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TITLE: DYNAMICAL THRESHOLDS FOR COMPLETE FUSION

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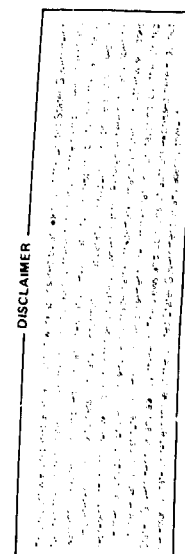
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## DYNAMICAL THRESHOLDS FOR COMPLETE FUSION

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### INTRODUCTION

It is especially appropriate to discuss dynamical thresholds for complete fusion at our winter meeting in Bormio, since the first dynamical calculation of complete-fusion cross sections was presented in Bormio 8 years ago, at the inaugural meeting in our new location.<sup>1</sup> At that time, the cross section for forming a compound nucleus from the fusion of an equal target and projectile was calculated by use of the criterion that the dynamical trajectory must pass inside the fission saddle point in a multidimensional deformation space. The dynamical trajectory for the fusing system was determined by numerically integrating classical equations of motion for nondissipative systems. These calculations illustrated that, even in the absence of dissipation, an additional bombarding energy  $\Delta E$  relative to the maximum in the one-dimensional interaction barrier is required to form a compound nucleus for sufficiently heavy nuclear systems and/or sufficiently high angular momentum. More detailed accounts of the results presented in Bormio in 1975 are published in Refs. 2 and 3.

In another approach, pioneered by Swiatecki,<sup>4-7</sup> the additional energy  $\Delta E$  required to drive a fusing system inside the conditional saddle point for fixed mass asymmetry was calculated from a schematic model that included one-body wall-and-window dissipation.<sup>8,9</sup> The expression for  $\Delta E$  is characterized by five constants, some of whose values have been estimated from the original

schematic model,<sup>4</sup> from improved models,<sup>10,11</sup> and from comparisons<sup>5-7</sup> with experimental data.<sup>12</sup> Although these comparisons have sometimes been interpreted as evidence for one-body wall-and-window dissipation, it must be borne in mind that the additional energy arises from the need to overcome both repulsive Coulomb and/or centrifugal forces on the one hand and dissipative forces on the other hand. No attempt has been made thus far to distinguish between conservative and dissipative forces.

#### APPROACH

It is our purpose here to study the effect of nuclear dissipation and shape parametrization on dynamical thresholds for compound-nucleus formation in symmetric heavy-ion reactions. This is done by solving numerically classical equations of motion for head-on collisions to determine whether the dynamical trajectory in a multidimensional deformation space passes inside the fission saddle point and forms a compound nucleus, or whether it passes outside the fission saddle point and reseparates in a fast-fission<sup>12,13</sup> or deep-inelastic reaction.

Specifying the nuclear shape in terms of smoothly joined portions of three quadratic surfaces of revolution,<sup>14</sup> we take into account three symmetric deformation coordinates. However, in some cases we reduce the number of coordinates to two by requiring the ends of the fusing system to be spherical in shape. The nuclear potential energy of deformation is determined in terms of a Coulomb energy and a double volume energy of a Yukawa-plus-exponential folding function,<sup>15</sup> with values of the constants determined in Ref. 16.

The collective kinetic energy is calculated for incompressible, nearly irrotational flow by means of the Werner-Wheeler approximation.<sup>14,17</sup> Four possibilities are studied for the transfer of collective kinetic energy into internal single-particle excitation energy: zero dissipation, ordinary two-

body viscosity,<sup>17</sup> one-body wall-formula dissipation,<sup>8,9</sup> and one-body wall-and-window dissipation.<sup>8,9</sup>

## RESULTS

To aid in interpreting the dynamical trajectories that will follow, we show in Fig. 1 a contour map in  $r$ - $\sigma$  space of the nuclear potential energy of deformation for the reaction  $^{110}\text{Pd} + ^{220}\text{Pd} \rightarrow ^{220}\text{U}$ , calculated in terms of a Coulomb energy and a double volume integral of a single-Yukawa folding function.<sup>18,19</sup> The two moments  $r$  and  $\sigma$  are defined by<sup>1-3,17,19</sup>

$$r = 2 \langle z \rangle \quad (1)$$

and

$$\sigma = 2 \langle (z - \langle z \rangle)^2 \rangle^{1/2}, \quad (2)$$

where  $z$  is measured along the symmetry axis and the angular brackets denote an average over the half volume to the right of the midplane of the reflection-symmetric shape. The moment  $r$  gives the distance between the mass centers of the two colliding ions, while  $\sigma$  measures the fragment elongation or the necking in the combined system.

In the reaction shown in Fig. 1, the two separated spherical  $^{110}\text{Pd}$  nuclei move up the binary valley near the bottom of the figure from right to left, and come into solid contact at the point indicated by the two adjacent solid circles. This point is slightly inside the maximum in the one-dimensional interaction barrier calculated as a function of  $r$  alone, but is on the side of a steep hill with respect to fragment elongation (increasing  $\sigma$ ). In order for a compound nucleus to be formed, the dynamical trajectory must pass inside the fission saddle point, located at the intersection of the dashed 6.3-MeV contours, and become trapped in the potential-energy hollow surrounding the sphere, whose location is indicated by the solid circle.

For the reaction  $^{110}\text{Pd} + ^{110}\text{Pd} \rightarrow ^{220}\text{U}$ , some typical dynamical trajectories are shown in Figs. 2-5. Figure 2 gives the dependence of the trajectories upon dissipation for a center-of-mass bombarding energy that exceeds the maximum in the one-dimensional interaction barrier by 20 MeV. In Fig. 3, similar trajectories are shown for the widely used approximation in which the end bodies are constrained to be spheres.<sup>10,11</sup> The dependence of the trajectories upon bombarding energy is shown in Fig. 4 for two-body viscosity and in Fig. 5 for one-body wall-formula dissipation.

