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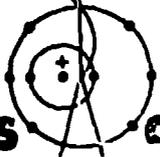
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## THERMAL X-RAYS AND DEUTERIUM PRODUCTION IN STELLAR FLARES

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### Abstract

The x-ray spectrum of flares is shown to be necessarily thermal up to  $> 200$  keV because the self magnetic field of any electron stream required for a thick or thin target source is inconsistently large. The resulting flare model can then be related to stellar luminosity, convection and magnetic fields to result in a maximum possible  $\gamma$ -burst (Mullan, 1976) and continuous x-ray flux. One of the most striking isotopic anomalies observed is the extreme enrichment of Helium ( $^3$ ) in some solar flares and the mysterious depletion of deuterium. We discuss how deuterium may be produced and emitted in the largest flares associated with  $\gamma$ -bursts but in amounts insufficient to support the tentative conclusion of Coleman and Worden (1976).

### Introduction

In a separate paper, we (Colgate et al. - these proceedings: "Helium ( $^3$ ) - Rich Solar Flares") discuss a model of solar flares based upon the enhanced dissipation of the current associated with a twisted helical flux tube convected to the surface of the sun. In the case of the August 4 flare, the theoretical thermal x-ray temperature  $T_7 = 2$  and x-ray emission measure  $n_e^2 \text{ Volume} = 6 \times 10^{49}$  derived from total energy, dimensions and time agree remarkably with observations. We thus extend these considerations to other stars by deriving in greater depth the arguments for expected magnetic fields.

### The Origin of Stellar Magnetic Fields

The generally accepted view of the origin of stellar magnetic fields, Parker (1971) is that a small dipole field is amplified by a large factor by differential rotation of a laminar, i.e., non-convective core. The resulting toroidal magnetic field (actually two or more oppositely directed toroids) lies entirely within the star. The outer boundary of the toroidal field is the base of the convective zone. Convection then carries some of this toroidal flux to the stellar surface. Additional flux can erupt due to excessive toroidal field strength (Leighton, 1969). Since convection takes place in a rotating system (the star must be rotating), the convection is partially cyclonic, i.e., a loop of flux will be twisted about a radius vector of the star. Hence, a loop of flux initially in the toroidal plane will be rotated-stochastically into the dipole, or orthogonal plane. The release of flux loops by resistive dissipation above the surface of the star in the dipole plane adds to the initial small dipole field and hence a generator occurs. The saturation of this generator occurs either because of a limited differential rotation stress or a limited convection stress.

The sun is a convective star which means that within a distance  $\Delta R$  from the surface at  $R$  all the heat must be carried by thermal convection. We will summarize the physical arguments for  $\Delta R$  rather than relying solely upon model calculations.

The boundary between convective and non-convective core is determined by the condition that the temperature-density distribution must be not less than adiabatic, otherwise an interchange between two fluid elements, vertically, can lead to more work done on the rising element than on the subsiding one and hence instability and hence turbulence. In the event of no turbulence and just radiative transport and pressure balance in the outer layers of a star where  $R$  and  $M$  are effectively constant lead uniquely to what is called the "radiative zero" solution (Schwarzschild 1958) where  $T \propto \rho^{\frac{1}{3.25}}$ . The exponent is quite insensitive

to various model parameters and so the convective-nonconvective zone boundary occurs where the effective gas ratio of specific heats,  $\gamma$ , falls below  $(1 + 1/3.25)$ . Since a free particle gas has  $\gamma = 5/3$  we only expect a lower  $\gamma$  where ionization occurs. Radiation pressure also lowers  $\gamma$ , but only asymptotically to  $(1 + 1/3)$ . Hence, in the sun where radiation pressure is not dominant, the convection zone boundary will occur near the onset of the highest helium ionization potential  $\approx 50$  eV depending upon density. Constituents of higher atomic number are fractionally too small to make a difference in  $\gamma$ .

