	2.7828	2.6793	1.0386	3.7989	27.952
7	2.34577	16.4204	1.71450	8.60494	2.92726	8.6048	1.71437	2.9273	2.8100	1.0417	3.9484	33.975
8	2.46769	19.7415	1.81610	10.3453	3.13659	9.8560	1.86876	3.0467	2.9174	1.0443	4.1957	41.353
9	2.57104	23.1394	1.91205	12.1260	3.31671	11.107	2.01185	3.1466	3.0071	1.0464	4.3451	48.261
10	2.65960	26.5960	2.00321	13.9374	3.47308	12.360	2.14599	3.2316	3.0831	1.0482	4.4719	55.272
12	2.80404	33.6485	2.17386	17.6331	3.73128	14.868	2.39262	3.3683	3.2050	1.0510	4.6750	69.508
14	2.91647	40.8306	2.33177	21.3969	3.93520	17.378	2.61677	3.4734	3.2983	1.0531	4.8306	83.946
16	3.00649	48.1038	2.47950	25.2083	4.10021	19.890	2.82363	3.5569	3.3721	1.0546	4.9535	98.525
18	3.08012	55.4422	2.61879	29.0539	4.23637	22.405	3.01675	3.6248	3.4319	1.0562	5.0532	113.22
20	3.14148	62.8296	2.75097	32.9252	4.35060	24.919	3.19829	3.6809	3.4813	1.0573	5.1355	127.97
30	3.34077	100.223	3.33387	52.5209	4.72527	37.505	3.98540	3.8608	3.6386	1.0611	5.3977	202.44
40	3.44968	137.987	3.82877	72.3108	4.93257	50.096	4.64161	3.9576	3.7227	1.0631	5.5379	277.43
50	3.51808	175.904	4.26649	92.1807	5.06395	62.691	5.21627	4.0182	3.7751	1.0643	5.6251	352.64

TABLE 3
 VARIABLES FOR CHOKED FLOW; AREA RATIO 5:1

y_0	$y_1 = p_4/p_1$	$y_0 y_1 = p_4/p_0$	$\frac{u_3}{c_0} = \frac{c_3}{c_0}$	p_3/p_0	ρ_3/ρ_0	$y_t = p_2/p_0$	u_2/c_0	ρ_2^*/ρ_0	ρ_2/ρ_0	ρ_2^*/ρ_2	$y_t' = p_2'/p_2$	$y_t' y_t = p_2'/p_0$
4	2.39371	9.5748	1.43247	4.71446	2.29751	5.6190	1.27901	2.55267	2.44059	1.04592	3.4010	19.110
6	2.91796	17.508	1.69697	8.62047	2.99351	8.5410	1.70615	2.97693	2.80392	1.06170	4.0065	34.220
10	3.51800	35.180	2.12444	17.3219	3.83798	14.4398	2.35229	3.44094	3.18654	1.07984	4.6442	67.061
16	3.96905	63.505	2.63625	31.2685	4.49914	23.3403	3.08552	3.77509	3.45136	1.09380	5.0856	118.70
20	4.14471	82.894	2.92771	40.8154	4.76175	29.2900	3.49169	3.90216	3.54941	1.09938	5.2490	153.74
30	4.40298	132.09	3.55273	65.0382	5.15279	44.1850	4.34587	4.08606	3.68870	1.10772	5.4812	242.19
50	4.63261	231.63	4.55167	114.0502	5.50492	74.005	5.68323	4.24672	3.80770	1.11530	5.6795	420.31

p_2' the pressure behind the reflected interstitial shock. These values constitute a really significant increase. (Of course, for fairly large y_0 's the pressures and temperatures attained are so high as to make the continued use of the ideal equation of state quite unjustifiable.) The interface region whose strength is given by the column labelled ρ_2^*/ρ_2 represents only an insignificant density change, varying between 4 and 12 percent; it will, therefore, be justified to neglect its influence upon the returning shock.

Of interest is the aforementioned fact that the transmitted shock is always stronger than the incident shock, and that the increase in strength becomes more significant as the constriction becomes more pronounced. That this would be so has often been conjectured by shock tube workers and it has been proposed as a means of overcoming the natural limitations of their instrument. However, the increase which an area reduction makes possible is by no means tremendous, being always less than 25 percent for the 2:1 and 50 percent for 5:1 case. About this more will be said in a later section.

