

LA-9040

C.3

Los Alamos National Laboratory is operated by the University of California for the United States Department of Energy under contract W-7405-ENG-36.

CIC-14 REPORT COLLECTION
REPRODUCTION
COPY

*Antel Calculations for the
Pulsed Generator-Driven
Dense Plasma Focus*



Los Alamos Los Alamos National Laboratory
Los Alamos, New Mexico 87545

Edited by Kyle Thorn
Photocomposition by Barbara J. Velarde

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. References herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

LA-9040

UC-34

Issued: February 1982

Model Calculations for the Explosive Generator-Driven Dense Plasma Focus

John Brownell
Rolf Landshoff*

This work was performed in August 1978.



*Consultant. 525 East Crescent Drive, Palo Alto, CA 94301.

MODEL CALCULATIONS FOR THE EXPLOSIVE GENERATOR-DRIVEN DENSE PLASMA FOCUS

by

John Brownell and Rolf Landshoff

ABSTRACT

A model was developed to aid in the design and interpretation of explosive generator-driven dense plasma focus experiments. Several models were investigated, but the one presented here employs a plane sheath propagating along the barrel of a Mather-type gun, entraining a constant fraction of the swept-up gas and carrying the return current between the electrodes. The motion of the sheath is determined from the momentum equation using the integrated magnetic force on the sheath. The solutions are obtained both numerically and by an approximate analytic procedure, and the performance of the system has been determined as a function of the switching time when the generator is connected to the dense plasma focus.

The model presented in this report gives a zero-order approximation to the operating conditions of the explosive generator-driven dense plasma focus and has been used by Group M-6 at Los Alamos to establish a starting point for their experiments.

The arrangement under consideration consists of an explosive generator G, a ballast inductor B, and a Mather-type coaxial gun F, with an equivalent circuit as shown in Fig. 1. The inductance of G is assumed to be

$$L_G = L_{G0} \left(1 - \frac{t}{\tau} \right) \quad (1)$$

independent of the back pressure of the compressed magnetic field. The initial inductance can be calculated from the expression

$$L_{G0} = \mu_0 \ell \eta \left[1 - \frac{\eta}{\pi} \left(\ell n \frac{1}{\eta} + 1.5 \right) \right] \quad (2)$$

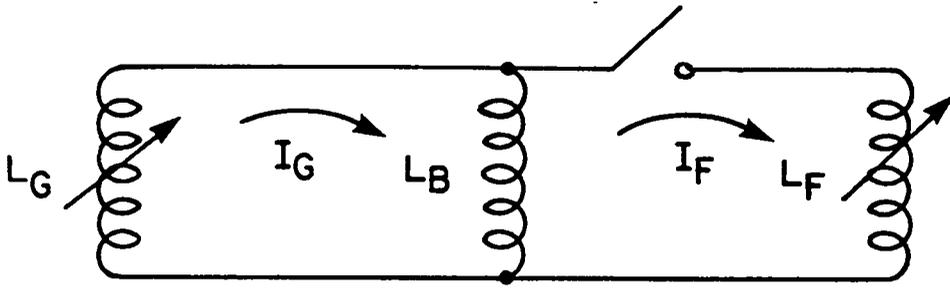


Fig. 1. Equivalent circuit of generator, ballast, and Mather gun.

where $\eta = x_0/w$, x_0 is the initial distance between the generator plates, and w and l are the width and length of the plates.* The length is the dimension in the direction of the current flow. The time τ depends on the strength of the explosive charges and construction details of the generator; it is typically 1-10 μ s.

