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Relation Between Shock Strength and Strain-Rate Plasticity at Maximum Deviatoric Stress

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Using Wallace's analysis for steady weak shocks, I establish for Cu, Ur, and 6061 T6Al approximate relations between the shock strength and the maximum deviatoric stress, τ_m, and plastic strain at τ_m. In addition, I show that the plastic strain rate is very nearly proportional to the total normal strain rate at τ_m. I use these results and the universal shock strength/strain rate relation of Swegle and Grady to draw conclusions about the general plasticity constitutive relation.

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I. Introduction

Recently, Swegle and Grady[1] have noted the following power law relation between $\dot{\epsilon}_m$, the maximum total normal strain rate, and σ_s, the normal stress at the shock end:

$$\dot{\epsilon}_m \sim \sigma_s^x. \quad (1)$$

This relation seems to hold for a wide variety of materials, for which the stress exponent, x, is about 4. The quantity ϵ, whose time derivative appears above, is the compression or $1 - V/V_0$, where V and V_0 are the current and initial specific volumes, respectively. It would be interesting to discover what meaning this power law has in terms of material properties, which must involve, for the metals at least, strain-rate plasticity. In order to do this however, connections are needed between the quantities of Relation (1), which are "volumetric" in nature, and their corresponding deviatoric quantites, which directly involve plasticity.

The purpose of this paper is to establish such connections for Copper, Uranium, and 6061T6 Aluminum, and to explore the consequences for a general power-law type of plastic strain-rate relation. I use Wallace's analysis for weak shocks to do this.[2] I find that ϵ^p_τ, the plastic strain at the point of maximum effective deviatoric stress, τ_m, and τ_m itself approximately depend on σ_s in the form of power laws. I further find that $\dot{\epsilon}^p_m$, the plastic strain rate at τ_m, is very nearly proportional to $\dot{\epsilon}_\tau$, which is $\dot{\epsilon}$ at τ_m. Using these results, I show that the exponents of my power laws, those of the general plasticity relation, and the exponent x of Swegle and Grady obey a certain relation. This relation should prove useful in high strain rate modeling.

II. Theoretical Treatment

The theoretical foundation of this work rests upon the weak shock theory of D. C. Wallace.[2] This theory allows one to calculate, for a wave with a steady plastic portion, the stresses and strains through the shock rise as a function of σ without knowing about the time dependence of the plastic steady wave at all. Besides thermomechanical parameters, only the shock velocity, D, and something of the precursor behavior need be known.

For the precursor behavior, I use the same model as Wallace,[2] who took the particle velocity for the "follower", the wave portion between the precursor rise and the bottom of the steady plastic wave, to be a linear function of time. This assumption is fairly well obeyed experimentally for 6061T6Al and Cr.[1] Its validity is not known for Cu since only one precursor was measured.[1] This assumption allows the equations of motion to be integrated to obtain Eqns.(14) and (15) of Wallace,[2] which I used here to obtain ϵ as a function of σ. The results do not depend on the slope of the follower, only on the particle velocities at the precursor rise and at the bottom of the plastic wave.

For the steady plastic wave, once the shock strength is specified, the jump conditions and the shock velocity can be used to relate ϵ to σ.

Once ϵ and σ are known for the whole shock, the effective deviatoric stress, τ, and the plastic strain, ϵ^p, can be determined from Eqns.(18) and (19) of Wallace[2] which give σ and τ in terms of the strains ϵ and ϵ^p, second and third order polycrystaline elastic constants, and the Gruneisen parameter. See Wallace[2] for the definitions of the effective deviatoric stress and strain.

At this point, one knows the stresses and strains throughout the shock path, but nothing about the time dependence. One can, however, calculate the ratio $\dot{\tau}/\dot{\epsilon}$, since the time differentials cancel to yield $d\tau/d\epsilon$. This quantity can be obtained from Eq.(18) of Wallace[2] by differentiating both sides and replacing $d\sigma$ with $\rho_0 D^2 d\epsilon$.

There remains only the task of finding the point of maximum τ. I found τ_m numerically, since the approximations necessary in analytical methods were not precise enough.