VII. APPROXIMATE FORMULAE FOR INFINITE AREA CHANGE

The last mentioned result, namely the possibility of enhancing the shock strength by letting it travel into a constriction, the amount of the strengthening being 25 percent to 50 percent for area ratio 2 and 5, respectively, serves as an inducement to investigate the case when

$$\frac{A_4}{A_3} \gg 1,$$

that is to say when the ultimate channel is mere crack. Since now Eq. (9) gives

$$M_* = 0 \tag{9''''}$$

Eq. (10) contains explicit solutions for the reflected shock

$$y_I = \frac{6y_0 - 1}{y_0 + 4} \tag{2'}$$

which is the law for the reflection from a solid wall. Using this, y_I may be eliminated from the three formulae (11), to give:

$$\frac{u_3}{c_0} = \frac{c_3}{c_0} = \frac{1}{2} \sqrt{\frac{3}{5}} \sqrt{\frac{(6y_0 - 1)(2y_0 + 3)}{4y_0 + 1}} \tag{17}$$

$$\frac{p_3}{p_0} = \left(\frac{\sqrt{3}}{2}\right)^5 y_0 \frac{6y_0 - 1}{y_0 + 4}; \quad \frac{\rho_3}{\rho_0} = \left(\frac{\sqrt{3}}{2}\right)^3 \frac{5y_0}{2y_0 + 3} \frac{4y_0 + 1}{y_0 + 4}.$$

The equation, which determines y_t , the strength of the transmitted shock, becomes:

$$\frac{(y_t - 1)^2}{4y_t + 1} = \frac{4}{3} \frac{(6y_0 - 1)(2y_0 + 3)}{4y_0 + 1} \left\{ 1 - \frac{\sqrt{3}}{2} \left(\frac{y_0 + 4}{y_0(6y_0 - 1)} y_t \right)^{1/5} \right\}^2 \tag{18}$$

After having obtained $y_t(y_0)$ the densities ρ_2 and ρ_2^* are calculated according to (16) and (16') with (2').

Table 4 contains the flow variables for this limiting case. In agreement with one's expectation the values of y_t run consistently higher than for $A_4/A_3 = 5$, but the increase is not very considerable.

Besides wishing to indicate the trend the flow variables show as A_4/A_5 is increased, the main reason for the inclusion of this case is the hope that it will ultimately account for a certain model which was studied in the University of Michigan report.¹⁾ This model consisted of a stack of wedges possessing no straight sections at all, so that its cross section was of pure zigzag shape. ("V-Wedge Model," see page 10 of that report.)

TABLE 4
 FLOW VARIABLES FOR INFINITE AREA REDUCTION $\frac{A_4}{A_3} \gg 1$

y_0	$y_{I=P_4/P_1}$	$y_0 y_{I=P_4/P_0}$	$\frac{u_3}{c_0} = \frac{c_3}{c_0}$	P_3/P_0	ρ_3/ρ_0	$y_t=P_2/P_0$	u_2/c_0	ρ_2^*/ρ_0	P_2/P_0	ρ_2^*/ρ_2	$y_t'=P_2'/P_2$	$y_t' y_t = P_2'/P_0$
4	2.8750	11.500	1.49411	5.60210	2.50951	6.2957	1.3885	2.6915	2.5430	1.0584	3.5718	22.487
6	3.5000	21.000	1.77482	10.2299	3.24760	9.6174	1.8403	3.1295	2.8984	1.0797	4.1641	40.048
10	4.21429	42.143	2.22814	20.5295	4.13514	16.3400	2.5264	3.6059	3.2625	1.1053	4.7709	77.957
16	4.75000	76.000	2.77003	37.0226	4.82500	26.5020	3.3075	3.9480	3.5282	1.1190	5.1804	137.29
20	4.95833	99.166	3.07829	48.3079	5.09797	33.2974	3.7406	4.0778	3.5978	1.1334	5.3297	177.47
30	5.26471	157.94	3.73895	76.9394	5.50363	50.3175	4.6523	4.2657	3.7238	1.1455	5.5397	278.74
50	5.53704	276.85	4.79404	134.865	5.86809	84.4023	6.0809	4.4295	3.8303	1.1564	5.7172	482.54

