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GRAPHICAL METHOD OF OBTAINING CRITICAL MASSES OF WATER-TAMPED

WATER BOILERS

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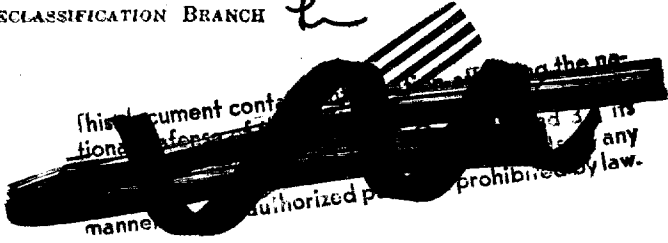
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ABSTRACT

An approximate method of calculating the critical mass of 25 as a function of spherical core volume of a wide variety of enriched uranium in water solutions surrounded by water is outlined. The results are expressed in such a form that one may read, after selecting a single parameter, the critical mass of 25 and the corresponding core volume from the intersection of two superimposed curves. By choosing a series of values for the parameter, one may easily compute the critical mass of 25 as a function of core involved.

This information is particularly useful for purposes of safety. One can rapidly obtain minimum critical masses and optimum water solution concentrations of several uranium compounds having various 25 isotope enrichment percentages. The approximations are in such a direction as to yield critical masses that are slightly low.

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GRAPHICAL METHOD OF OBTAINING CRITICAL MASSES OF WATER-TAMPED WATER BOILERS

In a previous report (LA-399) it has been shown that the variational treatment of the integral diffusion equation for a spherical water-tamped boiler yields the following critical equation:

$$N_H^T \sigma_a^H = N_{25} \sigma_a^c \left\{ \omega \left[H + 3\delta A_H / (5 - 3\delta) \right] - \left[G + 3\delta A_G / (5 - 3\delta) \right] \right\} \quad (1)$$

Here N_H^T and N_{25} are respectively the atomic hydrogen density in the tamper (water) and the atomic density of 25 in the core. The thermal absorption cross-section of an hydrogen atom is $\sigma_a^H = .31$ barns, and the absorption cross-section peculiar to the core that is associated with one 25 atom is σ_a^c . If the core is considered simply as a homogeneous mixture of uranium and water,

$$\sigma_a^c = \sigma_a^{25} + [(1-p)\sigma_a^{28} - D\sigma_a^H] p, \quad (2)$$

where p is the 25 enrichment fraction and D is the number of atoms of hydrogen (i.e., half molecules of water) displaced by one uranium atom in the core. If the core is a solution of some uranium compound in water, one must add to the numerator of the second term of Equation (2) the thermal absorption cross section of those atoms of the compound associated with one uranium atom, and the displacement D is that produced in water by the uranium plus its chemically bound atoms of the compound.

The number of thermal neutrons surviving the resonance capture of 28 during moderation that appear either in core or tamper per absorption by the material peculiar to the core associated with one 25 atom is the "effective γ " defined as follows:

$$\omega = \gamma \left(\sigma_a^{25} / \sigma_a^c \right) P \left(\sigma^3 / 28 \right) \quad (3)$$

The fraction of the fission neutrons that survive capture by 28 during moderation is the function of the effective scattering cross-section in barns per 28 atom, $\sigma^3 / 28$.

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shown in Figure 1.

$$P = \exp \left[- \left(\int \sigma_{28} dE/E \right) E_{eff} / (\sigma^{-3/28}) \right] \quad (4)$$

The circled points in Figure 1 correspond to the experimentally determined values of the effective $(\int \sigma_{28} dE/E)$ obtained by A. C. G. Mitchell and collaborators (CP-1676) for various moderating media.

