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ISOBAR PROPAGATION IN THE NUCLEAR MEDIUM

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ISOBAR PROPAGATION IN THE NUCLEAR MEDIUM*

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ABSTRACT

It is argued that introduction of the isobar degree of freedom in describing pion-nucleus interactions provides a convenient, unified framework within which to discuss both many-body corrections to the standard multiple scattering approach and the properties of the $\Delta(1232)$ in nuclear matter. Important aspects of isobar-nucleus dynamics, namely, isobar-hole interactions and Δ self-energy modifications, are discussed in the context of pion elastic scattering and incoherent pion production.

I. INTRODUCTION

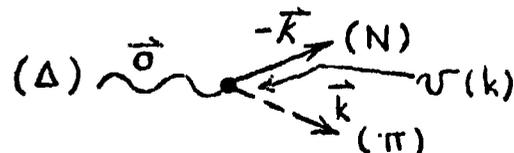
Meson-nucleon interactions at low to intermediate energies are dominated by resonances or, effectively, by the formation of short-lived "particles" with well-defined quantum numbers. For example, the pion-nucleon system exhibits resonances in all spin-isospin channels for invariant energies less than 2 GeV, while the K^-p interaction near threshold is controlled by the (subthreshold) $\Lambda(1405)$ resonance. Consequently, it is clear that any attempt to relate meson-nucleus and meson-nucleon interactions implicitly contains the isobar (i.e., in the energy variation of the elementary amplitudes), and furthermore it might seem natural to introduce explicitly the isobar degree of freedom in theoretical approaches to meson-nucleus interactions. However, the second step certainly is not necessary, as is evident from the fact that most treatments of pion-nucleus scattering are based upon fixed-scatterer multiple scattering theory, in which the isobar plays no explicit role. More precisely, since we never directly observe the resonance in our detectors, we can in principle project out these channels without altering the physics in any way. Nevertheless, it will be argued here that introduction of the isobar degree of freedom is very convenient in a many-body approach to the meson-nucleus problem, providing a unified framework within which both kinematical corrections to the standard multiple scattering approach and dynamical modifications of the elementary amplitudes can be discussed. We shall confine our attention to pion-nucleus interactions in the energy regime dominated by the $\Delta(1232)$ resonance and will discuss some specific reactions involving the important aspects of isobar-nucleus dynamics, namely, Δ -hole interactions and Δ self-energy interactions. A more detailed outline of the paper follows.

After a brief description of the free isobar propagator, we shall present some simple examples in which the dynamics of isobar propagation can lead to significant quantitative effects in the description of pion-nucleus scattering. First, the importance of nucleon recoil as a kinematical correction to the fixed-scatterer formalism will be emphasized, and then modifications due to the Pauli principle of the Δ self-energy in nuclear matter (and therefore of the πN amplitude in the medium) will be discussed. This is intended to motivate the ensuing discussion, in which these physical considerations are incorporated into somewhat more sophisticated approaches to pion elastic scattering and to the isobar propagator in nuclear matter. Specifically, we shall discuss a Tamm-Dancoff approach to $\pi^{-16}\text{O}$ elastic scattering, in which the isobar-hole states are diagonalized with the result that only one collective Δ -hole (doorway) state dominates the scattering in each partial wave. This is encouraging for a unified approach to several coherent reactions on the same nucleus. Finally, we calculate the Δ self-energy in nuclear matter and show how this enters directly in incoherent reactions such as total photoabsorption.

