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# SMOOTH PARTICLE HYDRODYNAMICS: THEORY AND APPLICATION TO THE ORIGIN OF THE MOON

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## Introduction

Some may wonder what the origin of the moon has to do in a workshop on the use of supercomputer in stellar dynamics. I am going to show that there is more in common between both than just the use of supercomputers. In fact, if one uses the so-called smooth particle hydrodynamics (SPH) method (Lucy, 1977, Monaghan 1985) which substitutes to the fluid a finite set of extended particles, the hydrodynamics equations reduce to the equation of motion of individual particles. These equations of motion differ only from the standard gravitational N-body problem insofar that pressure gradients and viscosity terms have to be added to the gradient of the potential to derive the forces between the particles. The numerical tools developed for "classical" N-body problems can therefore be readily applied to solve 3 dimensional hydrodynamical problems.

It is beyond the scope of this paper to go in the detail of the origin of the moon. I will only use it as an example of an application of the SPH technique. A reader more interested in the problem should consult Hartmann et al. (1986) and Benz et al. (1986).

## The smooth particle-hydrodynamics method.

Since the pioneer work of Lucy in 1977 the SPH method was greatly improved and got a more mathematical basis mainly through the work of Gingold and Monaghan (1982) and Monaghan (1986). One easy way of looking at it which is taken in part from Lucy's original paper is the following: Suppose we want to approximate the function.

$$\eta(r) = \int W(\underline{r}-\underline{r}';h)\, \xi(\underline{r}')n(\underline{r}')d\underline{r}'$$

by Monte-Carlo techniques. We need N points $\underline{r}_j$ distributed in space according to the probability density given by n(r)dr, then

$$\tilde{\eta}(\underline{r}) = \frac{1}{N} \sum_{j=1}^{N} W(\underline{r}-\underline{r}_j;h)\ \xi(r_j) \xrightarrow[N\to\infty]{} \eta(\underline{r})$$

$\tilde{\eta}(\underline{r})$ is an approximation of $\eta(\underline{r})$.

Now suppose that we require that at all times

$$\int W(\underline{r},\ h)\ d\underline{r} = 1$$

and let h (a measure of the width of the Kernel W) tend to zero. The result is obviously

$$W(\underline{r},h) \xrightarrow[h\to 0]{} \delta$$

which implies that:

$$\eta(\underline{r}) \xrightarrow[h\to o]{} \xi(\underline{r})\ n(\underline{r})$$

Therefore the final step is

$$\tilde{\eta}(\underline{r}) \xrightarrow[N\to\infty]{} \eta(\underline{r}) \xrightarrow[h\to o]{} \xi(\underline{r}) n(\underline{r}) \qquad \text{or}$$

$$\tilde{\eta}(r) = \frac{1}{N} \sum_{j=1}^{N} W(\underline{r}-\underline{r}_j;h)\ \xi(\underline{r}_j) \xrightarrow[\substack{N\to\infty \\ h\to o}]{} \xi(\underline{r}\ ) n(\underline{r})$$

which constitutes the basis of the SPH technique.

By taking $\xi(\underline{r}) = 1$ and multiplying the number density by the mass of each particle one gets the mass density:

$$\rho(r) = \frac{1}{N} \sum_{j=1}^{N} m_j W(\underline{r}-\underline{r}_j;h) \quad .$$

Using $\xi(\underline{r}) = \frac{1}{n(\underline{r})}$ for example would give an estimation of the pressure and so on. Gradients are easily taken, for example density gradient

$$\underline{\nabla}\rho(r) = \frac{1}{N} \sum_{j=1}^{N} m_j \, \underline{\nabla}.W(\underline{r}-\underline{r}_j;h) \quad .$$

The gradient of the kernel is a known analytical function. If W was chosen so that at least it's first derivatives are continuous the forces will be continuous also.

<u>Momentum and energy equation.</u>

Applying the above formalism to the Navier-Stokes equation one gets the equation of motion of the $i^{th}$ particle (See Benz et al., 1986 for details)

$$\left(\frac{dv}{dt}\right)_i = \sum_{j=1}^{N} m_j \left(\frac{P_j}{\rho_j^2} + \frac{P_i}{\rho_i^2}\right) \nabla W(r_{ij};h) - G\sum_{j=1}^{N} \frac{M(r_{ij})}{r_{ij}^2} \hat{r}_{ij}$$

$$+ F_{i\ visc} \qquad r_{ij} = |\underline{r}_i - \underline{r}_j| \, , \ \hat{\underline{r}}_{ij} = \frac{\underline{r}_i - \underline{r}_j}{r_{ij}} \, .$$

The first term of the right side represent the pressure gradients. The second is the gravitational term where $M(r_{ij})$ is the mass of particle j within a sphere of radius $r_{ij}$ of particle i (since all particles are spherically symetric). Finally the last term stands for viscous forces.