At the time the switch is closed the generator current is

$$I_1 = I_0 \left(\frac{L_{G0} + L_B}{L_{GS} + L_B} \right) , \quad (3)$$

where I_0 is the current at $t = 0$ and L_{GS} is the value of L_G at $t = t_s$. The ballast inductance L_B is assumed to be independent of time. After the dense plasma focus is connected, the return current between the two electrodes is assumed to be concentrated in a plane sheath propagating along the gun barrel where the sheath entrains a constant fraction k of the swept-up gas. If the distance of the sheath from the breech of the gun is Z and the radii of the outer and the central electrodes are R_o and R_c , the inductance of F is

$$L_F = fZ , \quad f = \frac{\mu_0}{2\pi} \ln \left(\frac{R_o}{R_c} \right) , \quad (4)$$

and the mass of the sheath is

$$M_F = mZ , \quad m = \pi k \rho_0 (R_o^2 - R_c^2) . \quad (5)$$

If we set $L_{G0} = \alpha \tau L_B$ and $f = \delta L_B$, the current in the gun as obtained from flux conservation is

$$I_F = D I_1 , \quad (6)$$

where

$$D = \frac{\alpha (t - t_s)}{\alpha (\tau - t) + \delta Z [1 + \alpha (\tau - t)]} \quad (7)$$

*Equation (2) is an approximation to a more elaborate expression developed by R. S. Caird, Los Alamos National Laboratory.

and

$$I_1 = \frac{1 + \alpha\tau}{1 + \alpha(\tau - t_s)} I_0 \quad . \quad (8)$$

The sheath motion is given by the momentum equation

$$\frac{d}{dt} (M_F \dot{Z}) = F \quad , \quad (9)$$

where

$$F = \frac{f}{2} I_F^2 \quad (10)$$

is obtained by entering the field

$$B = \mu_0 \frac{I_F}{2\pi R} \quad (11)$$

into the relation

$$F = \frac{1}{2\mu_0} \int B^2 2\pi R dR \quad . \quad (12)$$

The momentum equation thus assumes the form

$$\frac{d}{dt} (Z \dot{Z}) = \frac{f}{2m} I_F^2 \quad . \quad (13)$$

After Z has been determined (numerically or analytically as, for example, in the Appendix), various quantities of interest can be calculated, such as the energy deposited into F , the current I_F , and the rate of change of L_F .

The rate at which energy flows into F is

$$V_F I_F = I_F \frac{d}{dt} (L_F I_F) \quad . \quad (14)$$

Integrating with respect to time, the energy in F can be expressed as the sum of two terms

$$\int I_F \frac{d}{dt} (L_F I_F) dt = E_M + W \quad , \quad (15)$$

where

$$E_M = \frac{1}{2} L_F I_F^2 \quad (16)$$

is the stored magnetic energy, and

$$W = \frac{1}{2} \int I_F^2 \frac{dL_F}{dt} dt \quad (17)$$

is the work done by the magnetic pressure, which produces both thermal and directed kinetic energy.

In addition, the quantities I_F and \dot{L}_F can be calculated from

$$I_F = I_1 \sigma / \left\{ 1 - \sigma + \delta z \left[1 - \sigma + \frac{1}{\alpha(\tau - t_s)} \right] \right\} \quad (18)$$

with I_1 as given by Eq (8), and

$$\dot{L}_F = f \sqrt{\frac{f}{3m}} I_1 \sigma \frac{(1 + \frac{3}{2} \sigma + \frac{3}{2} \xi \sigma^2)}{\sqrt{1 + 1.2\sigma + \epsilon \sigma^2}}, \quad (19)$$

where σ and ξ are defined in the Appendix. These quantities are of particular importance when the generator plates make contact and the voltage drops to zero, at which time no more power is delivered to the load F. At the time t_c at which this contact occurs, the inductance L_G still has a finite value

$$L_{GC} = \mu_0 \frac{\ell x_c}{w} \quad (20)$$

The residual x_c in this expression is made up of two parts, the thickness x_1 of a sheet of insulating material placed between the two plates and the skin depth x_s by which the flux has diffused into the generator plates (twice the skin depth per plate). Typically, x_1 is $10^{-3}m$, and for aluminum,

$$x_s \approx 1.4 \times 10^{-4} \sqrt{\frac{I_0 \tau}{w}} m, \quad (21)$$

where all quantities are given in mks units.*

The residual inductance can be accounted for by terminating the calculation at

$$t_c = \tau \left(1 - \frac{L_{GC}}{L_{GO}} \right) \quad (22)$$

or

$$\sigma_c = \left(1 - \frac{L_{GC}}{L_{GO}} - \frac{t_s}{\tau} \right) / \left(1 - \frac{t_s}{\tau} \right) \quad (23)$$