For symmetric projectile and target nuclei that lie along Green's approximation to the valley of  $\beta$ -stability,<sup>20</sup> the dependence of the trajectories upon the nuclear system is shown in Fig. 6 for two-body viscosity and in Fig. 7 for one-body wall-formula dissipation. In each case, as the value of  $Z^2/A$  for the combined system increases, the location of the fission saddle point moves inward and the dynamical trajectory moves outward.

As shown in Fig. 8, for systems with  $Z^2/A$  larger than a threshold value  $(Z^2/A)_{\text{thr}}$  which depends somewhat upon dissipation, the center-of-mass bombarding energy must exceed the maximum in the one-dimensional interaction barrier by an amount  $\Delta E$  in order to form a compound nucleus. For both types of one-body dissipation our calculated values of  $\Delta E$  are in general an order of magnitude larger than those for zero dissipation and ordinary two-body viscosity. The values of  $\Delta E$  for wall-formula dissipation are larger than those for wall-and-window dissipation primarily because the surface normal velocities measured relative to the stationary center of mass of the entire system in the former case are larger than the normal velocities measured relative to the moving centers of mass of each half of the system in the latter case.

### COMPARISON WITH EXPERIMENTAL DATA

In order to compare the additional energy  $\Delta E$  required for compound-nucleus formation that we have calculated for symmetric systems with experimental values, it is necessary to scale the asymmetric nuclear systems that have been studied experimentally into symmetric ones. Near the contact region, the effective value<sup>4-7,10-12,21</sup>

$$(Z^2/A)_{\text{eff}} = 4 Z_1 Z_2 / [A_1^{1/3} A_2^{1/3} (A_1^{1/3} + A_2^{1/3})] \quad (3)$$

defined in terms of the atomic numbers and mass numbers of the projectile and target provides an approximate scaling. Because the dynamical trajectory of a fusing system moves from the contact region, where  $(Z^2/A)_{\text{eff}}$  is appropriate, to the saddle-point region, where  $Z^2/A$  for the combined system is appropriate, scaling in terms of the geometric mean<sup>7,10,21</sup>

$$(Z^2/A)_{\text{mean}} = [(Z^2/A)(Z^2/A)_{\text{eff}}]^{1/2} \quad (4)$$

should be approximately valid.

Figure 9 compares calculated values of the additional center-of-mass bombarding energy  $\Delta E$  required for compound-nucleus formation with existing experimental values. Solid symbols denote values extracted from measurements of evaporation residues,<sup>21-23</sup> which require the formation of true compound nuclei. Open symbols denote values extracted from measurements of nearly symmetric fission-like fragments,<sup>5,24,25</sup> where fast-fission processes involving significant mass transfer but not true compound-nucleus formation also contribute.

Taken together, all experimental values of  $\Delta E$  in Fig. 9 agree much better with results calculated for underdamped motion arising from two-body viscosity

than with results calculated for overdamped motion arising from either type of one-body dissipation. However, because the solid symbols usually lie somewhat above the open symbols, and because the error bars for the three solid symbols with the largest values of  $(Z^2/A)_{\text{mean}}$  extend to  $\infty$ , this conclusion must be regarded as tentative.

#### OUTLOOK

We are entering a new era in the study of large-amplitude collective nuclear motion. Up to now, theoretical approaches with vastly different pictures of the underlying nuclear dynamics have proved capable of reproducing many of the gross experimental features of heavy-ion reactions with fair accuracy because they include correctly the dominant nuclear, Coulomb, and centrifugal forces. However, with the arrival of a new generation of experiments and calculations testing specific predictions such as the additional energy required for compound-nucleus formation, we are on the brink of determining the magnitude and mechanism of nuclear dissipation.

#### ACKNOWLEDGMENTS

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## FIGURE CAPTIONS

Fig. 1. Potential-energy contours, in units of MeV, for the reaction  $^{110}\text{Pd} + ^{110}\text{Pd} \rightarrow ^{220}\text{U}$ , calculated with a single-Yukawa macroscopic model (Ref. 19). The moment  $r$  is the distance between the centers of mass of the two halves of the system, and the moment  $\sigma$  is the sum of the root-mean-square extensions along the symmetry axis of the mass of each half about its center of mass, both measured in units of the radius  $R_0$  of the combined system. The location of the sphere is given by the solid point, and the location of two touching spheres is given by two adjacent solid points.

Fig. 2. Effect of dissipation on dynamical trajectories in the  $r$ - $\sigma$  plane for the reaction  $^{110}\text{Pd} + ^{110}\text{Pd} \rightarrow ^{220}\text{U}$  at  $\Delta E = 20$  MeV in the full three-quadratic-surface parametrization. The interval  $\Delta E$  is defined as the difference between the bombarding energy in the center-of-mass system and the maximum in the one-dimensional interaction barrier. Solid circles indicate the single-sphere and tangent-spheres configurations, and the open circle indicates where the neck radius is 3.0 fm. The saddle-point configuration for the combined system is indicated by a cross ( $\times$ ).

Fig. 3. Effect of dissipation on dynamical trajectories in the  $r$ - $\sigma$  plane for the reaction  $^{110}\text{Pd} + ^{110}\text{Pd} \rightarrow ^{220}\text{U}$  at  $\Delta E = 20$  MeV when the end bodies are constrained to be spherical.

Fig. 4. Effect of bombarding energy on dynamical trajectories in the  $r$ - $\sigma$  plane for the reaction  $^{110}\text{Pd} + ^{110}\text{Pd} \rightarrow ^{220}\text{U}$ , calculated for two-body viscosity with coefficient  $\mu = 0.02$  TP.

Fig. 5. Effect of bombarding energy on dynamical trajectories in the  $r$ - $\sigma$  plane for the reaction  $^{110}\text{Pd} + ^{110}\text{Pd} \rightarrow ^{220}\text{U}$ , calculated for wall-formula dissipation.