The surface temperature of the sun,  $T_{\text{sur}} \approx 1/2$  eV and the scale height  $h_{\text{sur}} = T/gM_p \approx 2 \times 10^7$  cm.  $g = M_{\odot}G/R_{\odot}^2$ , and  $M_p =$  mean molecular weight  $\approx$  mass of the proton, (nonionized at the surface temperature). Therefore, the scale height at the convective zone boundary,  $h_{\text{con}} = h_{\text{sur}}T_{\text{con}}/T_{\text{sur}} \approx 2 \times 10^9$  cm. The depth of the convective zone  $\Delta R_{\text{con}} \approx h_{\text{con}}/(\gamma-1) \approx 6 \times 10^9$  cm in reasonable agreement with model calculations where  $\Delta R_{\text{con}} \approx 10^{10}$  cm, (Schwarzschild 1958). The density at the convective zone boundary scales as

$$\rho_{\text{con}} = \rho_{\text{sur}} \left( \frac{T_{\text{con}}}{T_{\text{sur}}} \right)^{\frac{1}{\gamma-1}} \approx \rho_{\text{sur}} \left( \frac{T_{\text{con}}}{T_{\text{sur}}} \right)^3 \approx 2 \times 10^6 \rho_{\text{sur}}. \quad (1)$$

We note that this result is very sensitive to  $\gamma$  and hence a crude estimate.

Since  $\rho_{\text{sur}} \approx 1/(K_{\text{sur}}h_{\text{sur}}) \approx 5 \times 10^{-8}$  g/cm<sup>3</sup>, where  $K_{\text{sur}}$  is the surface opacity, then

$$\rho_{\text{con}} = 0.1 K_{\text{sur}}^{-1} L^{-1} M T_{\text{con}}^3 \text{ g/cm}^{-3} \quad (2)$$

where we have included the scaling in terms of surface opacity, and luminosity,  $L$ , radius and mass in solar units, i.e.,  $\sigma T_{\text{sur}}^4 4\pi R^2 = L$  etc.

### Turbulent Convection

In a fully convective region, i.e., where heat transport other than by convective motions is negligible, the buoyancy force per unit volume in pressure equilibrium is  $F_b = \Delta T/T \rho g$ . If this force is converted into kinetic energy of the buoyancy element rising one scale height,  $h = T/g$ , and the energy is divided equally between kinetic and potential energy, then  $F_b h = \rho v^2 = \Delta T \rho$  or  $v^2 = \Delta T$ . The convective heat flux  $\phi$  should be a fraction ( $\approx 1/2$ ) of the mean convective velocity times  $(\rho \Delta T)$  so that the convective heat flux, which must also be the luminosity, becomes

$$\phi = 1/2 (v \rho \Delta T) = 1/2 \rho v^3 = L/4\pi R^2. \quad (3)$$

The maximum convected fluid stress is  $\approx \rho v^2/2$  which must be larger than the stress of the magnetic field that presumably is to be convected to the stellar surface. Therefore  $B_{\text{max}}^2/8\pi = \rho v^2/2$ . Since the convected heat flux in Eq. 3 must be constant, the turbulent stress  $\rho v^2 \propto v^{-1} \propto \rho^{-1/3}$ . In other words as we go radially outwards from the convective zone boundary the turbulent stress increases so that once a flux loop is "torn" loose, we would expect further magnetic flux convection to take place not limited by the turbulent stress. We must also check that our scaling does not violate our convection assumption of local pressure equilibrium; namely, the convection velocities should not exceed sound speed as we approach the surface. If we use equations 2 and 3

$$v_{\text{con}} = 1.1 \times 10^4 K^{1/3} L^{2/3} T_{\text{con}}^{-1} M^{-1/3} R^{-2/3} \text{ cm sec}^{-1} \quad (4)$$

where again solar units are used for  $K, L, R$  and  $M$ . At the surface the convective velocity will be larger by  $(\rho_{\text{con}}/\rho_{\text{sur}})^{1/3} = T_{\text{con}}/T_{\text{sur}} = 100$ , or  $v_{\text{sur}} \approx 10^6$  cm sec<sup>-1</sup>. This just slightly exceeds sound speed at  $T_{\text{sur}} = 1/2$  eV in partially

ionized hydrogen so that it agrees with the solar surface convective motions observed where line widths  $(\Delta\lambda/\lambda) c \approx 5 \times 10^5 \text{ cm sec}^{-1}$ . If we use this velocity distribution, then the maximum magnetic field that can be convected is

$$B_{\text{max}} = 1.2 \times 10^4 \left( \frac{ML}{K} \right)^{1/6} R^{-2/3} T_{\text{con}}^{1/2} \text{ gauss} . \quad (5)$$