*This form is due to an investigation by C. M. Fowler, Los Alamos National Laboratory.

The model developed above was applied to a setup in which the dimensions and the collapse time of the generator and the dimensions of the Mather gun remain fixed. The actual values as suggested by Group M-6 are

Generator: $l = 0.528$ m, $w = 0.1524$ m, $x_0 = 0.0762$ m
 $x_c = 0.002$ m, $\tau = 10^{-5}$ s

Mather Gun: $R_0 = 0.0762$ m, $R_c = 0.0508$ m, $d = 0.3$ m.

The last quantity is the length of the center electrode; we will describe later how this is to be used. The switch time t_s , the ballast inductance L_B , and the initial gas density ρ_0 in the gun are considered as tuning parameters. The entrainment factor, also called the plowing efficiency, was arbitrarily taken to be $k = 1$ because the results depend only on the product $k\rho_0$. Actually, the value of k is not known and may depend on time, but in the present model, the possibility is not considered. In principle, the initial generator current I_0 could be used for tuning, but a look at the equations shows that the performance depends on the ratio $I_0^2/\rho_0 k$ and there is no need for changing both ρ_0 and I_0 . The current, therefore, is considered as fixed at a value $I_0 = 7.5 \times 10^5$ A.

We have assumed that the best results could be obtained if the plasma sheath reaches the end of the center electrode at the same time the power flow into the gun ends; that is, if $Z(t_s) = d$. This relation has been used to determine t_s for given values of ρ_0 and L_B . The set of curves in Fig. 2 shows the results for different values of ρ_0 plotted against L_B . The curves are labeled by the gas pressure of room-temperature deuterium expressed in torr. The ratio was taken to be $p/\rho_0 = (3/7) \times 10^4$ torr/(kg/m³). In Figs. 2, 3, and

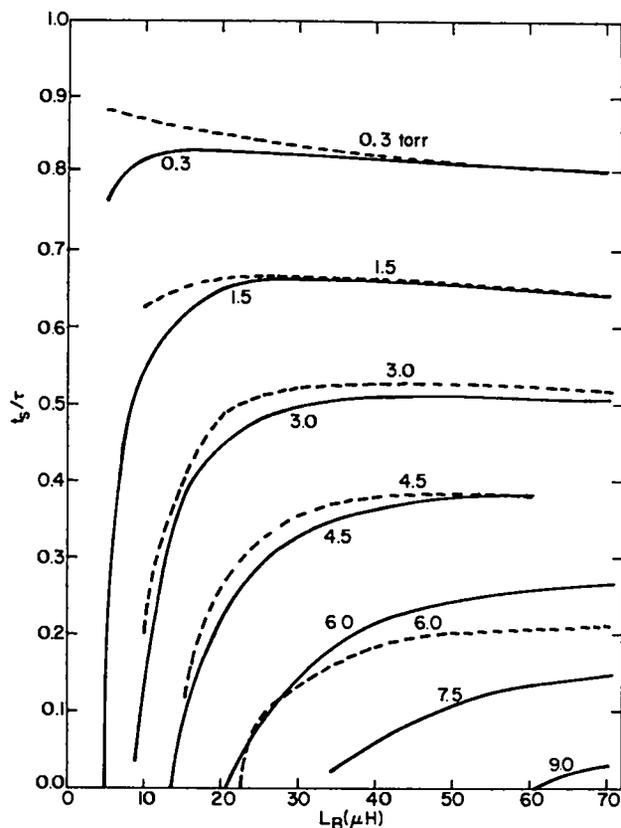


Fig. 2. Switch time (dashed lines represent analytic approximation, solid lines are numerical integration).