Fig. 6. Effect of the nuclear system on saddle-point configurations and dynamical trajectories in the  $r$ - $\sigma$  plane for  $\Delta E = 0.5$  MeV, calculated for two-body viscosity with coefficient  $\mu = 0.02$  TP. Each saddle point and trajectory is labeled by the value of  $Z^2/A$  for the combined system, with the symmetric target and projectile chosen to lie along Green's approximation to the valley of  $\beta$ -stability (Ref. 20).

Fig. 7. Effect of the nuclear system on saddle-point configurations and dynamical trajectories in the  $r$ - $\sigma$  plane for  $\Delta E = 0.5$  MeV, calculated for wall-formula dissipation.

Fig. 8. Effect of dissipation on the additional center-of-mass bombarding energy  $\Delta E$  relative to the maximum in the one-dimensional interaction barrier required to form a compound nucleus in a head-on collision. The smooth curves are drawn by hand through the calculated points.

Fig. 9. Comparison of additional energy  $\Delta E$  required for compound-nucleus formation calculated for symmetric systems with experimental values for asymmetric systems characterized by  $(Z^2/A)_{\text{mean}}$ , defined by Eq. (4). Values extracted from evaporation-residue measurements are represented by solid symbols ( $\bullet$ , Ref. 22;  $\blacksquare$ , Ref. 23; and  $\blacktriangle$ , Ref. 21), whereas values extracted from measurements of nearly symmetric fission-like fragments are represented by open symbols ( $\circ$ , Ref. 5;  $\square$ , Ref. 24; and  $\Delta$ , Ref. 25).