The magnetic fields interpreted for some white dwarfs (Angel et al., 1974) are as large as  $5 \times 10^7$  to  $10^8$  gauss. These fields are not obviously within the scaling of Eq. 5 unless we recognize that in white dwarfs the convection zone is likely to start at the boundary between a carbon-oxygen core and a helium atmosphere. Hence  $T_{\text{con}}$  (white dwarf) becomes that of an oxygen boundary  $\approx 16 T_{\text{con}}$  (solar). Then if L white dwarf  $\sim 10^{-2} L_{\odot}$ , and  $K = \text{compton at } T_{\text{surface}}$  white dwarf  $\approx 15,000$  degrees, then  $B_{\text{max}} \approx 2 \times 10^7$  gauss. A helium burning shell will force convection to initiate deeper yet and so may explain the somewhat larger observed fields.

#### The Topology of Convected Fields

The absolute maximum average field that could be convected to the surface of the sun by these arguments is  $10^4$  gauss. The average fields are very much less than this, 100 gauss, but typical sun spot fields extend up to 5000 gauss. We do not believe there is yet a strong physical argument leading to a prediction of the ratio of maximum/average but we note that for the August 4 large flare  $B \approx 10^3$  gauss which is comfortably less than the maximum value. Furthermore, the magnetic pressure of this field,  $B^2/8\pi$  is approximately the same as the gas pressure of the photosphere so that below the photosphere the magnetic pressure will be everywhere small compared to the gas pressure so that the field will only be a small perturbation on the turbulent motions. We expect that a large stellar flare will be that turbulent extremum that convects a loop of flux of the size of the largest eddy and hence scale height,  $h_{\text{con}}$ , of the convective zone. Such eddies reach the stellar surface only rarely without breaking up roughly each scale height in agreement with the rarity of flares. Finally since vorticity is a stochastic variable in turbulence, we expect such loops to have an arbitrary twist one end relative to the other. The scale size,  $h_{\text{con}} = 2 \times 10^9 T_{\text{con}} \mu R^2 / M \text{ cm}$ , where  $\mu = \text{molecular weight}$ , and the twist are both in agreement with the topology of solar flares.

We now consider the question whether using this model of the origin of flare fields and the associated model of the x-rays from stellar flares (Colgate et al. - these proceedings) whether  $\gamma$ -bursts are a reasonable extrapolation as suggested by Mullan (1976). Mullan argues for flare densities based upon chromospheric densities. On the other hand we have pointed out that the photosphere, heated by the thermal conduction flux, expands along the flux tube rapidly and reaches pressure equilibrium in a time short compared to the flare duration. The equilibrium pressure is determined by the condition that the flare radiates at the ends mostly in the XUV, at the rate that energy is released. At constant pressure the bremsstrahlung radiation from the major length of the flare is a small fraction of the total heat.

#### White Dwarf $\gamma$ -Bursts

The maximum magnetic field strength derived on the basis of the convective stress is  $B_{\text{max}} \approx 10^8$  gauss, and indeed such fields are observed, but we believe a more conservative value analogous to the sun is 1/10 of this or  $10^7$  gauss. This gives a pressure large ( $\times 100$ ) compared to the photosphere, but small ( $10^{-16}$ ) compared to the base of the convective zone pressure. The size is determined by  $\ell = h_{\text{con}} \approx 4 \times 10^8 \text{ cm}$  where  $R = R_{\odot}/70 \approx 10^9 \text{ cm}$ ,  $\mu = 2$  and  $T_{\text{con}} = 2.5 \times 10^8$  degrees. This flare size agrees with the spot size derived from light variation (Mullan, 1976). Then if we choose the same topology of a twisted flux loop of diameter = 1/10 length, then the total flare energy  $W_T = 2 \times 10^{36}$