The bracketed expression in equation (1) is a function of w and the diameter of the core. For a given value w the leakage functions $[H + 3\gamma A_H / (S - 3\gamma)]$ and $[G + 3\gamma A_G / (S - 3\gamma)]$ can be obtained numerically. In (LA-399) the following formula was derived for the parameter γ appearing in the parabolic trial function, $n = n_0 [1 - \gamma (r/R)^2]$, for the thermal neutron density in the core; R is the radius of the core.

$$3\gamma = 2.5 \left[1 - \sqrt{1 - 3.36C(1-C)} / (1-C) \right] \quad (5)$$

where

$$C = (w A_H - A_G) / (w B_H - B_G).$$

The three functions H , A_H , and B_H , depending on the core diameter, d , and the shape of the total slowing down kernel, $K(r)$, (c. f. Fig. (6) of LA-399) were obtained by numerical integrations of the following equations:

$$H(d) = \int_0^d (4\pi r^2 K) \left[1 - (3/2)(r/d) + (1/2)(r^3/d^3) \right] dr$$

$$A_H(d) = \int_0^d (4\pi r^2 K) \left[(r/d) - (10/3)(r/d)^2 + 3(r/d)^3 - (2/3)(r/d)^5 \right] dr \quad (6)$$

$$B_H(d) = 2 \int_0^d (4\pi r^2 K) \left[(r/d) - (14/3)(r/d)^2 + 2(r/d)^3 - (1/5)(r/d)^5 \right] dr$$

The expressions for the three other functions, G , A_G , and B_G , are given by the above

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integrals (6) upon replacing K by the diffusion kernel, $(e^{-r/L}) / (4 \pi r L^2)$, where $L = 2.88$ cm is the thermal neutron diffusion length in water.

For a given value of w the critical mass of 25 as a function of core volume in liters V , is given by Equation (1), namely:

$$M_{25} = \frac{235 N_{25}}{9 N_H^T} V (1 \text{ Kg / liter}) = (\sigma_a^{25} / \sigma_a^c) F(w, V)$$

$$\text{where } F(w, V) = \frac{235 \sigma_a^H}{9 \sigma_a^{25}} V / \left\{ w \left[H + \frac{3}{5-3V} A_H \right] - \left[G + \frac{3}{5-3V} A_G \right] \right\} \quad (7)$$

$$\text{and } \sigma_a^H = .31 \text{ barns, } \sigma_a^{25} = 640 \text{ barns.}$$

The quantity $F(w, V)$, which is proportional to the critical mass in Kg is plotted as a family of curves on the transparent sheet for various values of w . For a given boiler $\sigma_a^{25} / \sigma_a^c$ is known. In order to obtain the critical mass as a function of core volume, one must be able to evaluate w , which for the given boiler is a known constant (i.e., $\nabla (\sigma_a^{25} / \sigma_a^c)$ times the survival fraction $P (\sigma^S / 28)$). One may obtain the effective scattering cross-section per 28 atom as follows.

Let us define a "sphere of moderation" having a radius R^0 given by

$$R^0{}^2 = R^2 + \overline{r^2} \quad (8)$$

where R is the actual core radius and $\overline{r^2}$ is the mean square thermalization displacement for neutrons in the water tamper ($\overline{r^2} = 200 \text{ cm}^2$). During slowing down a neutron may suffer collisions in the tamper as well as in the core, survive 28 capture, and still produce fission in the core. The sphere of moderation is the estimated volume in which we shall assume scattering may complete with 28 resonance absorption. The effective hydrogen density, N_H^0 , in the sphere of moderation is given by

$$d^{-3} N_H^0 = N_H^T (d'^3 - d^3) + N_H^C d^3 \quad (9)$$

where N_H^C is the hydrogen density in the core. Now, since $N_H^C = N_H^T - (D/p)N_{25}$,

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the ratio N_{25}/N_H^T is given by

$$N_{25}/N_H^T = (d'^3/d^3) / \left[\frac{N_H d'^3}{N_{25} d^3} + \frac{D}{P} \right] \quad (10)$$

Thus an alternate expression for the mass of 25 in a volume V is

$$M_{25} = \frac{235}{4} \frac{\pi}{6} (d^2 + 800)^{3/2} \gamma \quad (11)$$

where $\gamma = 1 / \left[(N_H d'^3 / N_{25} d^3) + D/P \right]$

Figure 2 shows M_{25} plotted as a function of V for various values of the parameter γ .