The discussions will, of necessity, be brief and unfortunately much work in the field cannot be presented. A number of relevant references can be found in the proceedings of this conference and in the recent review article of Brown and Weise.¹

II. THE FREE ISOBAR PROPAGATOR

Before discussing the properties of the isobar in nuclei, it is useful to start with a dynamical model for the free Δ . Our approach is to introduce an elementary or bare isobar, with bare mass M_Δ , coupled to the πN channel with a vertex function $v(k)$, as depicted below. The assumption is that, since the πN channel is the only open



channel in the energy region of interest, the important physics will be included if we explicitly handle only this channel. Then, the "dressed" Δ -propagator is given simply in terms of the πN "bubble" self-energy:

$$\begin{aligned}
 \text{Diagram: } & \text{A wavy line with a shaded circle at its end} = \text{A wavy line} + \text{A wavy line with a bubble loop} \\
 \Delta^{-1}(s) &= \Delta_0^{-1}(s) - \Sigma(s) \\
 &= (s - M_\Delta^2) - \int_0^\infty \frac{dq q^2}{16\pi^2} \frac{\omega_q + E_q}{2\omega_q E_q} \frac{q^2 v^2(q^2)}{s^+ - (\omega_q + E_q)^2}
 \end{aligned} \tag{1}$$

where s is the invariant energy squared and where a Blankenbecler-Sugar prescription for the propagator has been assumed² in writing Eq. (1). Our task now is to choose the available parameters so that

the observed Δ -propagator is reproduced.³ This is easily done by noting that πN elastic scattering in the 3-3 channel proceeds through the Δ -propagator:



$$\Rightarrow \Delta(s)^{-1} = |\Delta(s)^{-1}| e^{i\delta_{33}(s)}$$

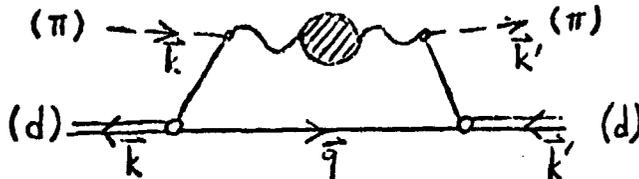
For example, a Yukawa form factor $v(k) = g/(\alpha^2 + k^2)$ with the parameters $g = 3.14 M_\Delta^2$, $\alpha = 1.8 \text{ fm}^{-1}$, and $M_\Delta = 6.83 \text{ fm}^{-1}$ yields an extremely accurate representation of the 3-3 phase shift. The important point is that Eq. (1) leads to an explicit form for the real and imaginary parts of the Δ self-energy; we shall return to this point later.

III. SOME SIMPLE EXAMPLES

A. Kinematical corrections: the effect of nucleon recoil.

In the conventional multiple scattering approach to pion scattering, the nucleon coordinates are "frozen" at fixed positions during the scattering process, thereby eliminating propagation of the isobar. However, precisely because the two-body interaction is resonant, the interaction time is long and we can expect the resonating πN system (i.e., the isobar) to propagate, on the average, about 2/3 fm. This is not negligible compared to internucleon separations and, in fact, simple exercises demonstrate the importance of this effect.

First consider pion-deuteron elastic scattering in the single-scattering approximation. The amplitude is given by



$$T(\vec{k}, \vec{k}'; s) = \int \frac{d\vec{q}}{(2\pi)^3} \psi_d^* \left[\frac{v(m^2) \vec{m} \cdot \vec{m}' v(m'^2)}{\Delta^{-1}(\sigma_q)} \right]^{(2)} \psi_d$$

where \vec{q} is the spectator nucleon momentum, \vec{m} and \vec{m}' are relative momenta at the $\pi N \Delta$ vertices, ψ_d is the deuteron wavefunction, and $\Delta(\sigma_q)$ is the isobar propagator evaluated at the invariant sub-energy

$$\sigma_q = (\sqrt{s} - E_q)^2 - \vec{q}^2 \quad (3)$$

The details can be found in the paper of Woloshyn, Moniz, and Aaron,² but we wish to emphasize here only the fact that the momentum dependence in σ_q corresponds to spatial propagation of the Δ . In the fixed nucleon approximation, σ_q would be fixed at some average value, and the result is shown in Figure 1: at large momentum transfers (i.e., back angles), the approximation is rather poor. In Reference 2, it is seen that this remains true when the full three-body problem is

solved and indeed that the multiple scattering series converges much more rapidly when the isobar kinematics is correctly accounted for. In the language of the optical potential, the factorization approximation is not adequate for precise quantitative predictions.