ergs. This energy places the median size  $\gamma$ -burst at roughly  $D \approx 30$  pc as discussed in Mullan (1976). Both his discussion and Strong et al. (1974) conclude that  $\gamma$ -bursts are more likely at  $D \approx 300$  to  $3000$  pc which requires  $10^2$  to  $10^4$  times our suggested energy. Mullan models these values by choosing  $B_0 = 10^8$  gauss and  $\ell = R_0 = 10^9$  cm. The time scale of the flux dissipation is an uncertain modeling parameter but if we use the resistive instability as the basis of the filamentation and enhanced resistance, then  $t = t_0 \ell / \ell_0 (V_A / V_{A0})^{2/3}$  where  $\ell$  = length of the flare,  $= h_{\text{con}} = 4 \times 10^8$  cm and  $V_A$  = the Alven speed.  $t_0 \approx 100$  seconds for typical solar flares,  $\ell / \ell_0 = 1/5$  and  $(V_A / V_{A0})^{2/3} \approx 1/10$  so that  $t \sim 1$  to  $2$  sec in agreement with typical  $\gamma$ -bursts (Cline et al., 1973). Then the temperature determined by thermal conductivity alone becomes

$$T = 2 \times 10^7 \left( \frac{W_T \ell}{\pi R_0^2 t K_0} \right)^{2/7} \text{ degrees where the quantities are scaled}$$

to the August 4, 1972 solar flare  $W_T = 10^{31}$  ergs,  $\ell = 2.5 \times 10^9$  cm,  $R_0 = 10^8$  cm,  $t = 10^3$  seconds, so that for the white dwarf flare  $T_{\text{conduction}} = 8 \times 10^9$  deg  $\approx 800$  keV. This is higher than estimated for  $\gamma$ -bursts  $\approx 150$  keV (Cline and Desai, 1975) but the thermal conduction loss solution is unrealistic for these high temperatures. In particular the density distribution based upon thermal conduction, pressure balance and radiation primarily in XUV from the ends assumes that the bremsstrahlung contribution to the radiation loss is small. This assumption is valid in the case of solar flares where  $T_{\text{max}} \approx 1$  to  $2 \times 10^7$  degrees and the XUV radiation loss  $\propto T^{-1/2}$  dominates over bremsstrahlung  $T^{+1/2}$  up to  $10^7$ . In this latter case the above three restrictions result in a total radiation loss per logarithmic temperature interval  $\propto T^{1/2}$ . In the case of non-relativistic bremsstrahlung this becomes  $\propto T$  and for relativistic bremsstrahlung ( $kT_e \gg 1/4 mc^2$ ) the radiation loss per logarithmic temperature interval becomes proportional to  $T^{3/2}$ . Thus the major energy loss will occur in the region of  $T_{\text{max}}$  for the white dwarf flares as opposed to lower temperature XUV in solar flares. This is simply because high temperature bremsstrahlung is a more efficient radiation mechanism than XUV, given the thermal conduction solution. Then the surface layers of the star will continue to expand up the flux tube until the radiation rate equals the heating rate. We note that the sound speed  $C_s$  at 150 keV is  $C_s \approx 10^9$  and the time to reach pressure equilibrium is  $\ell / C_s \approx 2/10$  seconds. Thermal stability considerations limit the temperature to the value separating the relativistic and non-relativistic bremsstrahlung, i.e.,  $kT_e \approx 1/4 mc^2 \approx 100$  to  $150$  keV because only below this temperature is the region of  $T_{\text{max}}$  stable. Then the bremsstrahlung radiation loss rate in this region equals the heating rate and using the results of Maxon (1972) for electron-electron and relativistic bremsstrahlung we obtain

$n_e \approx 7 \times 10^{10} \left( \frac{W_T}{\text{Vol } t} \right)^{1/2} = 10^{17} \text{ cm}^{-3}$ . This is somewhat higher than estimated by Charikov and Starbunov (1975) but they simply took the limiting dimensions given by  $\Delta x = c\Delta t$  and derived a density of  $n_e \approx 10^{16} \text{ cm}^{-3}$  assuming bremsstrahlung. Thus we have a description of a  $\gamma$ -burst as the largest stellar flare - following the original suggestion of Mullan, but substituting a solar flare model limited by stellar convection, and based upon conduction, pressure balance, and XUV radiation.