The effective scattering cross-section per 28 atom may now be expressed in terms of the parameter γ , namely:

$$\sigma_{28}^s = \left[\sigma_s^H (d' N_H^T) + \sigma_s^U (d^3 N_U) \right] / (d^3 N_{28}) = \left[\sigma_s^H (p/\gamma - D) + \sigma_s^U \right] / (1-p) \quad (12)$$

Here the important term is the scattering cross-section of hydrogen $\sigma_s^H = 20$ barns.

σ_s^U is the scattering cross-section associated with one uranium atom in the compound used.

The procedure for calculating the critical mass as a function of volume is now simple. The constants necessary are $(\sigma_a^{25}/\sigma_a^c)$, p , \bar{v} , and D . One places the horizontal index on the sheet of F curves labelled (σ_{25}/σ_c) over the value of $(\sigma_a^{25}/\sigma_a^c)$ on Figure 2. Selecting any value of γ enables one to compute $\sigma^s/28$ and in turn w by using Figure 1 to determine the survival fraction P in Equation 3. The critical mass and volume are read from Figure 2 at the point of intersection of the particular γ curve and its corresponding w curve.

As an example, consider a boiler containing a solution of 14 o/o enriched solution of UO_2SO_4 . The displacement produced by UO_2SO_4 in water is $D \approx 6.1$. The oxygen absorption is negligible and that of sulphur is .53 barns; thus $(\sigma_a^{25}/\sigma_a^c) = 0.99$. Selecting the values of γ listed below, one calculates $\sigma^s/28$ by Equation 12 and the survival fraction is read from Figure 1. Using the value $\bar{v} = 2.09$, one then obtains w .

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y	$\sigma^s/28$	P	w	V	M ₂₅
.0016	(1.9)10 ³	.955	1.97 ₅	12.1 _l	1.41 Kg
.0014	(2.2)10 ³	.959	1.98 ₄	15.3	1.30
.0012	(2.6)10 ³	.962	1.99 ₀	15.3	1.19
.0011	(2.9)10 ³	.965	1.99 ₇	16.5	1.15
.0010	(3.1 ₅)10 ³	.967	2.00 ₁	19.0	1.12
.0009	(3.5)10 ³	.969	2.00 ₅	22.5	1.12
.0008	(4.0)10 ³	.971	2.00 ₉	29.0	1.17

