

TITLE: SENSITIVITY OF NEUTRON MULTICROUP CROSS SECTIONS TO THERMAL BROADENING OF THE FUSION PEAK

AUTHOR(S): Douglas W. Muir

SUBMITTED TO: American Nuclear Society CTR Meeting, San Diego
April 15-18, 1974

By acceptance of this article for publication, the publisher recognizes the Government's (license) rights in any copyright and the Government and its authorized representatives have unrestricted right to reproduce in whole or in part said article under any copyright secured by the publisher.

The Los Alamos Scientific Laboratory requests that the publisher identify this article as work performed under the auspices of the U. S. Atomic Energy Commission.



Los Alamos
Scientific Laboratory
of the University of California
LOS ALAMOS, NEW MEXICO 87544

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

pey

SENSITIVITY OF NEUTRON MULTIGROUP CROSS SECTIONS TO
THERMAL BROADENING OF THE FUSION PEAK

by

D. W. Muir
University of California
Los Alamos Scientific Laboratory
Los Alamos, New Mexico 87544

ABSTRACT

Thermal motion of the ions in a thermonuclear plasma imparts a considerable energy spread to otherwise monoenergetic products of fusion reactions. Although this phenomenon has become a useful tool in plasma diagnostics, the implications for fusion reactor blanket neutronics have not been explored. Previous theoretical studies of the reaction-product energy spectrum are reviewed, and the results are given for the particular case of neutrons from the D-T reaction in a Maxwellian plasma. Various approximations to the spectrum are examined numerically. Using a simple Gaussian formula, spectrum-averaged cross sections are calculated for several nuclear reactions over a wide range of plasma temperatures. While some cross sections are found to be relatively insensitive to the plasma temperature, a few cross sections of potential interest increase by several orders of magnitude as the plasma temperature is increased from 10 to 40 keV.

I. INTRODUCTION

Most concepts proposed for the first generation of power-producing fusion reactors are based on the D-T cycle. That is, the plasma consists of an equimolar mixture of deuterium and tritium, with the reaction $T(d,n)^4\text{He}$ accounting for nearly all of the energy production. An important part of fusion reactor studies is the calculation of the transport of the neutrons released in the D-T reaction throughout the material regions adjacent to the plasma. In addition to the space-energy distribution of the transported neutrons, one needs to know the nuclear cross sections for the production of important neutron effects, such as tritium breeding, energy deposition, gamma-ray production, activation, transmutation, helium production, and others. These calculations are usually performed in a multigroup environment. In this approach, the cross section for a particular process is represented by a single spectrum-averaged number, for all neutrons in the neighborhood of the 14-MeV peak. Since many of the cross sections of interest (as well as the energy spectrum itself) are rapidly varying functions of energy in this region, it is important to know the detailed shape of spectrum.

In the following, previous theoretical studies of the neutron energy spectrum are reviewed and the results summarized. Various approximations to the energy spectrum are compared numerically with an accurate expression at 20 keV, a typical plasma temperature. Using a simple Gaussian formula,

spectrum-averaged cross sections are calculated for several nuclear reactions over a wide range of plasma temperatures.

II. GENERAL EXPRESSION FOR THE SPECTRUM

An excellent discussion of the energy spectra of the products of nuclear reactions in plasmas has been given by Lehner.¹ In that work, a wide variety of reactions and ion velocity distributions are considered, and the usefulness of the peak-broadening phenomenon in plasma diagnostics is shown. For the case of an isotropic Maxwellian distribution of ion velocities, Lehner's result reduces to

$$S(E) = C \int_0^{\infty} \exp[-b(v+v_0)^2 - cg^2] \cdot \sinh(2bv v_0) \frac{g^3 \sigma(g)}{v_0(g)} dg \quad (1)$$

Here $S(E)$ is the probability that the neutron will be emitted with a laboratory energy between E and $E + dE$, C is a normalization factor, v is the velocity of the neutron in the laboratory system, v_0 is the velocity of the neutron in the ion center-of-mass system, g is the relative velocity of the ions, and $\sigma(g)$ is the cross section for the D-T fusion reaction. The coefficients b and c are equal to $M/2kT$, where M is taken to be total mass of the reacting ion system for b and the reduced mass for c , and T is the plasma temperature.