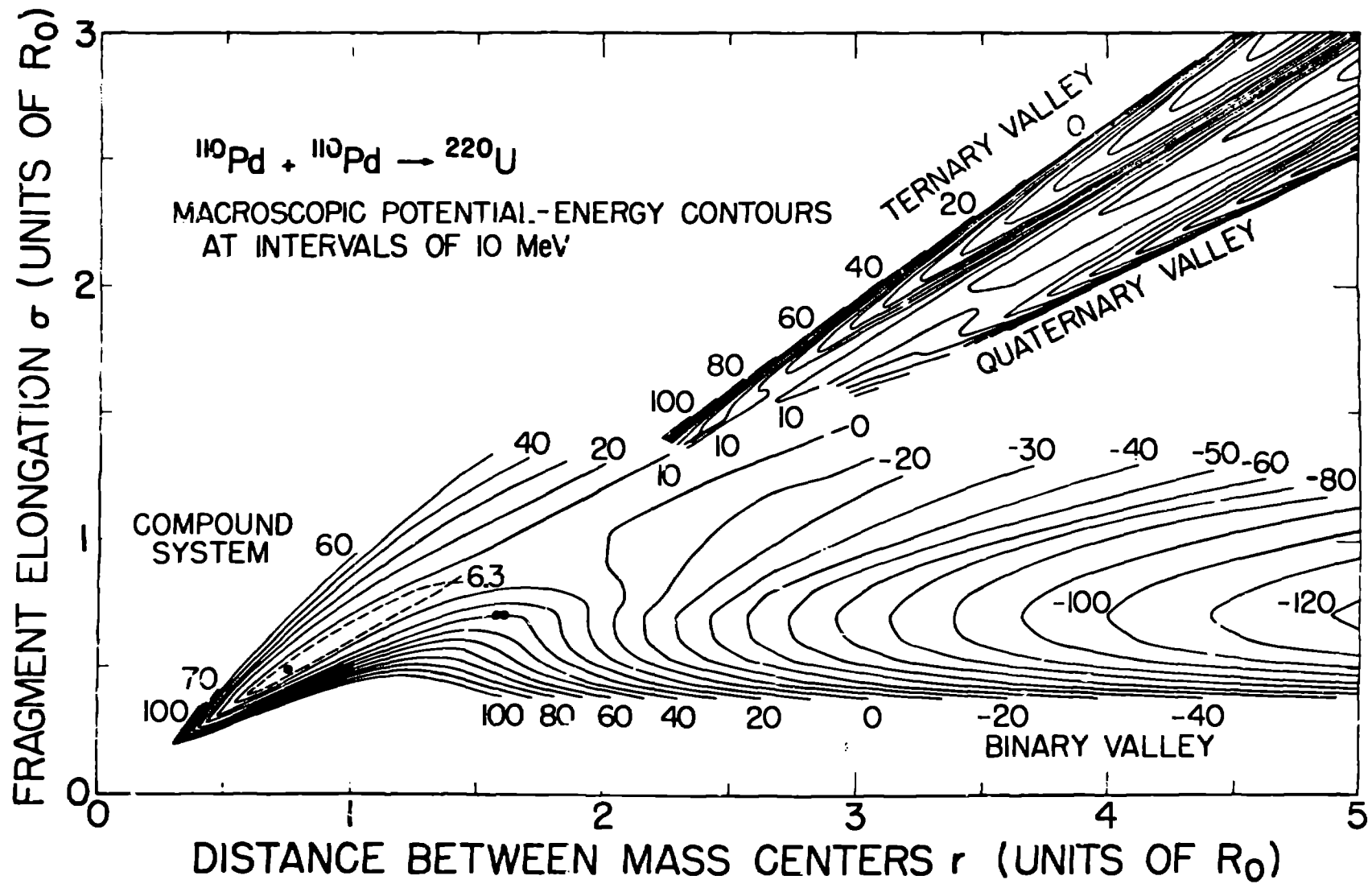


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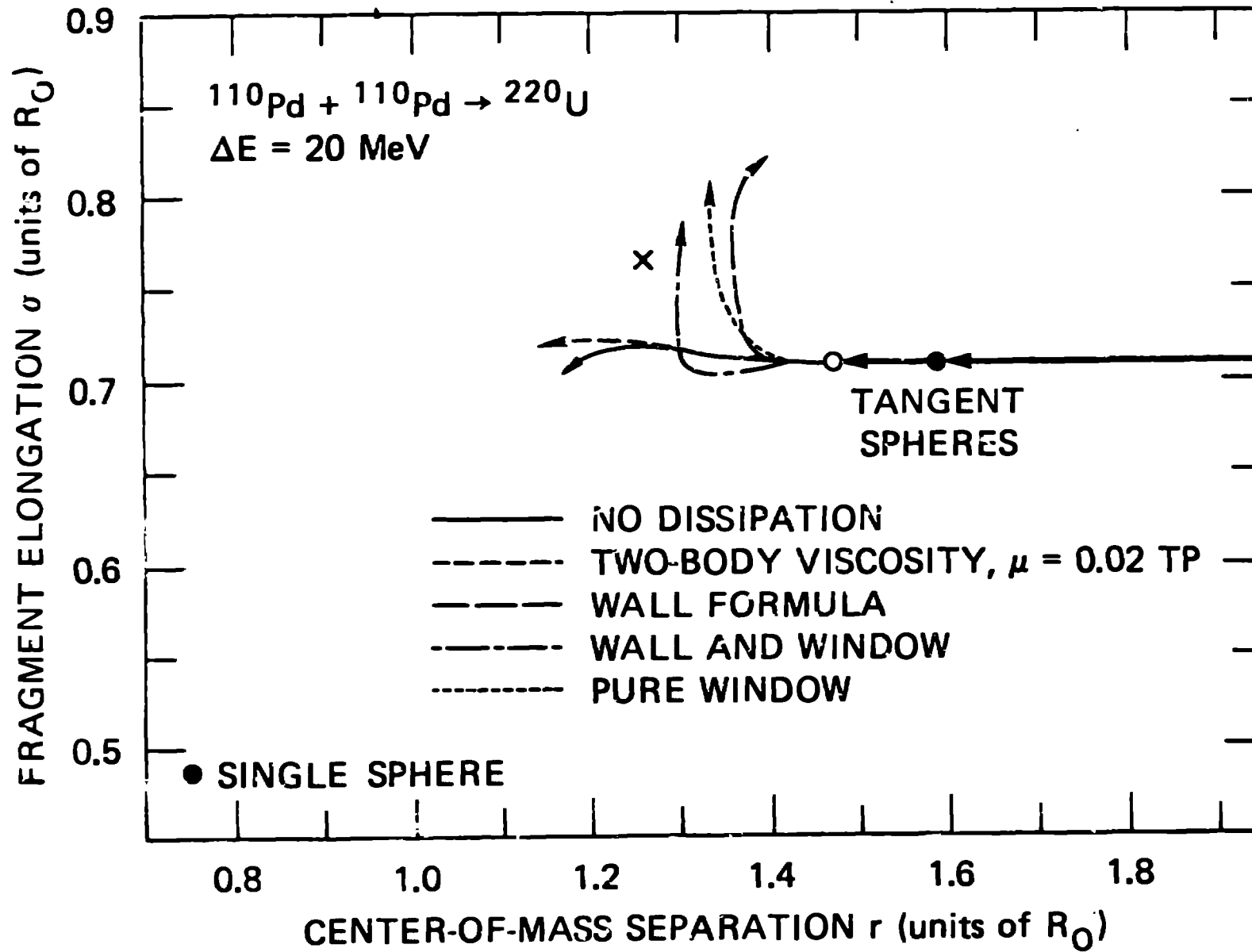


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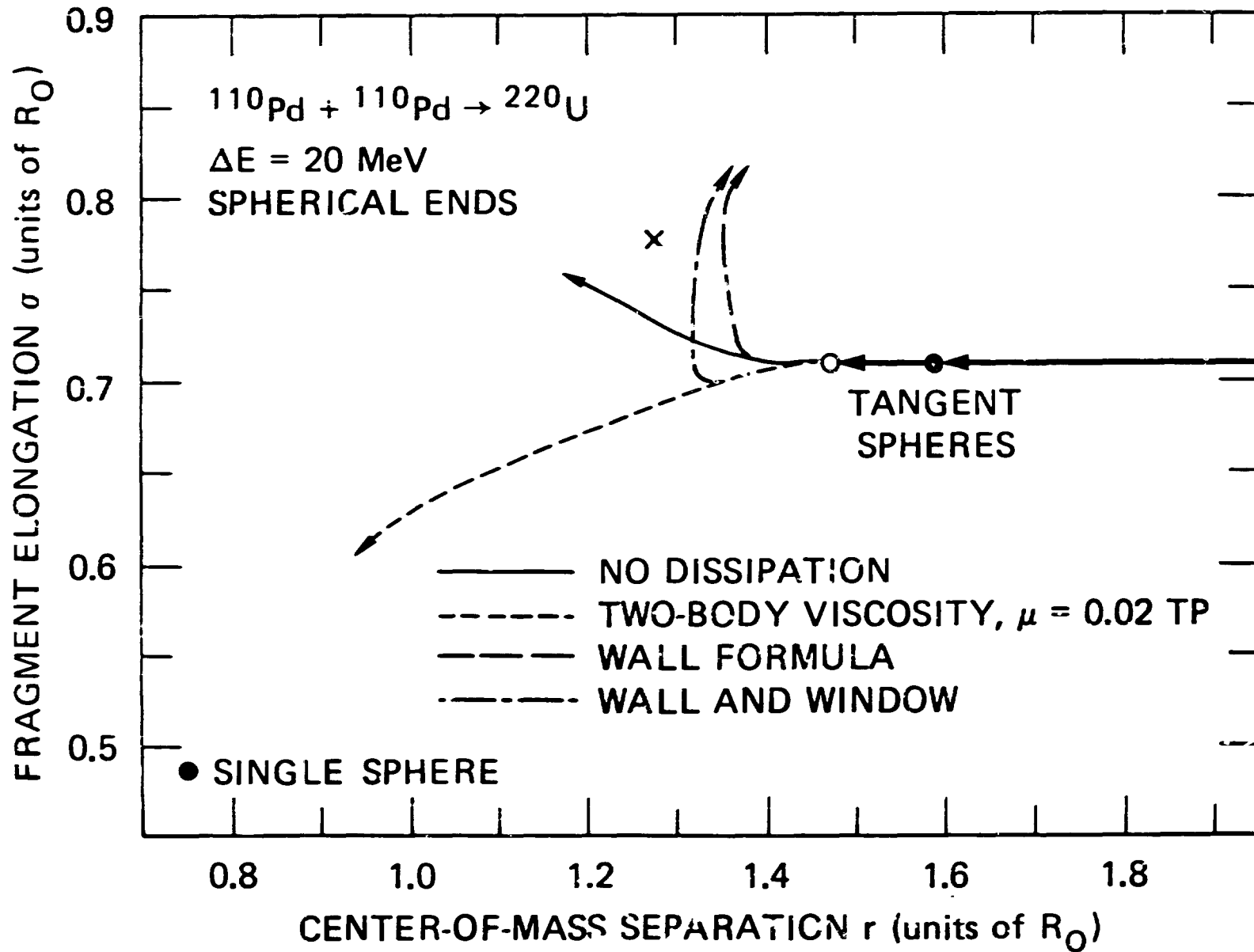


