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TITLE ON SIZE SCALING IN SHOCK HYDRODYNAMICS AND THE STRESS-STRAIN BEHAVIOR OF COPPER AT EXCEEDINGLY HIGH STRAIN RATES

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SUBMITTED TO Hypervelocity Impacts in Space

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# **On Size Scaling in Shock Hydrodynamics and the Stress-Strain Behavior of Copper at Exceedingly High Strain Rates**

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## **Summary**

In recent years the Hypervelocity Microparticle Impact (HMI) project at Los Alamos has utilized electrostatically accelerated iron spheres of microscopic dimensions to extend the range of controlled hypervelocity impact experiments to about  $100 \times 10^5$  cm/sec, well beyond the meteoroid velocity range and about an order of magnitude beyond the data range for precisely controlled impact tests with ordinary macroscopic particles. But the extreme smallness of the micro impact events brings into question whether the usual shock-hydrodynamic size scaling can be assumed. It is to this question of the validity of size scaling (and its refinement) that the present study is directed.

Hypervelocity impact craters are compared in which the two impact events are essentially identical except that the projectile masses and crater volumes differ by nearly 12 orders of magnitude--- linear dimensions and times differing by 4 orders of magnitude. Strain rates at corresponding points increase 4 orders of magnitude in the size reduction.

Departures from exact scaling, by a factor of 3.7 in crater volume, are observed for copper targets---with the micro craters being smaller than scaling would predict.

This is attributed, using a well-established relation for the dependence of crater volume upon target yield stress, to a factor 4.7 higher effective yield stress occurring in the micro cratering flow. This, in turn, is because the strain rate there is about  $10^8$ /sec as compared to a strain rate of only  $10^4$ /sec in the macro impact. This pronounced strain rate effect in copper is compatible with recent theoretical models by Follansbee, Kocks, Rollett and others.

Aluminum targets are found to behave similarly, though primary emphasis has been placed on the copper data because the high strain rate properties of copper have been discussed more fully in recent literature.

Work in this area may be of interest for several reasons: The classical laws of shock-hydrodynamic size scaling, as applied to condensed media, are put to a much more stringent test than hitherto. The departure from strict size scaling is quantified and explained in terms of basic material properties. Also the measurement of impact craters for very small impact events leads to the determination of metal yield stresses at strain rates more than two orders of magnitude greater than have been obtained by other methods. The determination of material strengths at these exceedingly high strain rates is of obvious fundamental importance.

# **On Size Scaling in Shock Hydrodynamics and the Stress-Strain Behavior of Copper at Exceedingly High Strain Rates**

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## **Abstract**

In recent years the Hypervelocity Microparticle Impact (HMI) project at Los Alamos has utilized electrostatically accelerated iron spheres of microscopic dimensions to generate hypervelocity impact experiments to about  $100 \times 10^5$  cm/sec, about an order of magnitude beyond the data range for precisely controlled impact tests with ordinary macroscopic particles. But the extreme smallness of the micro impact events brings into question whether the usual shock-hydrodynamic size scaling can be assumed. It is to this question of the validity of size scaling (and its refinement) that the present study is directed.

Hypervelocity impact craters are compared in which the two impact events are essentially identical except that the projectile masses and crater volumes differ by nearly 12 orders of magnitude---linear dimensions and times differing by 4 orders of magnitude. Strain rates at corresponding points increase 4 orders of magnitude in the size reduction. Departures from exact scaling, by a factor of 3.7 in crater volume, are observed for copper targets---with the micro craters being smaller than scaling would predict. This is attributed to a factor 4.7 higher effective yield stress occurring in the micro cratering flow. This, in turn, is because the strain rate there is about  $10^8$ /sec as compared to a strain rate of only  $10^4$ /sec in the macro impact.

The measurement of impact craters for very small impact events leads to the determination of metal yield stresses at strain rates more than two orders of magnitude greater than have been obtained by other methods. The determination of material strengths at these exceedingly high strain rates is of obvious fundamental importance.