We now consider the likely particle acceleration and spallation in such a flare. We have already shown in solar flares how it is consistent to assume that all the current of the flare is carried by run-away ions, and the electrons are immobilized by instabilities. The current and potential drop in the present case of a largest white dwarf flare are:  $I = 5 R B_0 = 10^{15}$  amps and  $V = d/dt$  (inductance  $\times$  current)  $= W_T / I = 4 \times 10^{14}$  volts.

### Deuterium Production

There are two sources of deuterium from such flares, the one from the spallation of  $^4\text{He}$ , (about 1/3), and the second from the capture on hydrogen of neutrons resulting from partial and more complete spallation of the  $^4\text{He}$ . In the case of  $^3\text{He}$ -rich flares, the neutron capture process would be the only source of deuterium but in order to contribute to the galactic deuterium abundance the deuterium formed by neutron capture must be ejected from the stellar surface before convection carries it to stellar regions where it will be destroyed. In this paper we consider only the direct spallation production of D and leave for later the possible survival and ejection of deuterium formed by capture in the envelope. In direct production, the life-time for survival of deuterium in a white dwarf large flare of the  $\gamma$ -burst type becomes  $\tau_D = 1/n_1\sigma_1v_1 = 1/10$  second, for the derived parameters  $n_1 = 10^{17} \text{ cm}^{-3}$ , and  $kT = 150 \text{ keV}$ ,  $\sigma_1 \approx 10^{-25} \text{ cm}^2$ . This means that a deuteron can just escape if it is formed by spallation and directly escapes out of the flare by the tangled flux model. The total deuterium production in the case of a helium-rich envelope (Angel and Landstreet, 1975) is  $N_D \approx \phi_1 \sigma_D n_1 \text{ Volume}$  where  $\phi_1$  is the current, I, in appropriate units - assumed in  $100 \text{ MeV/nucleon } ^4\text{He}$ ;  $\sigma_D \approx 10^{-25}$  is the spallation production cross section of D, and  $\int n_1 \times \phi$  (Volume) is the total plasma bombarded by the flux  $\phi_1$ . Then  $N_D \approx 10^{35}$  deuterons per flare. If all these deuterons were to escape, the energy invested per deuteron becomes  $\approx 25$  ergs. This is a prohibitively large value to create the deuterium of the ISM. In order to estimate an upper limit to the contribution to the deuterium of the ISM, we note that Herzo et al. (1976) have extended the  $\gamma$ -burst integral number distribution  $S^{-1.5}$  down to a magnitude  $\approx 10^{-6} \text{ ergs cm}^{-2}$  or  $\approx 1/500$  of the large event size implying that these events are isotropic out to the galaxy thickness of 100 pc and so large event size could occur within a radius of  $100/(500)^{1/2} \approx 5 \text{ pc}$  at a rate of 10 per year. This makes our typical large flare too large by  $6^2$  and so is an absolute upper limit. Therefore, 1/50 event occurs  $(\text{pc})^{-3} \text{ y}^{-1}$  or  $\approx 10^{40}$  deuterons per  $(\text{pc})^3$  per (1/10) age of the Universe - the likely astration time of the ISM. If the fractional deuterium density is  $\approx 2 \times 10^{-5}$  at  $n_p = 1/10$ , the contribution to the deuterium of the ISM by spallation production and ejection in  $\gamma$ -burst type flares is  $4 \times 10^{-10}$  and therefore negligible. It is not likely that there is a greater source of direct flare production because the uniform density isotropic distribution of  $\gamma$ -bursts extends over such a large dynamic range that a large contribution by very small flares seems unlikely.