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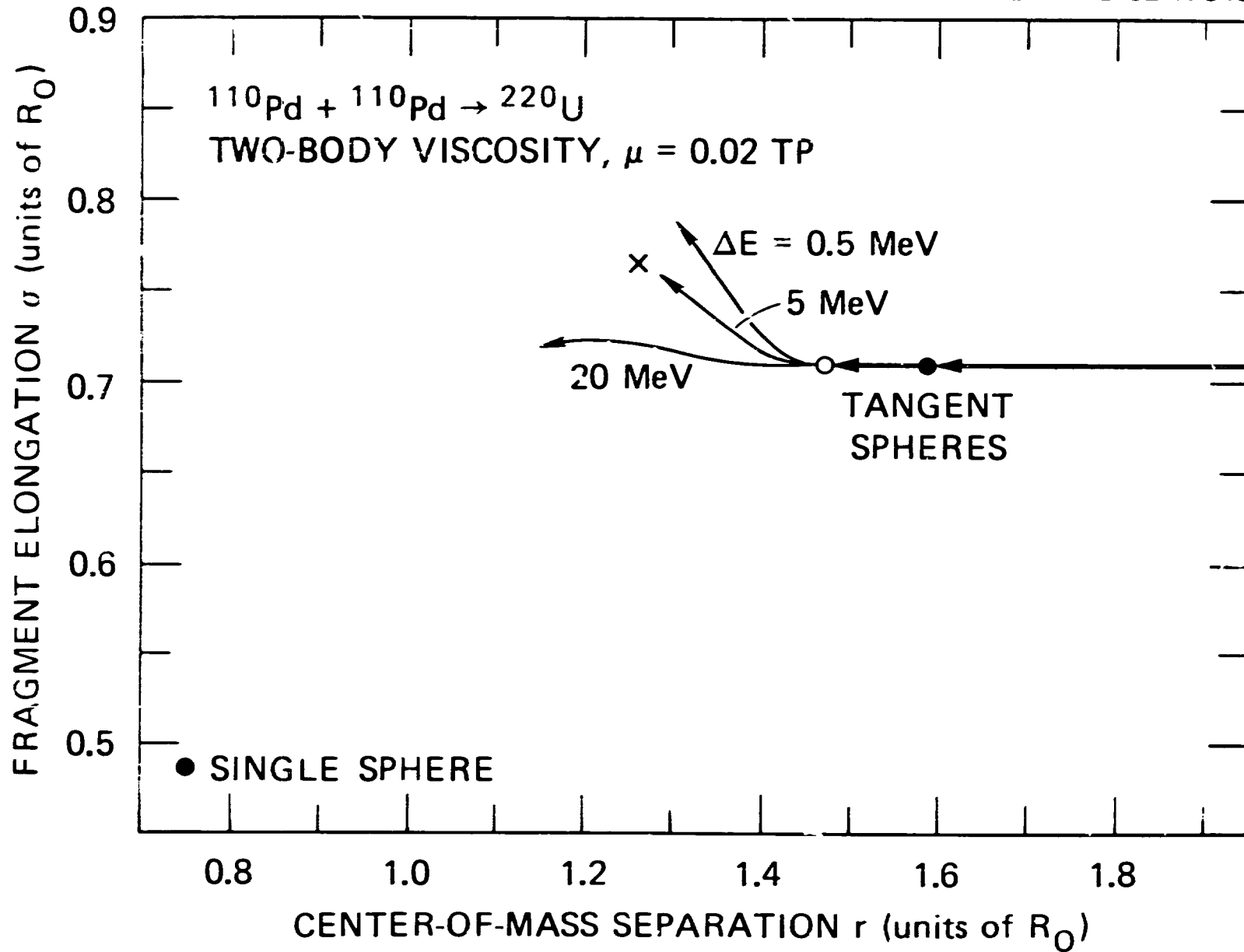