The only approximation involved in Lehner's derivation of Eq. (1) is that all particles may be treated non-relativistically. This is questionable only for the high-energy neutrons. At 14 MeV the relativistic factor γ is 1.015, where

$$\gamma = \frac{1}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}}$$

The factor varies negligibly over the range of interest (say, 13 to 17 MeV), so for our purposes it is sufficient to invoke relativistic mechanics in defining the location of the 14-MeV peak but not in discussing the shape. This effect moves the peak toward lower neutron energies, but only by about 20 keV. With this one reservation in mind, we will hence refer to Eq. (1) as an "exact" expression. It should be remembered that it is exact only for the case of an isotropic Maxwellian distribution of ion velocities, which is the only velocity distribution we will consider here. Although the expression for $S(E)$ in Eq. (1) is accurate, it has the disadvantage of requiring a numerical integration at each point E in the energy spectrum. For this reason, we consider what simplifications can be made without serious loss of accuracy.

III. THE VELOCITY EXPONENTIAL

In the energy range around the 14-MeV peak, the product $2bv v_0$ in Eq. (1) has a numerical value of about 7000. Thus the hyperbolic sine can be replaced by just a positive exponential term. If we make this change in Eq. (1), we can write the following expression for $S_1(E)$, the first approximation to $S(E)$.

$$S_1(E) = C_1 \int_0^{\infty} \exp[-b(v-v_0)^2] P(v_0) dv_0 \quad (2)$$

Here we also have inverted the function $v_0(g)$ and changed the integration variable. Now we see that the spectrum is a linear superposition of exponentials. For low plasma temperatures the velocity distribution $P(v_0)$ is very narrow, since the expression for v_0 is dominated by the nuclear Q-value (17.586 MeV) rather than the contribution from the ion kinetic energies (typically around 50 keV). Thus it seems a reasonable approximation to approximate $P(v_0)$ as a delta function

$$P(v_0) \approx \delta(v_0 - v_p)$$

This gives a second approximate form,

$$S_2(E) = C_2 \exp[-b(v-v_p)^2] \quad (3)$$

where v_p has the obvious meaning of the laboratory neutron velocity at the center of the peak. We shall refer to this as the velocity exponential form of the neutron energy spectrum. An expression essentially identical to Eq. (3) was given in an early paper² by Nagle and coworkers at Los Alamos.

In order to examine the accuracy of the velocity exponential form we have calculated $S(E)$ from Eq. (1) and $S_2(E)$ from Eq. (2) at 20 keV, a typical plasma temperature in current reactor concepts. In performing the numerical integration over g in Eq. (1), we used numerical values for the D-T fusion cross section taken from the compilation by Jarmie and Seagrave.³ In evaluating $S_2(E)$ using Eq. (3), a value of v_p was chosen so as to force agreement between $S(E)$ and $S_2(E)$ at 17 MeV, which is the upper end of our range of interest. The results of a calculation of the error in $S_2(E)$, relative to $S(E)$, are given in Fig. (1). The agreement over the range from 13.5 to 17 MeV is remarkably good, the maximum error being about 2%. The value of v_p thus derived corresponds to a peak-center energy E_p of 14.07 MeV. This value includes the small (-20 keV) relativistic correction mentioned in Section II.