Another simple example of the importance of properly treating isobar propagation is provided by studying pion propagation in infinite nuclear matter. The wave equation is just

$$G^{-1}(p) \psi(p) = [p^2 - k^2 - \pi(p; k)] \psi(p) = 0 \quad (4)$$

where k is the on-shell pion momentum (fixed by the energy), p is the (complex) wavenumber in the medium, and π is the pion self-energy or optical potential. Any nonlocality will be reflected by a p -dependence in π and will result in multiple solutions p_i of Eq. (4). Therefore, we can write a multiple eigenmode representation of the Green's function⁴

$$G(r) = \sum_i g_i \frac{e^{ip_i r}}{r} \quad (5)$$

with g_i the residues ($\sum_i g_i = 1$), and expect a strong mixing between the eigenmodes whenever the pion mean free path becomes comparable to the nonlocality.⁴ For example, isobar propagation corresponds to a nonlocality in the pion coordinate and, following the paper by Lenz,⁵ we can study this by writing⁶

$$\pi(p; k) = \frac{k \int \sigma_{\pi N} \Gamma/2}{E - E_R + i\Gamma/2 - p^2/2M^*} \quad (6)$$

This corresponds to the lowest order optical potential being generated by resonant scattering, and the isobar kinetic energy term in the Breit-Wigner denominator to isobar propagation. Inserting this into Eq. (4), it is clear that two eigenmodes emerge, which, in the low density limit, can be readily identified as "pion" and " Δ -hole" modes. At resonance, the solutions are

$$p_i^2 = k^2 + \frac{iM^*\Gamma}{2} \left[1 \pm \sqrt{1 - l_\Delta/l_\pi} \right], \quad i=1,2 \quad (7)$$

$$l_\pi = (\int \sigma_{\pi N})^{-1}, \quad l_\Delta = 2 \frac{k/M^*}{\Gamma/2}$$

where l_π and l_Δ are the pion mean free path and the isobar propagation distance, respectively. Clearly, when $l_\pi \gg l_\Delta$ (i.e., $\rho \rightarrow 0$), the conventional pion mode dominates, whereas when l_Δ and l_π are comparable, the eigenmodes are mixed. The net effect of all this is that, at large densities, the pion is much less damped in the medium when the Δ -hole channel is offered as a means of propagation.

It should be clear from these examples that isobar propagation is an important ingredient in the description of pion-nucleus interactions. We repeat that this could be incorporated without explicit introduction of the Δ but that recoil is most easily conceptualized in terms of isobar propagation.

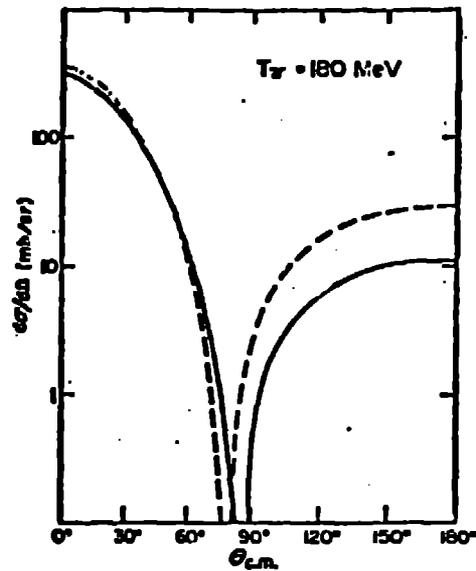


Figure 1. Impulse approximation for πd scattering: dashed and solid curves are without and with isobar propagation, respectively. Curves taken from Reference 2.