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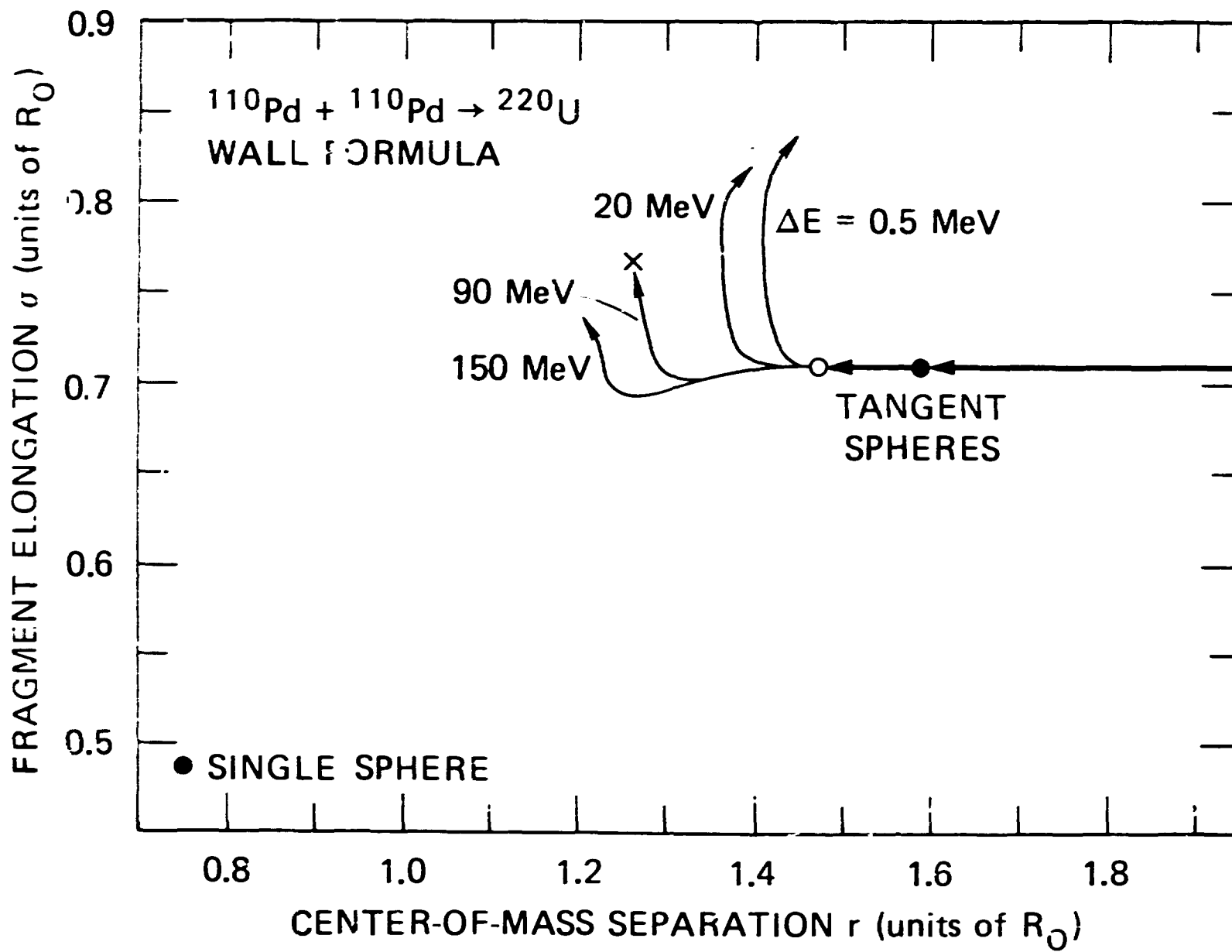


Figure 5.