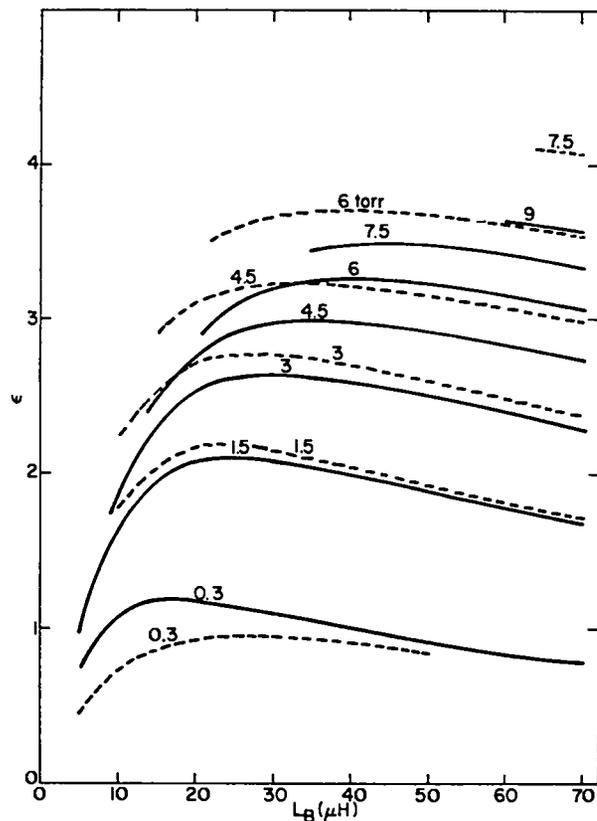


Fig. 3. Energy-gain factor (dashed lines represent analytic approximation, solid lines are numerical integration).

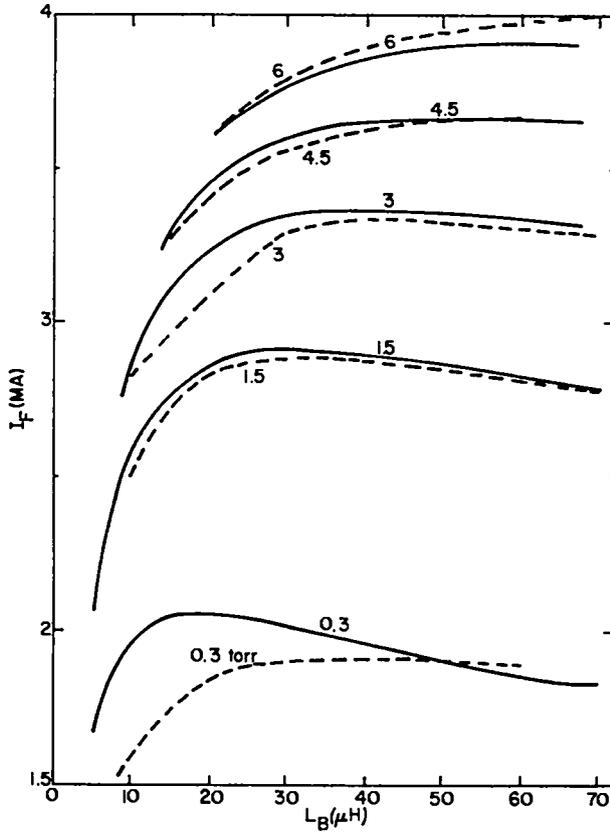


Fig. 4. Peak current (dashed lines represent analytic approximation, solid lines are numerical integration).