The variation of internal energy is given by the well know first law of thermodynamics

$$\frac{du}{dt} = - P\frac{dV}{dt} + \frac{dQ}{dt}$$

which writes using the SPH formalism:

$$\left(P\frac{dv}{dt}\right)_i = 0.5 \sum_{j=i}^{N} m_j \left(\frac{P_j}{\rho_j^2} + \frac{P_i}{\rho_i^2}\right) \underline{v}_{ij} \ \underline{\nabla} W(r_{ij};h)$$

$$\left(\frac{dQ}{dt}\right)_i = 0.5 \sum_{j=1}^{N} m_j \ \pi_{ij} \ \underline{v}_{ij} \ \underline{\nabla} W(r_{ij};h) \qquad , \ \underline{v}_{ij} = \underline{v}_i - \underline{v}_j$$

dQ/dt represents the energy dissipated in shocks due to the artificial vicosity (see Gingold and Monaghan 1982 for details).

The system of equations is completed with an appropriate equation of state. Since we modelled solid bodies and strong shocks leading to vaporization of parts of the rocks we used a form of the Tillotson (1962) equation of state (Benz, et.al. 1986)

Practical considerations

a) Kernel

So far we did not specify the Kernel W(r,h). Several types have been proposed in the literature ranging from gaussian to exponential and polynomes. In the present moon computation we used the exponential Kernel originally proposed by Wood (1981) which writes:

$$W(r,h) = \frac{1}{8\pi h^3} e^{-r/h}$$

However, recently Monaghan and Lattanzio (1985) proposed a new form that is based on spline function and that has the form

$$W(r,h) = \frac{1}{\pi h^3} \begin{cases} \frac{3}{2}\left(\frac{2}{3} - v^2 + \frac{1}{2} v^3\right) & 0<v<1 \\ \frac{1}{4}(2 - v)^3 & 1<v<2 \\ 0 & v>2 \end{cases}$$

where v = r/h.

The advantage of this Kernel is that it has a contribution only on a compact support. By superposing a grid with mesh size 2h one can easily find the nearest neighbors using linked list (Hockney and Eastwood, 1981).

b) integrator

We found that by using the leapfrog algorithm substantial errors can occur. We therefore prefer using a low order Runge-Kutta-Fehlberg integrator (Fehlberg, 1969). This allows also to compute the approach of the two body prior to collision much more efficiently since we are not limited by the courant condition for the timestep. In fact our code switches between the Runge-Kutta-Fehlberg integrator and a first order predictor-corrector scheme whatever is more efficient.

## Tests

Since the first time it was proposed the SPH technique was shown to do remarkedly well in many different problems. It was shown to treat shocks correctly (Gingold and Monaghan, 1982), and to reproduce many known analytical solution whithin a few percents. We run several test collisions computing the same problem with a completely different code, the results always agreed to within a few percents. Moreover, we confirmed earlier findings by Durisen et al. (1986) that there is a ratio of 30 to 50 between the number of particles used in a SPH code and the number of cells used in a classical finite difference code to achieve same accuracy in the results. All this and taking into account the relative simplicity of the method and the ease of programming makes the SPH technique a very powerful tool.

## The Origin of the Moon

There is no point reviewing in details all the theories of lunar origin. Let me just say that they can be put into 4 main groups depending on the basic assumptions made. These groups are fission, capture, binary formation and planetary collision. The first three have well known problems with either dynamical (angular momentum, dissipation rate) or chemical (siderophile abundances, etc) observations or theoretical considerations. An Earth-planet collision was proposed (Hartman and Davis 1975, Cameron and Ward 1976) and thought to take care of those problems. These simulations show for the first time that this theory really works; therefore, transforming the giant impact from just an idea to a plausible scenario.

With W. L. Slattery and A.G.W. Cameron we simulated the events taking place between the collision of a projectile with the protoearth and the subsequent formation of a disk around the protoearth; the evolution of such a disk to form the moon has been discussed by Ward and Cameron (1978) and by Thompson and Stevenson (1983). Different impact velocities, impact parameters, and initial internal energies were considered. Particular care was taken in the choice of the equation of state to model as accurately as possible the thermodynamics of the material during and after the collision.