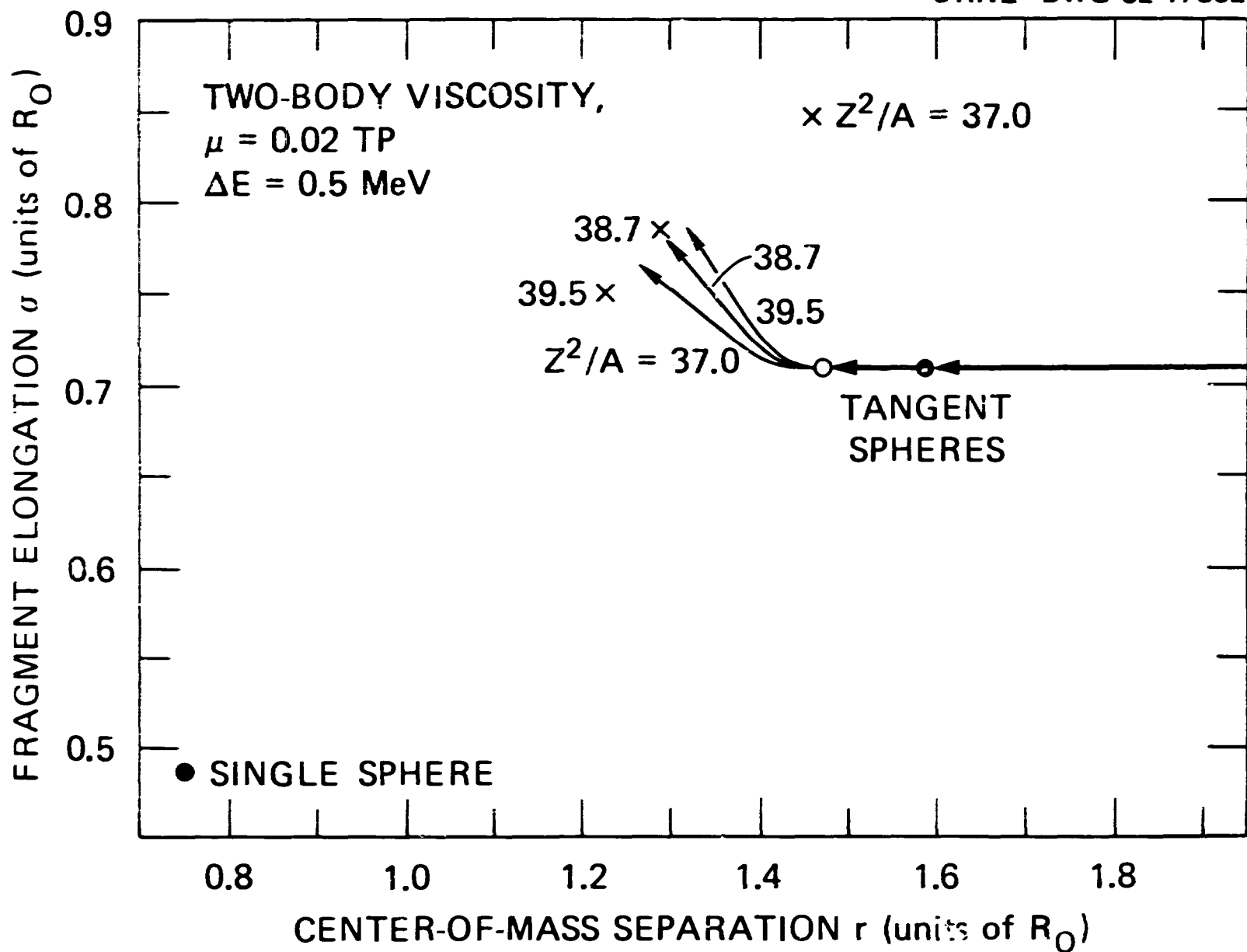


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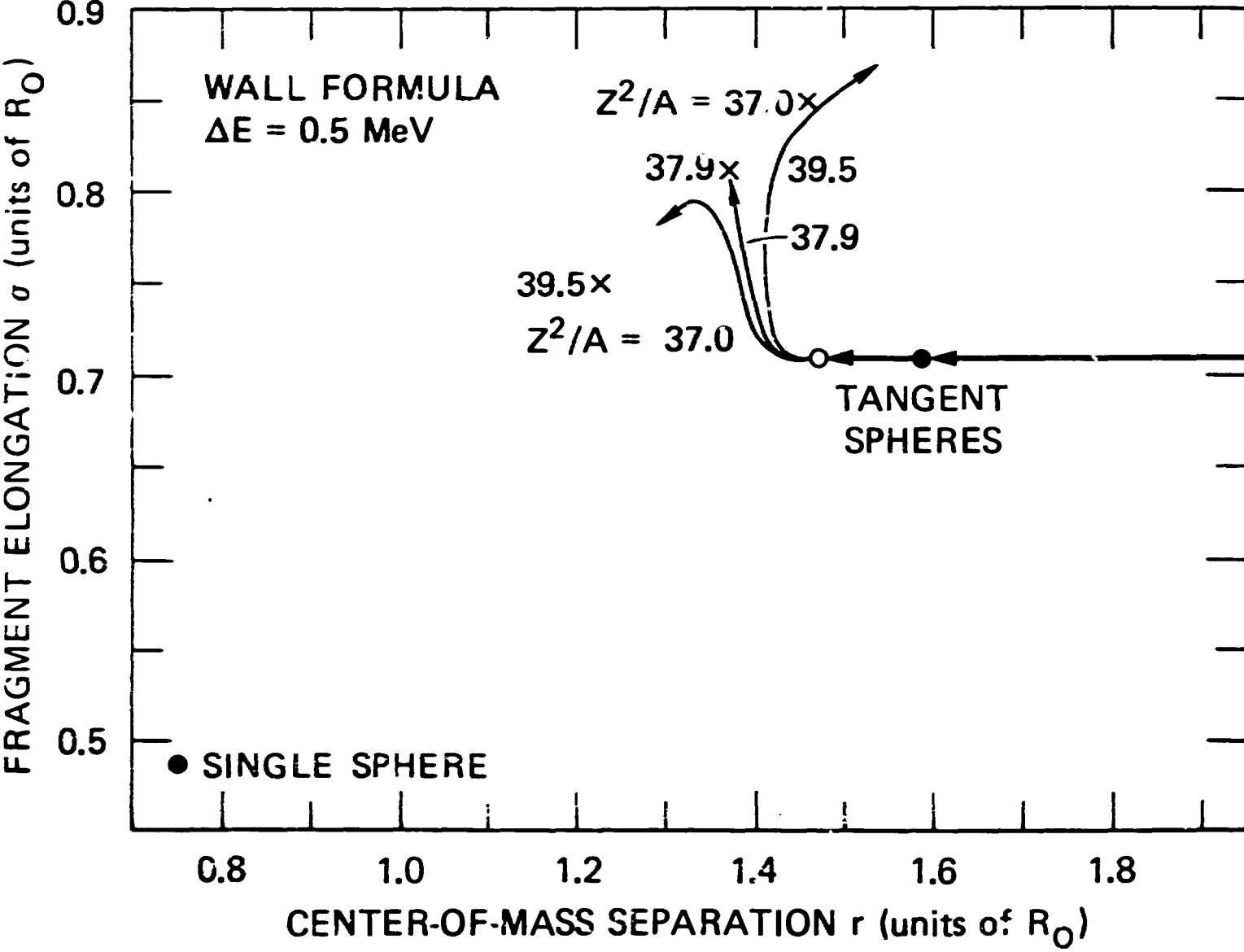