VIII. THE CASE OF VERY STRONG INCIDENT SHOCKS AND
REMARKS ON THE PRODUCTION OF INTENSIFIED TRANSMITTED SHOCKS

In the limiting case of

$$y_0 \rightarrow \infty$$

Eq. (10) has the roots:

$$\begin{array}{lll} y_I = 3.8202 & \text{for} & A_4/A_3 = 2 \\ y_I = 5.0231 & \text{for} & A_4/A_3 = 5 \\ y_I = 6.0 & \text{for} & A_4/A_3 = \infty. \end{array}$$

While u_3 and c_3 go to infinity as the square root of y_0 , ρ_3 approaches finite values which can immediately be calculated with the above y_I values. The transmitted shock y_t will go to infinity with y_0 , the proportionality factor being of special interest. Putting

$$y_t = ky_0 + \dots \quad (19)$$

where the dots indicate terms of lower order in y_0 than the first, the following equation for k results:

$$k = \frac{20}{9} (M_*^2 + 3) \frac{y_I(y_I + 4)}{4y_I + 1} \left\{ 1 - \frac{3}{2} \frac{1}{\sqrt{M_*^2 + 3}} \left(\frac{k}{y_I} \right)^{1/5} \right\}^2 \quad (20)$$

Simultaneously with y_I the proper values of M_* as given by (9'), (9'') and (9''') have to be used. The density ρ_2^* approaches the finite value

$$\frac{\rho_2^*}{\rho_0} = 4 \frac{4y_I + 1}{y_I + 4} \left(\frac{k}{y_I} \right)^{3/5} \quad (21)$$

The numerical values are summarized in the following Table 5.

TABLE 5
ASYMPTOTIC VALUES OF FLOW VARIABLES FOR INFINITE INCIDENT SHOCK
STRENGTHS AND VARIOUS AREA RATIOS

	A_4/A_3		
	2	5	∞
y_I	3.8202	5.0231	6.0
P_3/P_0	5.6512	6.1124	6.4953
y_t/y_0	1.2602	1.4924	1.7061
P_2^*/P_0	4.2808	4.5142	4.7023
P_2/P_0	4.0	4.0	4.0
P_2^*/P_2	1.0702	1.1285	1.1756

It can be seen from this table that the strengthening of a shock by a converging channel can never exceed about 70 percent. It appears at first quite disappointing until one remembers that what is calculated here is not the focussing effect of a wedge shaped region on a cylindrical shock but the value of the ultimate plane shock emerging from the vertex of the wedge shaped region. To obtain stronger enhancements one needs only to employ two or more constrictions in series, separated to be sure, by a suitable length of straight channel. For instance, in a shock tube of 50 cm height the use of two reductions to one fifth of the previous area would result in an amplification factor of 2.22 at a channel height of 2 cm or the use of four 2:1 reductions in a factor of 2.52 at channel height of 3.1 cm. By reducing the width of the tube as well, this result can even be bettered.

XI. CONCLUDING REMARKS

The tables of the previous sections take us, with the calculations of p_2' , just past the moment when the transmitted shock is reflected from the end of the channel. No further calculations are presented at this time, although it would offer no great difficulty to carry them out, to obtain the strength y_{II} after emergence. It was thought to be of lesser interest here.

The amalgamation of y_I and y_{II} is being studied by the University of Michigan shock tube group⁹⁾ and has been found to be in agreement with the theory which goes back to von Neumann.

Interferometric measurements of densities inside the narrowed channel have been carried out also. They have up to now not been too successful since the space limitations of the complicated wedge stack model have made fringe observations difficult. These measurements are currently being repeated with a larger model. For schlieren pictures of the flow reference is made to the University of Michigan report.

⁹⁾ Otto Laporte and E. B. Turner, "On the interactions of Two Plane Shocks Facing in the Same Direction," J Applied Phys, 25, 678 (1954).

REPORT LIBRARY

DEC 6 1954