4, the analytic approximations are given as dashed lines and the results of the numerical integrations are given as solid lines. As shown, the analytic approximation agrees quite well. The purpose of the dense plasma focus arrangement is to produce a maximum fusion yield at the focus. At the end of the first stage (the rundown), it is not possible to predict precisely how to achieve this. It appears likely, however, that one should optimize the ratio of the energy transfer to the gun to the initially available electromagnetic energy

$$E_0 = \frac{1}{2} (L_{GO} + L_B) I_0^2 \quad (24)$$

We therefore define an energy-gain factor

$$\epsilon = \frac{E_M + W}{E_0} \quad (25)$$

Figure 3 shows graphs of this factor for the switch times shown in Fig. 2, and peak currents are shown in Fig. 4. The curves show that the peak values of ϵ rise with p , but one is limited by the fact that t_s has to be positive. Thus, for the chosen dimensions, it appears impractical to go beyond 6 torr, where one reaches a gain factor of about 3.7. In an arrangement where the electromagnetic energy is fed directly from a capacitor to the gun, the fraction of the energy fed to the gun has been estimated* to be at most $\epsilon = 0.586$; that is, the explosion generator setup is about six times as effective.

*This information provided by R. H. Gerwin, Los Alamos National Laboratory.

The energy gain is derived from the work done by the explosive and the efficiency in terms of that energy is quite small. However, that is not an important consideration here because the explosive energy is readily available.

When comparing the predictions of the model, consider that the quantity designated as the pressure is really the product of the true pressure and the entrainment factor. The effective value of this factor can be obtained by observing the sheath distance Z at generator burn-out time for a set combination of t_s , L_B , and p and dividing the kp for which one expects the observed Z by the true value of p . In Fig. 5, we have plotted curves of Z vs kp for $L_B = 30$ nH and three values of t_s/τ that can be used for such a determination.

Note that the relation between p and ρ_0 applies only to deuterium. For other gasses, one should represent the results in terms of density rather than pressure.

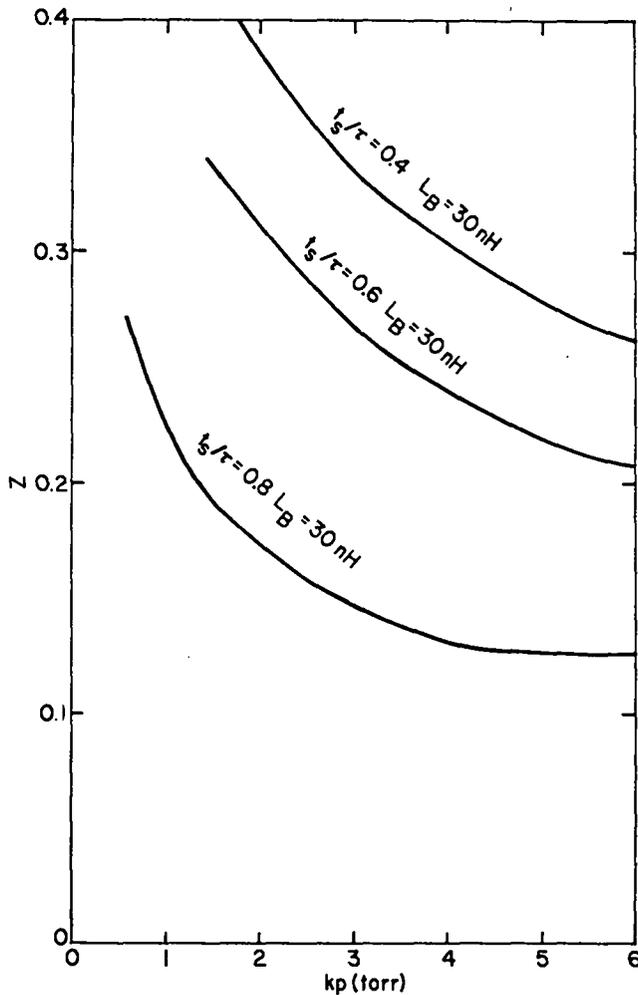


Fig. 5. Sheath distance at burn-out time vs kp .