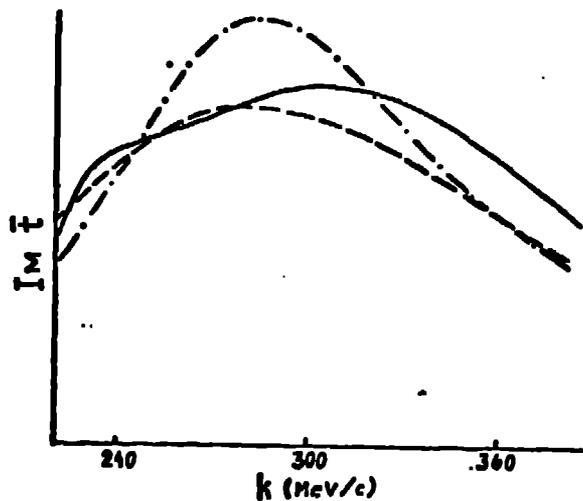


Figure 2. Imaginary part of the forward on-shell πN amplitude in the Fermi sea, as a function of the pion momentum k . Dot-dash, dashed, and solid curves are the free space, Fermi averaged, and full results, respectively.

B. Dynamical modification of the Δ -propagator: effect of the Pauli principle.

In considering propagation of the isobar in nuclei, we must expect dynamical modifications due to interaction of the constituents (i.e., the pion and nucleon) with the nuclear medium. In the simplest approximation, the pion and nucleon propagate in optical potentials and the nucleon is subject to restrictions imposed by the Pauli principle. We consider here only the Pauli blocking effect; that is, we consider the problem of computing the isobar propagator in a free Fermi gas:

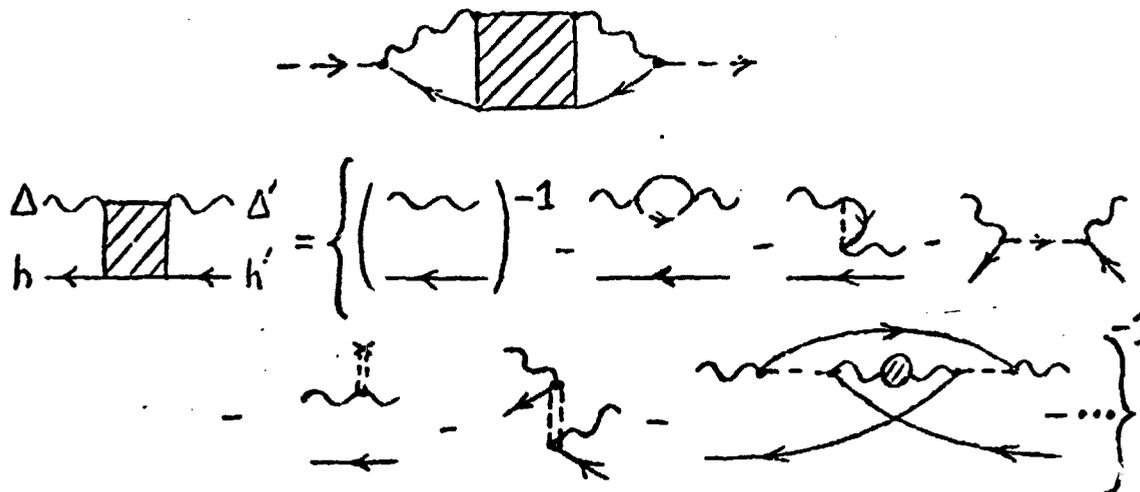
$$\Delta^{-1} = \Delta_0^{-1} - v G_0 Q v \quad (8)$$

where v represents the $\pi N \Delta$ vertex functions, G_0 is the free πN propagator, and Q prevents the intermediate nucleon from occupying states in the filled Fermi sea, thereby modifying the self-energy. In fact, this calculation can be carried out in closed form for simple vertex functions even with the inclusion of spin;⁷ here, we just present in Figure 2 the results for the imaginary part of the forward πN t-matrix in the medium (of course, this is directly proportional to $\text{Im } \Delta$). The dot-dash curve is the free space value for a stationary nucleon, while the dashed curve includes Fermi averaging over the target nucleon ($k_F = 270$ MeV/c, corresponding to nuclear matter density). The solid curve represents the full calculation including Pauli blocking. We see that there is a substantial modification of the amplitude in the medium, including a "shift of the resonance" towards higher energy. As emphasized in Reference 1, pion and nucleon interactions must be included before definite conclusions can be drawn, and this will be discussed in section V.