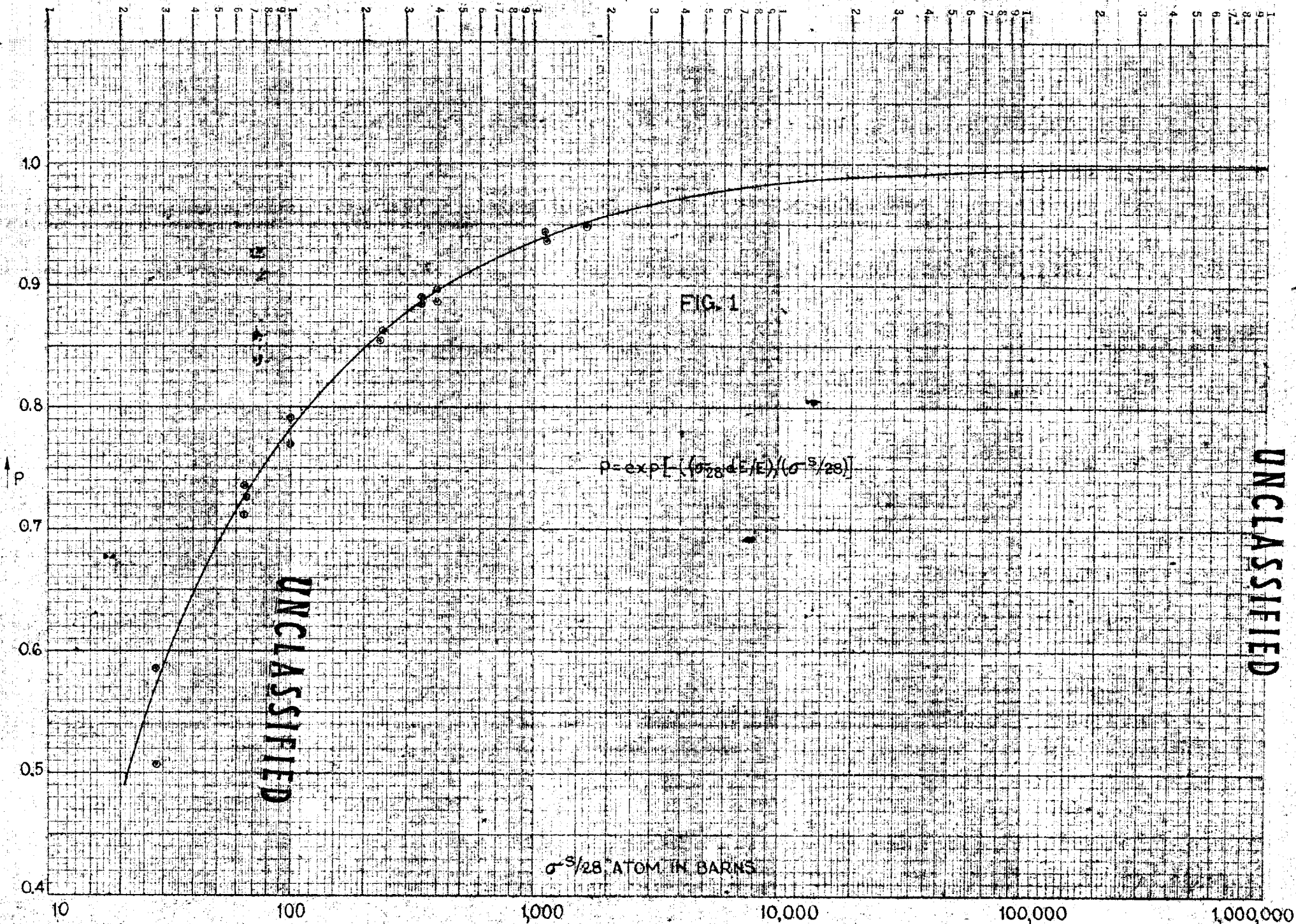
The last two columns are the value of M₂₅ and V read from Figure 2 at the intersection of the corresponding y and w curves.