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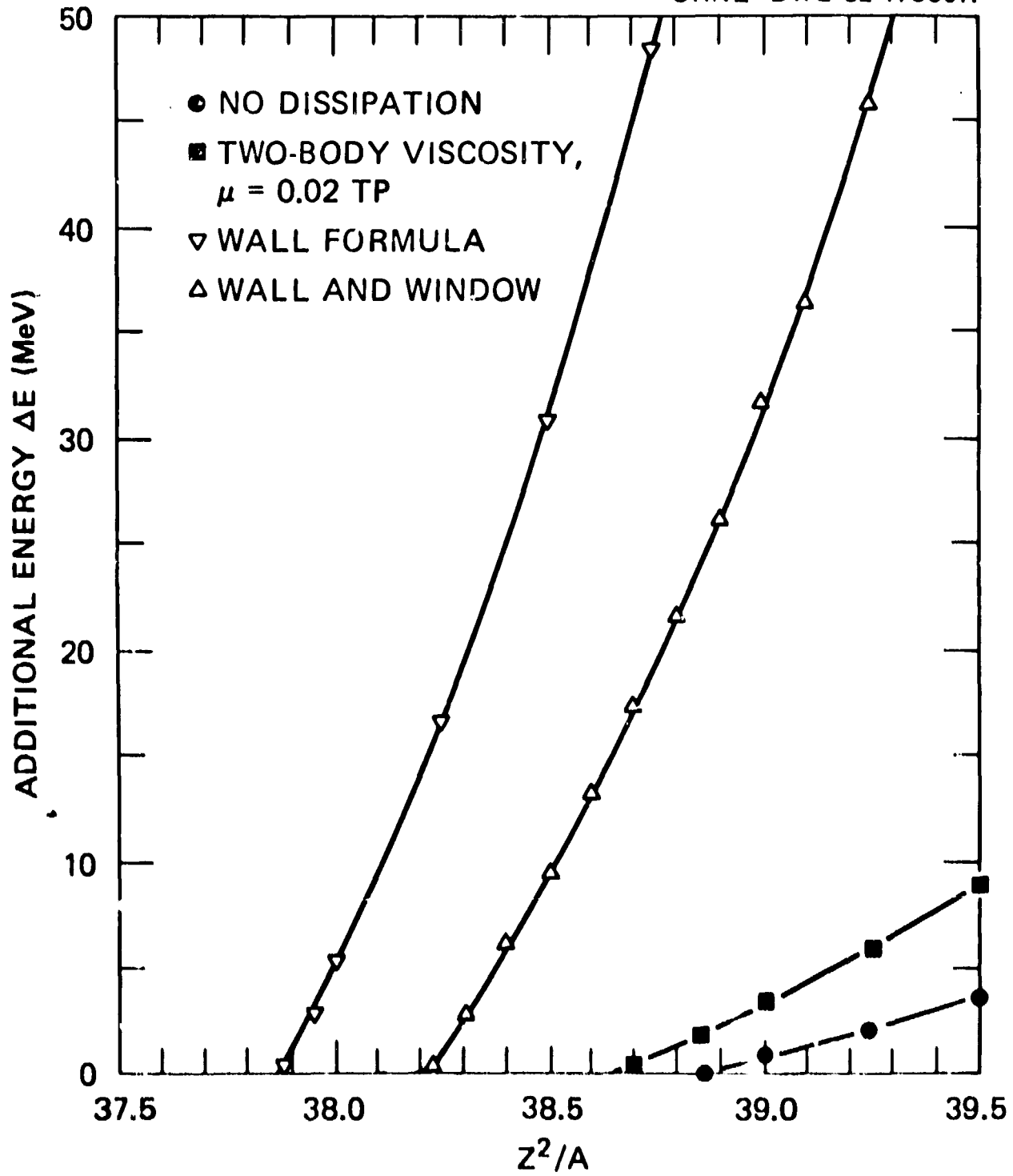


Figure 8.

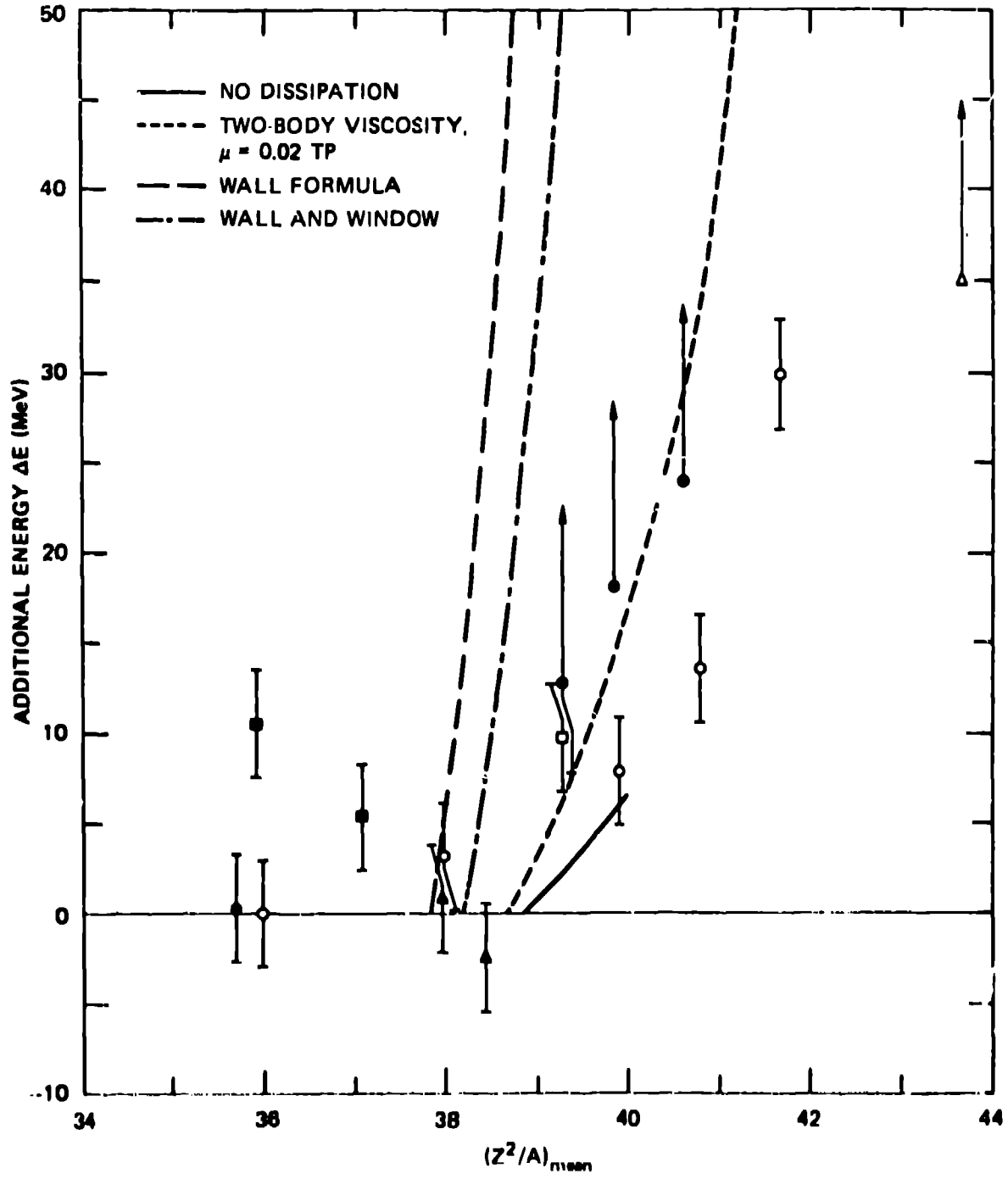


Figure 9.