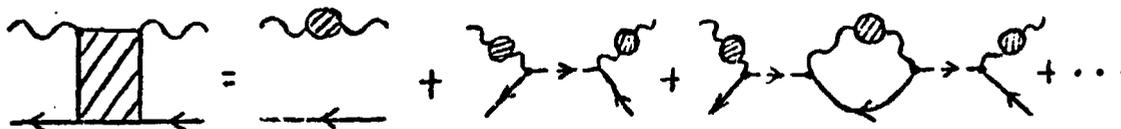
In ending this section, we repeat that both kinematical and dynamical effects arising from isobar propagation can be quantitatively significant and must be incorporated into a reliable theoretical approach to the pion-nucleus problem. We now go on to outline some preliminary calculations of coherent and incoherent reactions in which this has been attempted.

IV. π - ^{16}O ELASTIC SCATTERING IN THE Δ -h FORMALISM

We present here results of a many-body approach to pion elastic scattering which is very similar in spirit to Tamm-Dancoff calculations of nuclear excited states; this work, still in an early stage, is being carried out in collaboration with M. Hirata, J. Koch, and F. Lenz. Basically, Δ -hole states are treated as doorway states⁸ and a diagonalization is performed to find the eigenstates. This approach will be useful if, after diagonalization, only a few collective Δ -h states carry most of the strength in each partial wave. The calculation is carried out for ^{16}O , treated as a harmonic oscillator closed shell nucleus, with the intermediate Δ state also expanded in a harmonic oscillator basis.⁹ The approach is summarized diagrammatically below:



Therefore, the Δ -h states act as entrance and exit channels for the pion and the problem is to compute the intermediate Δ -h propagator. The first three terms inside the brackets correspond to the free isobar propagator with the Pauli blocking effect; the fourth term corresponds to pion propagation between Δ -h states (this should be distinguished from pion "exchange" in the sense that the pion here is "real"); the next term accounts for a background potential acting upon the isobar (e.g., creation and destruction of nuclear p-h pairs would contribute here); the sixth term represents some additional effective interaction between Δ -h states, such as exchange of a ρ -meson; and the last term shown is one example of a multi-hole contribution. To this point, we have included the first five diagrams, and the connection to standard multiple scattering theory can be seen by expanding the Δ -h propagator:



where the dressed propagator now contains both the Pauli effect and the effect of the background Δ potential; this potential is taken to have the shape of the nuclear density and the strength is treated as the only free parameter. It is clear that we have the coherent approximation to the multiple scattering series, but including effects beyond those usually treated in the lowest order optical potential, such as isobar propagation (Fermi motion), binding effects, and Pauli restrictions; in addition, other contributions to the Δ -h interaction are trivially added.

The next step is to diagonalize the Δ -h propagator:

$$\langle D_i | G_{\Delta h} | D_j \rangle = \frac{\delta_{ij}}{E - E_R + i\Gamma/2 - \epsilon_i}, \quad |D_i\rangle = \sum_j C_j^i |(\Delta-h)_j\rangle$$

$$T_{\pi-^{16}\text{O}} = \sum_i \frac{\langle 0 | \vec{k}' \cdot \vec{S}^\dagger | D_i \rangle \langle D_i | \vec{k} \cdot \vec{S} | 0 \rangle}{E - E_R + i\Gamma/2 - \epsilon_i} \quad (9)$$

where \vec{S} is the operator creating a Δ from a nucleon.¹ The results of the diagonalization are the complex eigenvalues ϵ_i and the coefficients C_j^i relating the eigenstates to the original Δ -h basis states. With a Δ background potential of strength roughly $(-70 - 70i)$ MeV,¹⁰ we obtain the total cross section results shown in Figure 3; furthermore, the fit to the differential cross section data is also quite good and we note that the partial wave decomposition of the nuclear amplitude is significantly different from that obtained with a standard optical potential code producing comparable results for the total cross section.