The following assumptions had to be made to keep the problem tractable. We neglected material strength. This is a fairly common hypothesis in hypervelocity impacts and is certainly justified during the impact itself, but obviously wrong a short time after. We do not think, however, that this assumption affects our result for the following reasons. First, most of the simulations were started with molten planets. Second, even when starting with solid planets the material put into orbit became very hot and therefore molten. Taking material strength into account would only affect the way the protoearth recovers its spherical shape after the impact.

Radiative transfer and radiative energy losses were not included either. This is justified by a timescale argument. The typical timescale for the shock to heat the material is about half an hour, whereas the timescale to transport the heat or to lose it by radiation is much longer. The resolution in the code is equivalent to a chunck of material of about $10^{24}$ grams, so the time needed to cool such a piece of rock greatly exceeds half an hour.

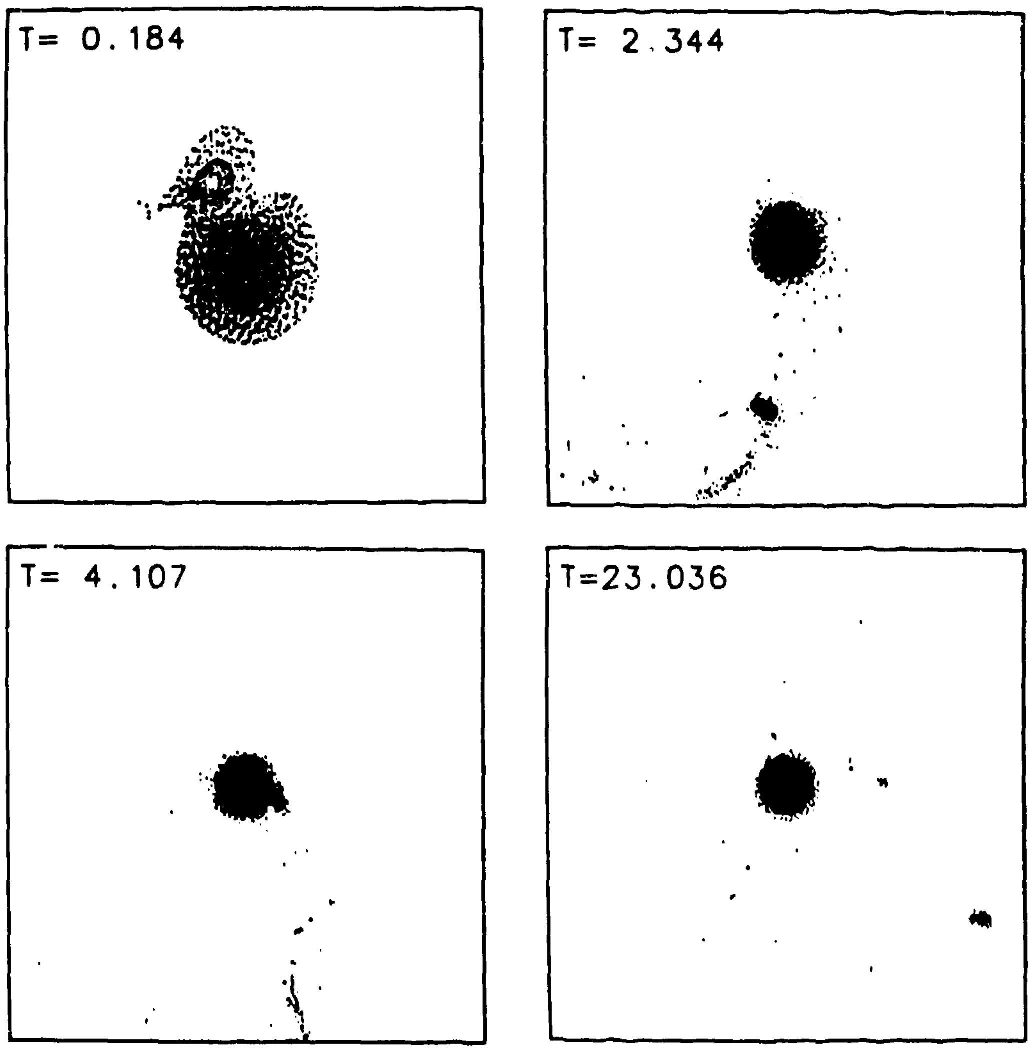
The equation of state we used is the Tillotson equation of state (1962). This equation of state has 10 material dependent constants that are defined by fitting the analytical formula to experimental data. The main property of this equation is that for cold and condensed matter the equation allows for negative pressures which simulate tension, whereas for hot and expanded matter the equation goes asymptotically toward the equation of state for perfect gases.