IV. THE GAUSSIAN DISTRIBUTION

An even simpler form for the spectrum is used occasionally. First we write

$$v-v_p = \sqrt{2/m} (E^{1/2} - E_p^{1/2})$$

where m is the mass of the neutron and E_p is $1/2 mv_p^2$. To first order in $(E-E_p)/E_p$, we can write

$$v-v_p \approx (E-E_p)/\sqrt{2mE_p}$$

Substitution into Eq. (3) gives our third approximation.

$$S_3(E) = C_3 \exp[-b(E-E_p)^2/2mE_p] \quad (4)$$

For plasma temperatures not too far from 20 keV, E_p can be taken to be a constant, 14.07 MeV. This Gaussian form, Eq. (4), is the same as that quoted in a recent review by Brysk.⁴ The Gaussian form is of limited accuracy. For example if E is 15 MeV and kT is 20 keV, $S_3(E)$ is lower than the more accurate $S_2(E)$ by about 10%. Ever so, the simplicity of $S_3(E)$ lends itself to quick hand-calculations of thermal broadening effects.⁵

V. NUMERICAL RESULTS

In order to illustrate the sensitivity of spectrum-averaged cross sections to the plasma temperature, we have chosen the five nuclear reactions listed in Table I. Since the results are only meant to give a general indication of the sensitivity, we have used the simplest form of the spectrum discussed above, namely $S_3(E)$, Eq. (4). In more accurate numerical studies, one should use either the general expression $S(E)$ from Eq. (1) or the velocity exponential $S_2(E)$ from Eq. (3). The cross sections for the reactions chosen for this example exhibit fairly strong energy dependence near 14 MeV. The cross sections, obtained from the sources referenced in Table I, were compiled in ENDF/B format and then processed in multigroup form using the new multigroup processing code MINX.⁶ Seven representative⁵ plasma temperatures were used in constructing the weighting functions. In addition, results were calculated for delta-function and constant-spectrum weighting functions. The MINX results for a neutron group extending from 13.5 to 17.0 MeV are given in Table I.

It should be noted that the neutron cross sections governing neutron transport generally have less structure in the 14-MeV region than the $^{12}\text{C}(n,\text{total})^{12}\text{C}$ and $^{19}\text{F}(n,\alpha)^{16}\text{N}$ cross sections. Thus it is clear that a careful treatment of the thermal broadening phenomenon is not essential in preparing multigroup cross sections for neutron transport.

As can be seen from Table I, the situation is quite different for certain "effects" cross sections, such as the high-threshold activation reactions $^{54}\text{Fe}(n,2n)^{53}\text{Fe}$, $^9\text{Be}(n,p)^9\text{Li}$, and $^{209}\text{Bi}(n,3n)^{207}\text{Bi}$. The cross sections for these reactions rise in rather parabolic fashion from thresholds E_t near 14 MeV. The sensitivity is large (factors of five) for the ^{54}Fe reaction ($E_t < 14$ MeV) and extreme (orders of magnitude) for the other two ($E_t > 14$ MeV). Accurate calculations of ^{207}Bi production in lead shielding, for example, will require careful attention to the detailed shape of the fusion spectrum.

ACKNOWLEDGMENTS

The author acknowledges helpful discussions of this subject with G. I. Bell, W. B. Riesenfeld, and R. J. Doyas.