## **I. Introduction**

The Hypervelocity Microparticle Impact (HMI) project,<sup>1</sup> has obtained impact data from microscopic iron spheres impacting targets at impact velocities from  $1 \times 10^5$  cm/sec to  $100 \times 10^5$  cm/sec. The iron spheres are charged and accelerated electrostatically<sup>2</sup> in a 6 MeV Van de Graaff accelerator. Each impact is characterized by simultaneous measurement of projectile charge and velocity using careful cross-correlation techniques<sup>3</sup>. Measurement of impact crater characteristics is performed using a scanning electron microscope. A typical crater in copper is shown in Fig. 1. Impact studies have been performed on a variety of materials relevant both to practical impacts in space and to the study of impact physics. In this discussion we focus on impacts in copper and aluminum in order to compare with existing libraries of data from macroscopic impact physics research.

## **II. Departure From Strict Size Scaling for Impact Craters in Soft Copper Targets**

Micro impacts, when compared to the same impacts at ordinary sizes, make it possible to put classical shock-hydrodynamic size scaling to severe tests in which corresponding masses (and other

extensive variables) are scaled down nearly 12 orders of magnitude---linear dimensions and times being scaled down 4 orders of magnitude. Strain rates increase four orders of magnitude in the size reduction.

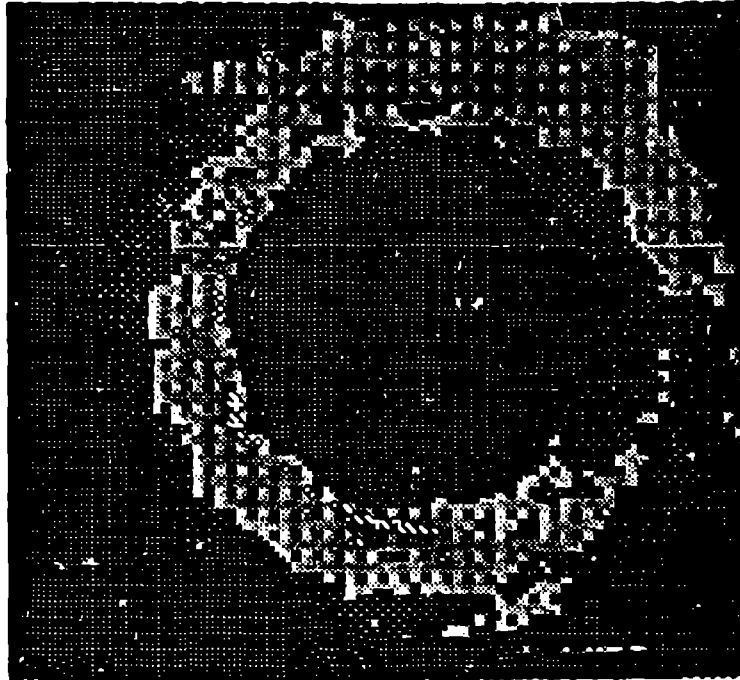


Fig. 1. A typical hypervelocity impact crater in copper produced by a microscopic iron sphere impacting at  $12.5 \times 10^5$  cm/s. The craters produced by microscopic impacts are axisymmetric and appear to be geometrically similar to the craters produced by macroscopic impacts.

As Fig. 2 we reproduce the pertinent data for copper, to call attention to the fact that the normalized target crater volume is a factor of 3.7 smaller for the IBF micro-impacts (projectile masses  $0.25 \times 10^{-12}$  gm and  $1.5 \times 10^{-12}$  gm) than for the large scale impacts (projectile masses 0.15 gm and 0.50 gm) at the same impact velocity ( $6 \times 10^5$  cm/sec). Exact size scaling would, of course, require that these normalized crater volumes be equal. Thus, the size reduction, by a factor of about  $0.3 \times 10^{12}$  in the projectile mass (or, equivalently, by a factor of  $0.7 \times 10^4$  in projectile diameter) has not only reduced the crater volume by a factor of  $0.3 \times 10^{12}$ , as it should in accord with strict scaling, but also by an additional factor of 3.7.