In order to compare my results with the experimental relation (1), which involves $\dot{\epsilon}_m$, I must assume that the max-τ point coincides with $\dot{\epsilon}_m$. This need not be so but seems plausible. In the case of 6061T6Al, the work of Wallace[4,5] shows for both 21kbar shocks and the one "clean" 37kbar shock that the two points exactly coincide. The other two 37kbar shocks have irregularities whose resolution suggests that the two points should coincide. At any rate, it seems that the two points should be close together.

It is of interest to compare our point of maximum τ with Swegle and Grady's point of "maximum viscous stress".[1] In their approximation, this point occurs a fraction of $f = .5$ along the steady wave, where f is the ratio between $\epsilon - \epsilon_p$ at this point and $\epsilon_s - \epsilon_p$. Here, ϵ_s is at the shock end and ϵ_p is at the base of the plastic steady wave. In my calculations, my point of τ_m occured at $f = .51, .48$, and $.63$ for 6061T6Al, Cu, and Cr, respectively. These values are averages for all but the first shock calculated. Hence, for the materials and shock strengths I calculated, my point of τ_m and the point of "maximum viscous stress" of Swegle and Grady closely coincide.

A practical word about third order adiabatic polycrystaline elastic constants is in order. A good set is available for 6061T6Al[5] but not for the other two materials I calculated. Those available for Cu, for example Kanemochi,[6] do not give the correct pressure derivatives of the shear and bulk modulii. The pressure derivatives[7-9] can be more reliably measured. Fortunatly, if one writes the third order constants ν and ξ in terms of these pressure derivatives and the third third order constant ζ: one finds that the weak shock analysis is sensitive mostly to the pressure derivitives.[10] I did this for Cu and Cr and then varied ζ over a wide range to insure that its actual value mattered little. This proved to be the case, as will be described in more detail in the next section. The equations used[11] were the following:

$$\nu = 1.5(-3B'B - 6\zeta)$$
$$\xi = (-3BG' - 3B - \mu + .5\nu)/3.$$

where B and G are the bulk and shear modulii, respectively, and the primed quantities are the corresponding adiabatic pressure derivatives.

The experimental pressure derivatives[7-9] are at constant temperature. To convert them to adiabatic derivatives, the following formula was used:

$$\left.\frac{\partial B}{\partial P}\right)_S = \left.\frac{\partial B}{\partial P}\right)_T + \left.\frac{\partial B}{\partial T}\right)_P \frac{T\beta}{\rho_0 C_P}.$$

where S is the entropy, T is the temperature, β is the volume coefficient of thermal expansion at constant P, ρ_0 is the initial density, and C_P is the heat capacity at constant pressure. The analogous formula was used for the shear modulus. Values of the above parameters were obatined, in part, from Simmons and Wang[12] and a thermophysical handbook.[13]

III. Results

In Figures 1 and 2. I show results for Cu and Ur. respectively. Those for 6061T6Al are similar. Notice from the upper panels that $\dot{\epsilon}/\epsilon$ rapidly drops to a constant for each shock. When, for a given family of shocks, one fits the τ_s point to a power law in σ_s, one finds that the exponent is about .025 for Cu and 6061T6Al and less than .05 for Ur. Hence, for all practical purposes, $\dot{\epsilon}_m$ is proportional to $\dot{\epsilon}_s$ for these materials and shock strengths.

The shock of lowest strength was not used in these or any of my fits, since it often visibly departed from the behavior of the rest of the shocks. This was due to the "follower" strength being too close to that of the plastic steady wave itself. The behavior reported here is due to the steady plastic portion of the shock, which dominates if the shock is not too weak.

The second panels of the figures show log-log plots of $\dot{\epsilon}_m$ versis σ_s, together with a least squares fit to the power law, σ_s^p. It is seen that the power law fits fairly well, the "error of fit" of the exponent p being roughly 1% for all three materials. The values found for p were .98, .97, and .78 for Cu, 6061T6Al, and Ur respectively.

The third panels of the figures show log-log plots of τ_s versis σ_s, with the corresponding least squares fits to the power law, σ_s^t. Again, the fits are good with the fitting error being about 3%. The values obtained for the τ-exponent t are 1.23, .58, and .78 for Cu, 6061T6Al, and Ur, respectively.

It should be emphasized that these fits are meaningful only for the weak shock regieme of stress and strain. The ranges of shock strength used here correspond closely with those of the data used by Swegle and Grady in deriving Eq.(1). Consequently, my fits can be meaningfully compared with their relation, but care should be exercised in using them outside the regieme of the data.