APPENDIX

APPROXIMATE ANALYTIC SOLUTION

We attempt to find an approximate solution to Eq. (13) by introducing the variable

$$\sigma = \frac{t - t_s}{\tau - t_s} \quad (\text{A.1})$$

and by setting

$$Z^2 = \left(\frac{f}{12m} \right) I_1^2 (\tau - t_s)^2 \sigma^4 (1 + 1.2\sigma + \xi\sigma^2) \quad (\text{A.2})$$

so that

$$\frac{d}{dt}(ZZ) = \frac{d^2}{dt^2} \left(\frac{Z^2}{2} \right) = \left(\frac{f}{2m} \right) I_1^2 \sigma^2 \left(1 + 2\sigma + \frac{5}{Z} \xi\sigma^2 \right) \quad (\text{A.3})$$

By substituting Z from Eq. (A.2) with D [see Eq. (6)], we obtain the relation

$$1 + 2\sigma + \frac{5}{2}\xi\sigma^2 = \{ 1 - \sigma + q_0 \sigma^2 \sqrt{1 + 1.2\sigma + \xi\sigma^2} [1 + \alpha(\tau - t_s)(1 - \sigma)] \}^{-2} \quad (\text{A.4})$$

with

$$q_0 = \frac{\sigma}{\alpha} \sqrt{\frac{f}{12m}} I_1 \quad (\text{A.5})$$

which is satisfied to terms in first order in σ . The value of ξ for which Eq. (A.4) is satisfied is not constant, which it would have to be if Eq. (A.2) were an exact solution. After comparing the approximate Z with one obtained by a numerical solution of Eq. (13), we decided to determine ξ by solving Eq. (A.4) for $\sigma = 0.8$; that is, by solving