### III. Strain-Rate Effect As a Reason for Scaling Failure

We believe that this failure to scale exactly is due to strain-rate effects within the copper. More fully we develop here the notion that the higher strain rates in the smaller flows<sup>5</sup> cause a higher effective flow stress in the smaller flows and a correspondingly smaller crater.

In Fig. 3 we reproduce (in addition to impact data for copper) a well-known correlation formula due to Sorensen<sup>6</sup> for hypervelocity impact data. It shows, in particular, that crater volume  $V$  varies with target shear yield strength  $s$  as  $s^{-0.845}$ . This dependence of  $V$  on  $s$  is shown by Sorensen to fit a wide range of impact data for metal targets, encompassing a variation of  $s$  from 0.13 kilobars for lead to nearly 10 kilobars for a steel. See Ref. 6 for a detailed discussion. Similar dependences of  $V$  on  $s$  have been established in hydrocode studies. Thus, adopting Sorensen's correlation, we find that the observed 3.7-fold reduction in crater volume could be caused by a yield stress increase by a factor of 4.7.

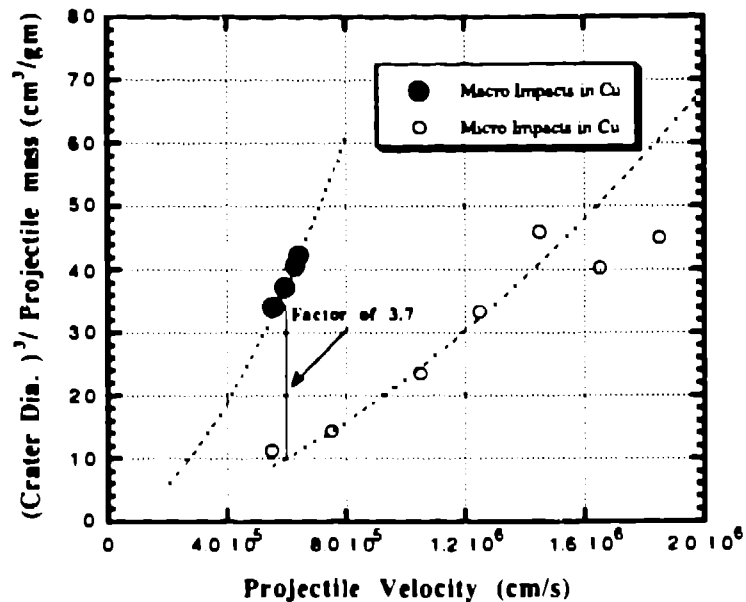


Fig. 2. Hypervelocity impact cratering data for copper. The upper curve, with four representative data points, is from Sorensen's empirical correlation of (macro) impact data for copper. See also Fig. 3 and Sorensen's paper (Ref. 6) for further information on Sorensen's work. The lower curve shows the (micro) impact data on copper. Of importance to the present discussion is the fact that the normalized crater volumes in the micro impacts are smaller by a factor of 3.7. The two curves would coincide if size scaling were exact.

#### IV. Strain Rates in Hypervelocity Impact

Next, we need a reasonably good estimate of the strain rates occurring in the cratering process. Specifically, it will suffice to estimate the average strain rate during the crater formation for the 0.3 gm (macro) impact at  $6 \times 10^5$  cm/sec since we already know the ratio of the strain rates in the micro and macro events. Making this estimate is the object of the present Section.

It is useful to recall the prominent features of such a hypervelocity impact. The initial shock pressure is given by

$$P \approx \rho U_s U_p = 1.9 \times 10^{12} \text{ dynes/cm}^2 = 1.9 \text{ megabars,}$$

since the shock particle velocity  $U_p$  is  $3 \times 10^5$  cm/sec from symmetry, the density  $\rho$  is 8.9 gm/cc and the shock wave velocity  $U_s$  associated with the given particle velocity is  $7 \times 10^5$  cm/sec. This is more than three orders of magnitude greater than material strength, implying that the early phases of the impact are hydrodynamic with strength playing a negligible role. This shock front and the attached pressure pulse propagate almost hemispherically into the thick copper target, and serve to set the engulfed copper into nearly hemispherical motion. Were it not for the finite yield strength of the copper the (nearly hemispherical) crater would grow without limit. What happens instead is that the 1.385 kilobar copper yield strength limits the crater volume to about 83 times the volume of the impacting projectile, in accord with Sorensen's correlation formula as applied to this impact.