The history of a typical collision is as follows. As the impactor approaches the Earth it becomes deformed by the tidal field. Following initial contact it is slowed down and there is a corresponding dissipation of kinetic energy into heat in the strong shock formed at the interface. As soon as this shocked material expands a little (in the jets for example) it turns into vapor. However, most of this vapor becomes solid again since during subsequent expansion the gas cools, and soon its internal energy is lower than the vaporization energy.

The evolution after that point depends strongly on the mass ratio between the impactor and the Earth, $M_r$. For large mass impactors ($M_r > 0.17$) less than half a moon's mass is left in orbit to form a disk. About the same amount escapes the system in form of high velocity jets. The bulk of the material was slowed down sufficiently that the Earth eventually accreted almost all of it. The disk left in orbit has very little iron in it. The iron core of the impactor is completely "swallowed" by the Earth and ended on top of the Earth's core. For small mass impactors ($M_r < 0.12$) the collision is too weak to destroy the impactor in the first hit. It gets into a very eccentric orbit that leads to a second collision after one revolution. At this time, the impactor is destroyed and spread out into a disk. The problem is that this disk contains almost all the iron initially in the impactor's core (more than one moon mass). It is hard to imagine how to form a moon which is severely depleted in iron out of an iron rich disk. So small mass impactors are ruled out!

There is an intermediate mass region ($0.12 < M_r < 0.16$) where the mass put into orbit is more than 1.5 moon masses. For $M_r = 0.14$ (see Fig. 1) the simulation ended with a clump in orbit having exactly a moon mass together with a disk of roughly half a moon's mass. For all cases in this intermediate region the amount of iron in the disk or in the clump is very small. Indeed the iron core of the impactor is separated from the granite mantle soon after the beginning of the collision. Individually both the core and mantle clump together due to their self-gravity. The iron core goes into a very eccentric orbit that collides with the

Earth again. During this time the gravitational torque exerted on the material in orbit by the Earth and the separated iron core transfers angular momentum outwards. This transfer is big enough that the material in orbit never collides with the Earth again. The subsequent evolution is not very clear: the clump may again come inside the Roche limit and be destroyed and spread out into a disk. The material put into orbit is hot enough so that all volatile elements are readily lost either during the collision itself or during the subsequent disk phase. Moreover, most (90%) of the material orbiting the Earth originates from the impactor, accounting for the chemical differences between the moon and the Earth.




After the collision, the impactor is partially destroyed and spread out in space. Due to their own gravitational attraction, however, part of the debris clump together; at 2.26 hours after the beginning of collision the iron core is completely separated from the granite mantle. About 4 hours after the impact the iron core hits the Earth again and is swallowed by it, leaving a granite clump with almost exactly a moon's mass in orbit. The time spent since the beginning of the collision is less than 24 hours! Since this clump is at the edge of the Roche limit subsequent evolution is not clear. Gas drag may slow it down inside the Roche limit, leading to its destruction and the formation of a prelunar accretion disk.

References:


1. Benz, W., Slattery, W.L., Cameron, A.G.W., 1986, Icarus, in press.

2. Cameron, A.G.W., Ward, W.R., 1976, Lunar Planet. Sci., VII, 120.

3. Durisen, R.H., Gingold, R.A., Tohline, J.E., Boss, A.P., 1986, Ap.J., 305, 281.

4. Fehlberg, E., 1969, NASA TR. R-315.

5. Hartmann, W.K., Davis, D.R., 1975, Icarus, 24, 504.

6. Gingold, R. A., Monaghan, J.J., 1982, J. Comput. Phys., 46, 429.

7. Hartmann, W.K., Phillips, R., Taylor, G.J., Eds, Origin of the Moon, Lunar Planetary Institute, Houston, 1986.

8. Hockney, R.W., Eastwood, J.W., 1981, Computer Simulations Using Particles, McGraw-Hill, New York.

9. Monaghan, J.J., 1985, Computer Physics Communications, 3, 72.

10. Monaghan, J.J., Lattanzio, J.C., 1985, Astron. Astrophys., 149, 135.

11. Tillotson, J.H., 1962, General Atomic Report, GA-3216, July 1962.

12. Wood, D., 1981, Mon. Not. R. Astron. Soc., 194, 201.