We would like to discuss the results in some more detail. As an example, consider the $\pi - ^{16}\text{O}$, $J^P = 0^-$ state. Truncating the Δ -space at $8 \hbar\omega$, we have a 17-dimensional space, and the relative contributions of the eigenstates to the nuclear amplitudes are listed in Table 1 for pion kinetic energy 140 MeV. While the pion coupling to Δ -h states is distributed over many states, we see from Table 1 that virtually all the strength is carried by one collective eigenstate (number 9 in the table), which we can characterize as a 0^- collective state¹¹ in ^{16}O . In Table 2, we list the energy eigenvalues for the important eigenstates in each partial wave, again (the imaginary parts of the eigenvalues are fairly energy dependent, while the real parts increase with pion energy). In the case of the 4^- and 5^+ states, more than one eigenstate contributes appreciably and all the relevant eigenvalues are given; i.e., the Δ -h strength is fragmented over a few states in the higher partial waves. Note that the imaginary parts of the eigenvalues are strongly L-dependent, corresponding to a large broadening in the 0^- case, and a slight narrowing in the 5^+ ; this of course points out the importance of having available partial wave decomposition of the pion-nucleus elastic amplitude. Further theoretical study will focus upon understanding the dynamical origin of the Δ single-particle potential, but the results are already quite encouraging. In particular, several dynamical effects have been included in the calculation and the dominance of one collective eigenstate in each partial wave implies both that a simple, theoretically motivated parameterization of the pion-nucleus amplitude may be possible⁸ and that we can consequently learn about Δ -nucleus dynamics. Furthermore, having obtained the collective Δ -h states, several coherent processes which go via the Δ can be calculated in a unified way with comparatively little additional effort; for example, a calculation of the high energy $\gamma(^{16}\text{O}, ^{15}\text{N}_{gs})p$ reaction is being carried out.¹²

V. THE Δ SELF-ENERGY IN NUCLEAR MATTER

Finally, we turn to the questions of the Δ self-energy in nuclear matter and of the manner in which we can expect to observe the modifications from the free space value. We start by addressing the second question.

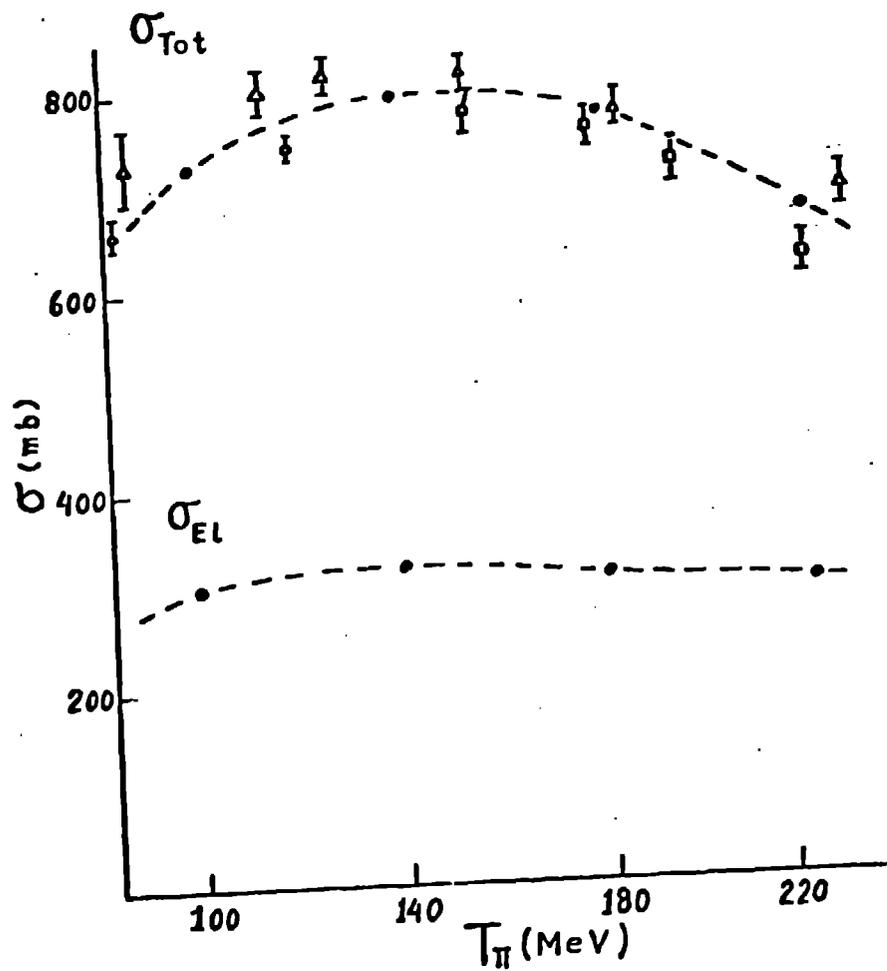
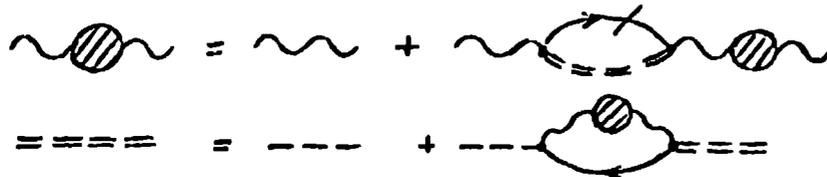
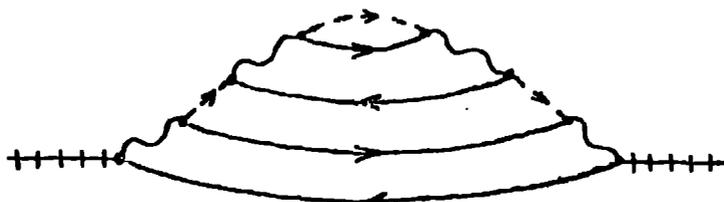