Two-dimensional finite difference hydrocode calculations (axisymmetric and time dependent, incorporating material compressibility and strength effects by utilizing available material property formulations) can provide us with a very detailed description of the impact process, and such calculations have been provided by a number of investigators over the past 30 years.

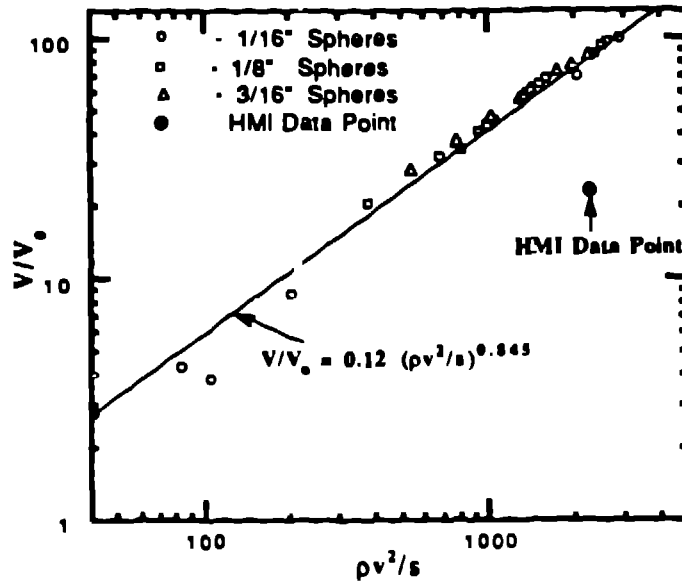


Fig. 3. Sorensen's data correlation for copper, from Ref. 6. The data extend to about  $7.5 \times 10^5$  cm/sec. The analytical correlation is for several metals and therefore does not fit copper exactly, although quite well. It may be noted that the factor 3 variation in projectile diameters (27 in masses) does not cause an apparent size effect in the macro data points. The point at  $v = 6 \times 10^5$  cm/sec on the micro curve (Fig. 2) has been transformed to this plot and is seen to be substantially below the macro data. (To transform this point the values  $\rho = 8.9$  gm/cm<sup>3</sup> and  $s = 1.385 \times 10^9$  dynes/cm<sup>2</sup> were used for the density and shear yield strength of annealed copper.) The projectile mass ratio between the micro and macro experiments is nearly twelve orders of magnitude, as explained in the fourth footnote.

While a specific computation has not yet been performed for our 0.3 gm,  $6 \times 10^5$  cm/sec impact into copper, suitable computed results from other impacts have been reported in the literature and can be used to deduce (using only Sorensen's correlation formula and dimensional analysis) useful estimates of the effective strain rate in our impact. Dienes<sup>7</sup> has reported calculations for a spherical aluminum projectile (diameter 0.476 cm) impacting a hard aluminum target (shear yield strength 2.39 kilobars) of density 2.7 gm/cc at a velocity of  $7.3 \times 10^5$  cm/sec. He finds for times of 2, 4, 8 and 16 microseconds that the crater depth is 0.4, 0.8, 0.9, and 1.0 of its final value. Hence for present purposes we can take this aluminum crater formation time to be 15 microseconds. Next we note that Sorensen's correlation formula:

$$V/V_0 = C_1(\rho v^2/s)^{.845}$$

is entirely equivalent to

$$T/T_0 = C_2(\rho v^2/s)^{\frac{.845}{3}} = C_2(\rho v^2/s)^{.282}$$

when re-expressed to give the time T for crater formation. Here  $T_0$  is any suitable measure of the impacting projectile size (such as the time it takes the free flying projectile to move one diameter) that must, of course, be the same measure for the two impacts under consideration. Thus  $T_0$  would be:

$$T_0 = 0.476 \text{ cm} / (7.3 \times 10^5 \text{ cm/sec}) = .65 \text{ microseconds}$$

for the aluminum problem and

$$T_0 = 0.400 \text{ cm} / (6.0 \times 10^5 \text{ cm/sec}) = .67 \text{ microseconds}$$

for our copper impact. Next for the two cases of aluminum and copper impacts, the quantities  $(\rho v^2/s)$  and  $(\rho v^2/s)^{.282}$  would be:

$$(\rho v^2/s) = 602; \quad (\rho v^2/s)^{.282} = 6.08$$

and for the copper impact;

$$(\rho v^2/s) = 2313, \quad (\rho v^2/s)^{.282} = 8.88.$$

Hence the 15 microsecond crater formation time for aluminum scales to

$$T = (8.88/6.08)(.67/.65)15 \text{ microseconds} = 23 \text{ microseconds}$$

for our copper impact.

In another problem from Ref. 7 a soft aluminum target (shear yield 0.75 kilobars) was used and total plastic work was reported instead of crater depth. (Other problem parameters were the same as in the hard aluminum impact.) At 4, 8 and 16 microseconds the total plastic work was 20%, 50%, and 95% of the final value when the flow was completely arrested. This again suggests a time like 15 microseconds for flow arrest. Scaling this over to our copper impact, by a calculation similar to that detailed for the hard aluminum, gives a time of 16 microseconds for the copper impact.

We need also to know the average strain occurring in the plastically deforming material when the crater is formed. Here both computational and experimental evidence (where targets thicker than about two crater depths react much the same as semi-infinite targets subjected to the same projectile impact) suggest that the target material within about one crater radius of the crater is effective in arresting the flow. For this material the strain field is a maximum, about 0.6 at the crater surface, dropping to essentially zero a crater depth into the material. A suitable average strain for this plastically deforming material is about 0.2. (This value may be reliable only to about a factor of two.) Dividing it by the above crater formation times of 23 microseconds and 16 microseconds implies average strain rates of  $0.86 \times 10^4/\text{sec}$  and  $1.25 \times 10^4/\text{sec}$ . Hence we adopt a value of  $1.0 \times 10^4/\text{sec}$  as an average strain rate in our 0.3 gm copper impact, recognizing that this value is uncertain by a factor of two. Surprisingly, perhaps, this uncertainty is tolerable in present considerations because of the weak dependence of yield stress on strain rate.

It may be noted that a more accurate determination of this average strain rate could be made as part of a hydrocode computation of our copper impact. For this purpose we suggest

$$\bar{\epsilon}_p = \frac{\sum_N \sum_K W_p(K,N) \dot{\epsilon}_p(K,N)}{\sum_N \sum_K W_p(K,N)}$$

where K is the cell number and N is the time step number. The formula gives an average strain rate, averaged over all (Eulerian) cells and all time steps, with each  $\dot{\epsilon}_p(K,N)$  weighted in proportion to the amount of plastic work  $W_p(K,N)$  occurring in the cell during the time step.

## V. Comparison of Results With Recent Theoretical Expectations

In Section II we saw that when the projectile mass was reduced by a factor  $0.3 \times 10^{12}$ , the crater volume was reduced not only by this factor, as expected from size scaling, but by an additional factor of 3.7.

In Section III we found, using a well-established empirical correlation, that the factor 3.7 crater volume reduction would be caused by a yield stress increase by a factor of 4.7.

In Section IV we used published computational results for the crater formation process, together with the Sorensen correlation formula, to establish that the average strain rate in the macro impact was about  $1.0 \times 10^4$ /sec. This means that the average strain rate in our micro impact [which must be greater by a factor of  $(0.3 \times 10^{12} \times 3.7)^{.333} = 1.03 \times 10^4$ ] is about  $1.0 \times 10^8$ /sec. So it remains to ask whether it is indeed reasonable to expect a factor 4.7 increase in the flow stress over this strain rate regime.