### M Dwarf Flares

For completeness we calculate the probably characteristics of a large M-dwarf flare on the same basis as the white dwarf flare. If  $T_{\text{con}} \approx 50 \text{ eV}$ , at the base of the convection zone corresponding to the helium ionization potential, then from Eq. 6,  $B_{\text{max}} \approx 50,000$  gauss as suggested by Mullan (1974 and 1975) and Worden (1974). The size of these flares using our estimate of  $h_{\text{con}}$  is  $\approx 4 \times 10^8 \text{ cm}$ , ( $R \approx 1/20 h_{\text{con}} \approx 2 \times 10^7 \text{ cm}$ ) smaller than estimated from the optical variations, (Mullan, 1976) so that probably we have underestimated the base of the convection zone. If we increase  $h_{\text{con}} \approx 10^9 \text{ cm}$ , and  $R = 10^8 \text{ cm}$ , and  $B_0 = 10^4$  gauss in agreement with the B Y Draconis interpretation, then  $W_T \approx 10^{32}$  ergs in agreement with the stellar flares observed in the optical and x-rays (Heise et al, 1975). The temperature should be higher than solar flares by the ratio

$\left(\frac{W_T \ell}{R^2}\right)^{2/7} \approx \times 4$ , or  $\approx 8 \times 10^7$  degrees. A typical small flare might be 1/2 of this temperature in rough agreement with Heiss et al. The electron density becomes

$n_e = n_0 \left( \frac{W_T}{R^2 t} \right)^{6/7} (T_7^{-1})$  where  $n_0 = 2 \times 10^{12}$  for the August 4 flare parameter and so  $n_e \approx 0.5 \times 10^{13}$ . The deuterium production by direct spallation becomes  $N_D = \phi_1 \sigma_D n_i \text{ Volume} \approx 10^{28}$  D/flare, or  $10^4$  ergs per deuteron. Coleman and Worden (1976) consider the production by the plasma shock wave of the flare which they believe gives a very much larger yield of spallation deuterium. At this point the solar flare model does not consider this effect, but we note that the filamentation instability in the case of the  $^3\text{He}$ -rich flares gives a yield of  $10^{29}$   $^3\text{He}$  for the expenditure of  $10^{29}$  ergs or 1 erg/ $^3\text{He}$ . This is still  $10^3$  times the energy limit required to explain the deuterium production by M dwarfs (Coleman and Worden 1976). We therefore conclude that a primary origin of deuterium from M dwarfs is unlikely.

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#### References

- Angel, J.R.P. and Landstreet, J. P., 1975 Ap. J., 196 E19.  
 Angel, J.R.P., Carswell, R. F., Strittmatter, P. A., Beaver, E. A. and Harms, R. 1974, Ap. J. (Letters) 194, L47.  
 Charikov, Yu.E. and Starbunov, Yu.N., 1975, 14th International Cosmic Ray Conference, Munich, Vol. 1 paper Og5-4.  
 Cline, T. L. and Desai, U. D., 1975 Ap. J. (Letters), 196, L43.  
 Cline, T. L., Desai, U. D., Klebesadel, R. W., and Strong, I. B., 1973, Ap. J. (Letters), 185, L1.  
 Coleman, G. D. and Worden, S. P., 1976 Ap. J., 205, 475.  
 Colgate, S. A., Audouze, J. and Fowler, W. A., these proceedings: "Helium (3)-Rich Solar Flares."  
 Heise, J., Drinkman, A. C., Schriver, J., Mewe, K., Gronenschild, E., Den Boggende, P., 1976 Ap. J. (Letters) 202, L73.  
 Herzo, D., Dayton, B., Zych, A. D., White, J. R., 1976, Ap. J. (Letters) 203, L115.  
 Leighton, R. B., 1969, Ap. J. 156, 1.  
 Maxon, S. 1972, Phys. Rev. A 5, 1630.  
 Mullan, D. J. 1974, Ap. J. 192, 149. 1975, Ap. J. 200, 641. 1976, Ap. J. 208, 199.  
 Parker, E. N., 1971, Ap. J., 164, 491.  
 Schwartzschild, M. 1958, "Structure and Evolution of the Stars," Princeton Univ. Press.  
 Strong, B., Klebesadel, R. W., Olsen, R. A., 1974, Ap. J. (Letters) 188, L1  
 Worden, S. P., 1974, Pub. A. S. P. 86, 595.