Figure 3. Total and total elastic cross sections for $\pi^{-16}\text{O}$ scattering. Circles connected by the dashed lines represent the theoretical predictions in the Δ -h model.

Calculation of the Δ -propagator in the nuclear medium involves a problem of self-consistency, since the Δ self-energy involves the pion propagator in the medium and the pion self-energy in turn involves the Δ propagator in the medium:



Here, the double-dashed line represents the pion propagator in the medium and the slash on the nuclear line denotes Pauli blocking. We present here only results for a noninteracting Fermi sea of nucleons (as is done in References 13 and 14 for quasi-elastic nucleon knock-out). Then solution of the coupled equations includes terms in the photon polarization propagator such as that shown below:



Note, however, that interference diagrams obtained, for example, by crossing the hole lines are not included. If we adopt the multiple eigenmode representation for the pion propagator (see Equation 5) and a Yukawa-like $\pi N \Delta$ vertex function, then the self-energy in the medium for a Δ at rest can be written as

$$\begin{aligned} \Sigma(E) = & -i \frac{g^2}{4\pi} \sum_j g_j \left\{ \frac{p_j^3}{(\alpha^2 + p_j^2)^2} \left[1 + \frac{i}{\pi} \ln \left(\frac{p_j - k_F}{p_j + k_F} \right) \right] \right. \\ & \left. - i \frac{\alpha}{2} \frac{(\alpha^2 + 3p_j^2)}{(\alpha^2 + p_j^2)^2} \left[1 - \frac{2}{\pi} \tan^{-1} \frac{k_F}{\alpha} \right] - \frac{i}{\pi} \frac{\alpha^2 k_F}{(\alpha^2 + p_j^2)(\alpha^2 + k_F^2)} \right\} \end{aligned} \quad (10)$$

where the eigenmodes p_j are functions of energy.¹⁶ The element of self-consistency resides in the fact that the p_j in turn depend upon $\Sigma(E)$ in a non-trivial way. However, we can start by considering the simpler case in which the eigenmodes are calculated with the first order optical potential including isobar propagation. We then find that there are two important eigenmodes (i.e., those with small imaginary parts) and their trajectories as a function of energy are shown in Figure 4. As in Section II.A, the rather weak damping is caused by a mixing between the pion and Δ -h eigenmodes. Inserting these results into Eq. (10), we obtain the imaginary part of the Δ self-energy shown in Figure 5. The significant features here are the