$$2.6 + 1.6\xi = (0.2 + q_1 \sqrt{1.96 + 0.64\xi})^{-2} \quad (\text{A.6})$$

where

$$q_1 = 0.64q_0 [1 + 0.2\alpha (\tau - t_s)] \quad (\text{A.7})$$

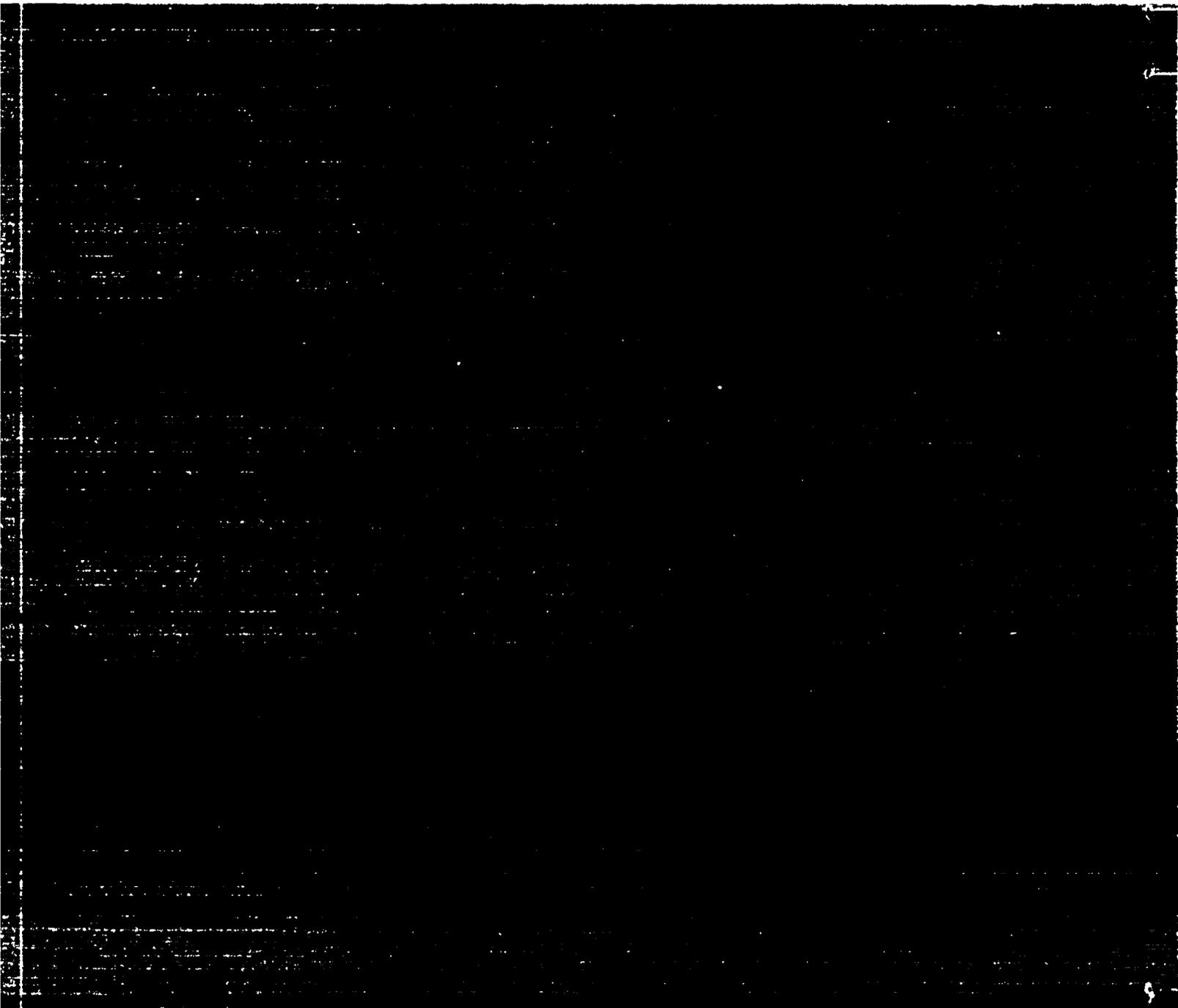
For large values of q_1 , this procedure of obtaining ξ must be modified because it would lead to negative values of $(d^2/dt^2)(Z^2)$, which are physically impossible. We therefore change the relation so that in the limit $q_1 \rightarrow \infty$, ξ approaches -1.2 , which is the value for which the factor in Eq. (A.3) becomes zero when $\sigma = 1$. A relation that does this and fits Eq. (A.6) for $q_1 < 0.4$ is

$$\xi = (0.2 + 0.25q_1 + 6.25 q_1^2)^{-1} - 1.2 \quad .$$

Printed in the United States of America
 Available from
 National Technical Information Service
 US Department of Commerce
 5285 Port Royal Road
 Springfield, VA 22161
 Microfiche \$3.50 (A01)

Page Range	Domestic Price	NTIS Price Code	Page Range	Domestic Price	NTIS Price Code	Page Range	Domestic Price	NTIS Price Code	Page Range	Domestic Price	NTIS Price Code
001-025	\$ 5.00	A02	151-175	\$11.00	A08	301-325	\$17.00	A14	451-475	\$23.00	A20
026-050	6.00	A03	176-200	12.00	A09	326-350	18.00	A15	476-500	24.00	A21
051-075	7.00	A04	201-225	13.00	A10	351-375	19.00	A16	501-525	25.00	A22
076-100	8.00	A05	226-250	14.00	A11	376-400	20.00	A17	526-550	26.00	A23
101-125	9.00	A06	251-275	15.00	A12	401-425	21.00	A18	551-575	27.00	A24
126-150	10.00	A07	276-300	16.00	A13	426-450	22.00	A19	576-600	28.00	A25
									601-up	†	A99

†Add \$1.00 for each additional 25-page increment or portion thereof from 601 pages up.



Los Alamos