Any such estimates must be theoretical because measurements have been limited to strain rates below about  $10^6$ /sec. Fortunately, the properties of copper at exceedingly high strain rates has been the subject of recent investigations by Follansbee, Kocks, Rollett and others. (See Refs 8 and 9 and literature cited there.) In Fig. 4 the theoretical stress *versus* strain rate curve is reproduced (from Fig. 2 of Ref. 9) for a constant strain of 0.1. This strain is taken to be an average strain during the cratering flow, corresponding to the estimate made in Section III that the average total strain is about 0.2. (Also the theoretical stresses were reduced by a factor of  $\sqrt{3}$ , in accord with the von Mises yield condition, because longitudinal yield stresses were used, whereas shear yield stresses are used throughout the present paper.)

Plotted also in Fig. 4 are our two experimental points  $\sigma = 1.385$  kilobars at  $\dot{\epsilon}_p = 10^4$ /sec and  $\sigma = 6.5$  kilobars at  $\dot{\epsilon}_p = 10^8$ /sec

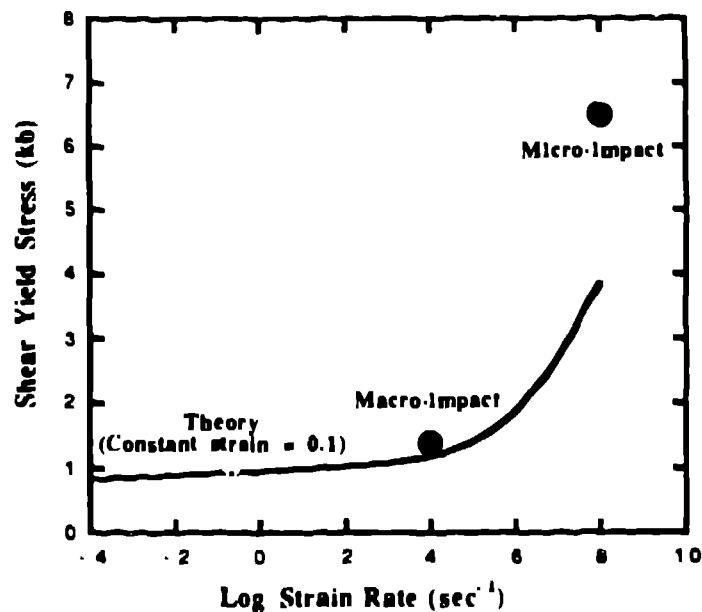


Fig. 4. Shear yield stress  $\sigma$  versus strain rate for copper strained to 0.1. The theoretical curve is from Ref. 9, the results from Fig. 2 of the reference being re-presented here in terms of shear yield stress at a constant strain of 0.1.

The most important conclusion to be drawn from the present comparison is that both the theory (Refs 8 and 9) and experiment are indicating a very substantial strain rate effect in copper in the  $10^4$ /sec to  $10^8$ /sec strain rate regime. The experimental effect is somewhat the larger, the yield stress increasing by a factor of 4.7 as compared to 2.8 for the theoretical curve. In the theoretical modelling<sup>9</sup> this strain rate effect has been attributed to a gradual transition, as the strain rate is increased, from thermally-activated to viscous-drag controlled deformation.

The experimental factor of 4.7 depends upon only the experimental volume ratio of 3.7 (Fig. 2) and the Sorensen correlation formula, and is estimated here to be reliable to 10% or less. Other aspects of the comparison are discussed in the next section.



## VI. Comments on Sources of Error

It was remarked in Section IV that the estimate of the average strain rate in the macro impact was uncertain by a factor of about two. In the Fig. 4 data plot the experimental points are represented as circles with diameters spanning a factor of four in the strain rate. It is readily apparent that a lateral shift of the macro data point to either of the extreme positions (causing an equal lateral shift of the micro point) would have only a very small effect on the comparison.