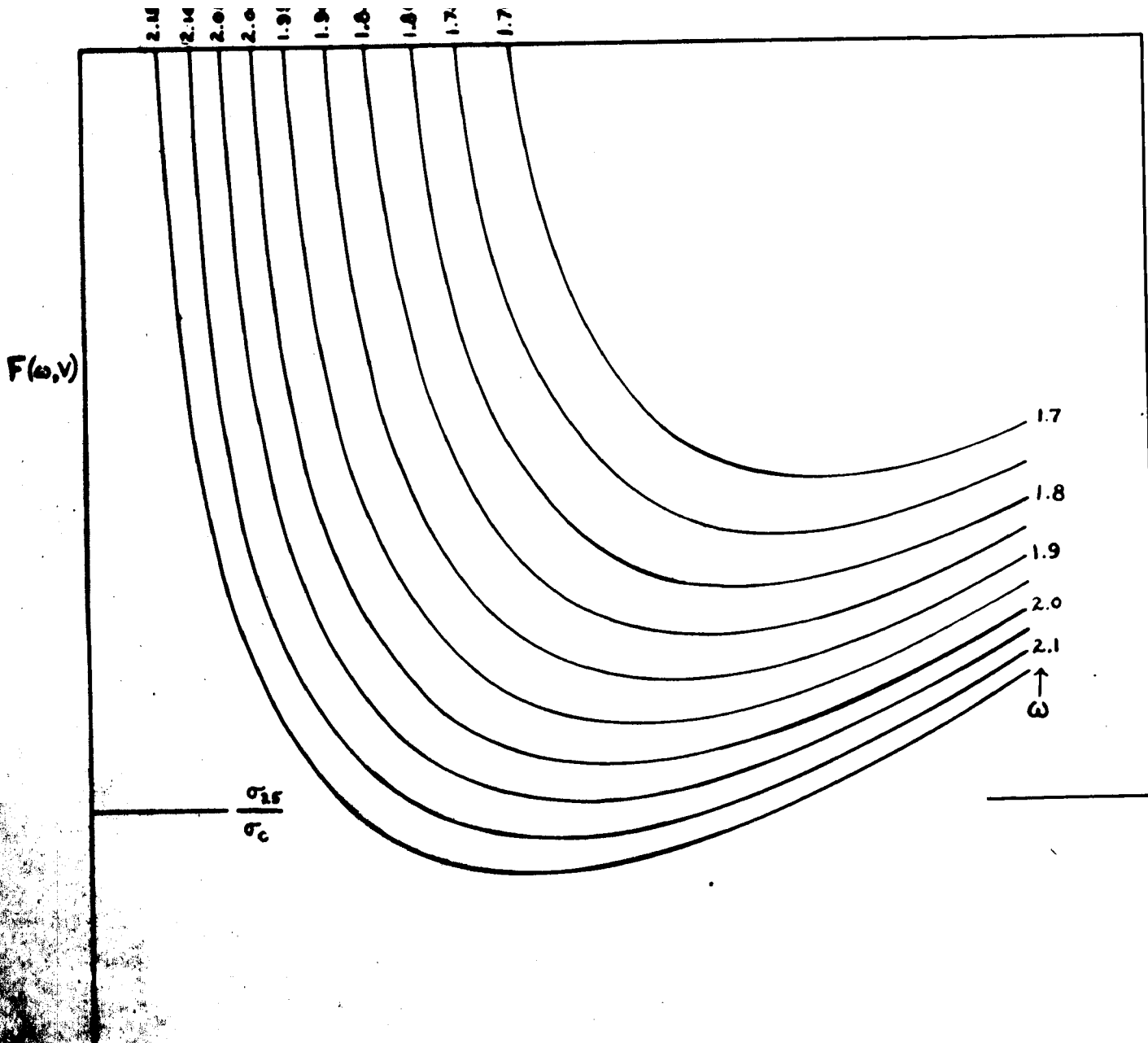
The experimental value of the critical mass of a 15 liter water tamped 14 o/o enriched uranyl sulphate boiler, as reported in LA-241, is 1.2[±] .05 Kg. An interpolation in the above table between 1.19 Kg and 1.30 Kg yields, for V = 15 liters, a mass of 1.20 Kg.

The graphical method is particularly useful in obtaining minimum critical masses and optimum 25 concentrations for purposes of safety. We have assumed in the treatment outlined here that slowing down and thermal diffusion lengths in the core solution are the same as in the normal density water tamper. Actually the effect of dilution of the water in the core by the uranium in solution is to increase the average slowing down displacement of fission neutrons. The leakage from the core is thus underestimated. Consequently, for a given w the function F(w,V), which is proportional to the critical mass, is a minimum estimate. In LA-399 the 14 o/o uranyl sulphate boiler was treated more accurately; displacement effects on the leakage were not ignored. This more involved theory which is not easily summarized in graphical form yielded, for the 15 liter volume boiler, a critical mass only 9 o/o greater than that obtained by the graphical approximation presented here.

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