REFERENCES

1. G. Lehner, "Reaction Rates and Energy Spectra for Nuclear Reactions in High Energy Plasmas," *Z. Physik* 232, 174 (1970).
2. D. E. Nagle, W. E. Quinn, W. E. Riesenfeld, and W. Leland, "Ion Temperature Scylla as Determined from the Reaction $D(d,p)T$," *Phys. Rev. Letters* 3, 318 (1959).
3. Nelson Jarmie and John D. Seagrave (editors), "Charged Particle Cross Sections," Los Alamos Scientific Laboratory report LA-2014 (1956).
4. H. Brysk, "Fusion Neutron Energies and Spectra," *Plasma Physics* 15, 611 (1973).
5. D. W. Muir, "Sensitivity of Fusion Reactor Average Cross Sections to Thermal Broadening of the 14-MeV Neutron Peak," Los Alamos Scientific Laboratory report LA-5411-MS (1973).
6. D. R. Harris, R. J. LaBauve, R. E. MacFarlane, P. D. Soran, C. R. Weisbin, and J. E. White, "MINX, A Modular Code System for Processing Multigroup Cross Sections from Nuclear Data in ENDF/B Format," Los Alamos Scientific Laboratory report LA-UR-1766 (1973).
7. F. G. Perey, private communication (1973), (ENDF/B-IV, MAT 1274).
8. J. D. Picard and C. F. Williamson, "Sections Effices (n,p), (n, α) et (n,2n) sur ^{19}F et ^{23}Na Entre 13 et 21 MeV," *J. de Physique* 24, 813 (1963).
9. R. Bass, K. Krüger, B. Stagginnus, and P. Hauck, private communication (1965).
10. J. Terrell and D. M. Holm, "Excitation Function for $^{56}\text{Fe}(n,p)^{56}\text{Mn}$," *Phys. Rev.* 109, 2031 (1958).
11. W. G. Cross, R. L. Clarke, K. Morris, G. Slinn, M. H. Ahmed, and K. Beg, private communication (1963).
12. S. R. Salisbury and R. A. Chambers, " $^{54}\text{Fe}(n,p)$, (n, α), and (n,2n) Cross Sections," *Phys. Rev.* 140, B305 (1965).
13. R. H. Augustson and H. O. Menlove, "Cross Section for Delayed-Neutron Yield from the $^9\text{Be}(n,p)^9\text{Li}$ Reaction at Neutron Energies Between 14.1 and 14.9 MeV," *Trans. Am. Nucl. Soc.* 17, 494 (1973).
14. D. E. Alburger, "Beta Decay of ^9Li ," *Phys. Rev.* 132, 328 (1963).
15. J. B. Wilhelmy, private communication (1973).

Table I

Spectrum-Averaged^a Cross Sections (barns) For Various Plasma Temperatures

Reaction	¹² C(n,total)	¹⁹ F(n, α)	⁵⁴ Fe(n,2n)	⁹ Be(n,p)	²⁰⁹ Bi(n,3n)
Threshold (MeV)	0.0	1.60	13.63	14.26	14.43
$\sigma_{av}(0 \text{ keV})^b$	1.27(+0) ^d	1.62(-2)	3.08(-3)	0.00	0.00
$\sigma_{av}(4 \text{ keV})$	1.27(+0)	1.67(-2)	3.38(-3)	9.26(-7)	5.91(-8)
$\sigma_{av}(10 \text{ keV})$	1.28(+0)	1.76(-2)	3.65(-3)	5.57(-6)	3.90(-6)
$\sigma_{av}(20 \text{ keV})$	1.28(+0)	1.86(-2)	4.19(-3)	1.55(-5)	6.63(-5)
$\sigma_{av}(40 \text{ keV})$	1.29(+0)	1.99(-2)	5.34(-3)	3.87(-5)	6.63(-4)
$\sigma_{av}(100 \text{ keV})$	1.32(+0)	2.22(-2)	8.30(-3)	1.16(-4)	6.89(-3)
$\sigma_{av}(200 \text{ keV})$	1.34(+0)	2.39(-2)	1.20(-2)	2.34(-4)	2.48(-2)
$\sigma_{av}(400 \text{ keV})$	1.36(+0)	2.50(-2)	1.63(-2)	3.78(-4)	5.46(-2)
$\sigma_{av}(\text{constant})^c$	1.30(+0)	2.09(-2)	5.88(-3)	4.33(-5)	1.73(-4)
Source of Data	Ref.7	Ref.8,9	Ref.10,11,12	Ref.13,14	Ref.15

a. The averages extend over neutron energies from 13.5 to 17.0 MeV.

b. In this case, the neutron spectrum is a δ -function at 14.1 MeV.

c. The spectrum here is a constant from 13.50 to 14.92 MeV and zero elsewhere.