In Section V we estimated an average strain in the cratering flow to be 0.1. This strain was used to select the appropriate constant-strain theoretical curve from Ref.(9). Had one used 0.05 or 0.2 instead of 0.1, the corresponding average-strain theoretical curve, in the two cases, would be below or above the macro experimental point and in somewhat poorer agreement with that point. Here, however, an alternative interpretation is useful: The properties of copper at strain rates around  $10^4$ /sec and below, where test data and theoretical understanding have been in accord for years, can be assumed known. One then selects that particular constant-strain curve from Ref. (9) that causes agreement with the macro data point. This constant-strain curve is the one for an average strain in the cratering flow of about 0.13, instead of our estimated value of 0.1 given above. (This might, in the present situation, be a better way to estimate the average strain in the cratering flow). In any event, the theoretical strain rate enhancement factor (taken to be 2.8 in the last paragraph of Sec. V) is a weak function of which constant-strain curve one uses and would not be substantially affected.

Finally, we recall that the impacting spheres in the micro experiments are actually iron instead of copper. In our comparison of the micro- and macro-events these iron projectiles are assumed to be equivalent to copper projectiles of equal mass. This equal-mass assumption has been investigated extensively in test work and in computer studies<sup>7</sup> and is found to be accurate for sufficiently high velocities and/or density ratios sufficiently close to unity. For the present application at  $6 \times 10^5$  cm/sec, with iron and copper projectiles, the cratering effects on thick copper targets are expected to be essentially identical.

## VII. Extension to Aluminum

The IBF data for aluminum target impacts exhibits essentially the same behavior as copper, i. e. the micro crater volumes are small by about a factor of 4, corresponding to a strain rate enhancement of yield stress by a factor of 5.

Attention here has been focussed on copper because its constitutive modelling appears to be more advanced, but it seems likely that aluminum (another FCC metal) will exhibit similar behavior to copper at high strain rates<sup>10</sup>.

## VI. Conclusion

The classical laws of size scaling, as applied to the shock hydrodynamics of condensed media, have been put to severe test. The size reduction spans four orders of magnitude in length or time dimensions, or 12 orders of magnitude in extensive variables, such as corresponding masses or volumes. The observed departure from exact scaling is by a factor of 3.7 in extensive variables, or by 1.5 in corresponding lengths or times.

The departure is attributed to strain rate enhancement of the flow stress in the copper targets. This dramatic rise in flow stress at very high strain rates had already been anticipated in the theoretical literature.

Work in this area is of interest for several reasons:

1. It validates and/or refines classical shock-hydrodynamic size scaling, and thus pertains directly to the important engineering area of scale model experimentation.

2. For velocities above about  $15 \times 10^5$  cm/sec, the only precisely controlled hypervelocity experiments have been performed, at Los Alamos and elsewhere, with electrostatically accelerated microparticles. Experimental data are available for velocities throughout the meteoroid velocity range (to about  $70 \times 10^5$  cm/sec) and beyond. To understand this valuable data source, and to be able to scale it with confidence to larger impact events we need, as done here for copper, to quantify the departures from exact size scaling and attribute such departures to appropriate material properties.

3. Strain rates attainable in microparticle impacts extend the present day test range by more than two orders of magnitude. The determination of material strengths at these exceedingly high strain is of obvious fundamental importance.

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- <sup>4</sup>The average mass of the bigger impacts is  $(0.5 + 0.15)/2 = 0.3$  gm. The average mass of the HMI impacts is  $(0.25 + 1.5) \times 10^{-12}/2 = 0.9 \times 10^{-12}$  gm. Thus the reduction is by a factor of about  $0.3 \times 10^{12}$  on mass, or  $0.7 \times 10^4$  on linear dimensions and times.
- <sup>5</sup>By the above factor of  $0.7 \times 10^4$  if the flows scaled exactly, and by an additional factor of  $3.70^{.333} = 1.5$  because of the smaller-than-expected craters--combining for a factor of  $10^4$ .
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