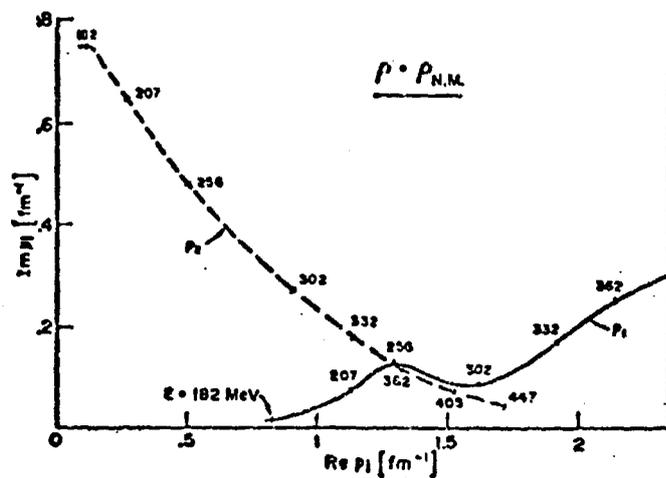


Figure 4. Trajectories of the two most important eigenmodes. Numbers along the trajectories represent the total pion energy.

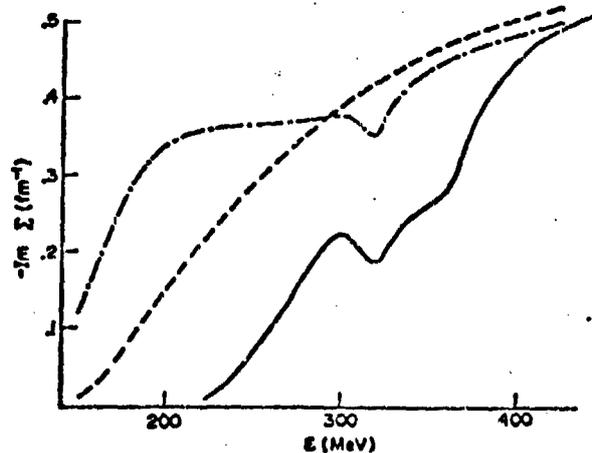


Figure 5. Imaginary part of the Δ self energy as a function of pion total energy. Dashed curve is the free space value, while the dot-dash and solid curves are the values in the medium both without and with the Pauli restriction, respectively. These have been evaluated at nuclear matter density.

important role played by the Pauli principle at the lower energies, the structure introduced into the self-energy through the eigenmode mixing, and the fact that $\text{Im } \Sigma(E)$ is reduced considerably in the medium (corresponding crudely to a narrowing of the resonance). It must be stressed that these results pertain only to nuclear matter density and to isobars at rest and that much more is to be done before any comparison to data would be sensible: nucleon-nucleon correlations, self-consistency, and background terms must be introduced and the integral over allowed hole momentum must be performed in evaluating $\sigma_T(YA)$. Nevertheless, these results point to the importance of a detailed theoretical treatment of Δ self-energy interactions in nuclei.

VI. CONCLUDING REMARKS

We have seen that introduction of the isobar degree of freedom in a many-body approach to the problem of pion-nucleus interactions provides a unified framework within which both kinematical corrections to the standard multiple scattering approach and dynamical modifications of the elementary amplitudes can be discussed. Further, these effects are quantitatively important and more refined calculations of the isobar-nuclear dynamics are called for.

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10. In contrast to the results in Reference 9, the real part of the Δ potential is weakly energy-dependent here.
11. It must be remembered that the eigenvalues are energy-dependent, since the Δ -h interaction is itself energy-dependent (in contrast to the situation in nuclear structure calculations¹).
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