d. To be read as $1.27 \times 10^{+0}$

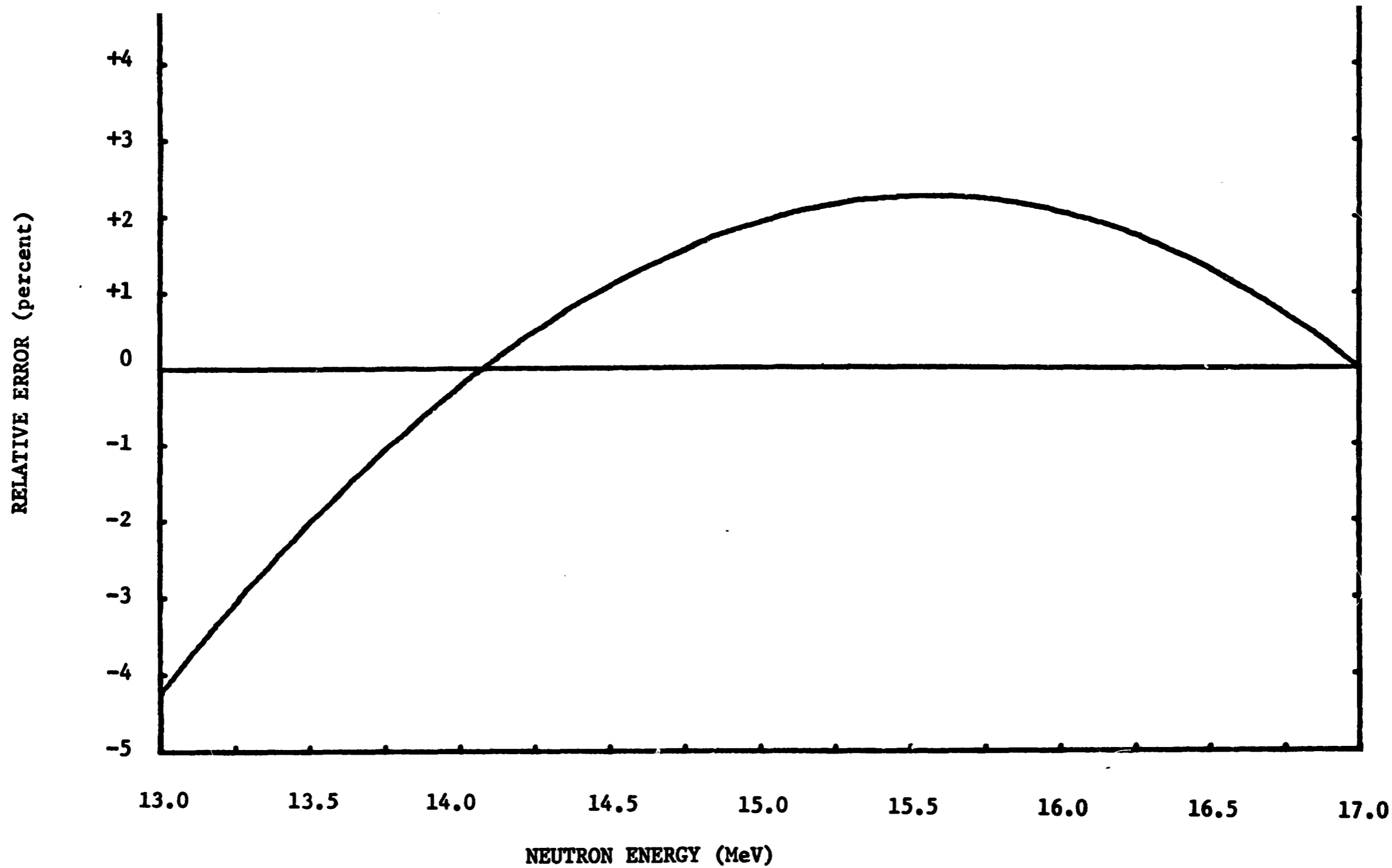


Fig. 1. Relative Error of the Velocity Exponential Approximation.