To asses the uncertainty in the Cu and Ur calculations due to not knowing ζ, I varied ζ from -5500kbar, the value used for the results above, to +5500kbar. The value of p for both materials changed by 2% or less and the value for t changed by less than .5%. Hence, the results are, indeed, sensitive only to B' and G' and not to ζ. For the two materials, I also reduced the value of v_c, the particle velocity at the beginning of the steady shock, by 20% to see how sensitive the results are to the follower and the assumptions made about it. The changes in p and t were about 5%, except for the Cu t-value which changed by about 11%. Significantly, the Cu shocks calculated were the weakest compared to the follower. These v_c uncertainties are not significant.

I used the v_c-values .0108 and .05 km/sec for Cu and Ur, respectively. For Cu and Ur, I used 0., the value observed experimentally,[3,4] for the velocity at the precursor rise. For the shock velocity, I used the relation $D = c + sv$, where v is the particle velocity. For Cu[11] and Ur[13] respectively, I used the values 3.917 and 2.51 for c and the values 1.52 and 1.51 for s. For Cu and Ur, respectively, I used 8.94 and 18.9 for the initial densities and 1.99 and 1.56 for the Gruneisen parameters.[1] For 6061T6Al, I used the same parameter values as Wallace.[2]

I will now outline the consequences of my power law fits and the Swegle-Grady Eq.(1) for a general power-law strain-rate constitutive relation. These consequences constitute my interpretation of the Swegle-Grady relation in terms of plasticity. I assume a relation for the plastic strain rate of the form: $\dot{\epsilon} \sim \epsilon_c^{-1} \tau^{k}$, which is assumed to hold at the τ_m point of the shock.

and, presumably, for other states if they are not too far "away". I can use this form and my results above to obtain a relation to compare with Eq.(1) of Swegle and Grady in the following way. First, apply the form above to the τ_m shock points, to obtain $\dot{\epsilon}_m \sim \epsilon_m^a \tau_m^b$. Then, replace $\dot{\epsilon}_m$ with $\dot{\epsilon}_m$, ϵ_m with σ_e^p, and replace τ_m with σ_e^t, as justified above. The result is $\dot{\epsilon}_m \sim \sigma_e^{pa+tb}$. This result is analogous to Eq.(1). Comparing the two yields

$$x = pa + tb, \tag{2}$$

with x being roughly 4. This relation is the condition imposed on the general strain-rate power law constitutive relation by the Swegle-Grady relation and my results in this paper.

It is of interest to check out relation (2) above for the case of 6061T6Al, for which a shock-wave power law strain rate relation is available.[18] For this relation, $a = 3$ and $b = 4$ so that $pa + tb = 5.2$. By my own calculation, the exponent x of the Swegle-Grady relation lies in the range 4.5 to 5.2 for this material. I used the 21kbar shocks and the one clean 37kbar shock to obtain these numbers. These values of x agree fairly well with the 5.2 obtained via Eq.(2) and are not inconsistent with the rough value of 4 quoted by Swegle and Grady.

IV. Conclusions

For the realm of plasticity encountered in weak shocks, the results of this paper, together with the Swegle-Grady strain-rate relation, appear to support a power law relation for the plastic strain rate as a function of plastic strain and effective deviatoric stress in which the exponents obey the relation (2) of this paper.

References

[1] J.W. Swegle and D.E. Grady, J. Appl. Phys. **58**, 692 (1985).

[2] D.C. Wallace, Phys. Rev. **22**, 1487 (1980).

[3] D.E. Grady, ***Metallurgical Applications of Shock-Wave and High-Strain-Rate Phenomena***, edited by L.E. Murr, K.P. Staudhammer, and M.A. Meyers (Dekker, New York, 1986).

[4] D.E. Grady, private communication.

[5] D.C. Wallace, private communication.

[6] S. Kanemochi, M. Akaboshi, and K. Mizuno, J. Soc. Mater. Sci. Japan **27**, 974 (1978).

[7] N. J. Trappeniers, S. N. Biswas, and C. A. ten Seldam, Physica **85B**, 20 (1977).

[8] P. van't Klooster, N. J. Trappeniers, and S. N. Biswas, Physica **97B**, 65 (1979).

[9] A. E. Abey and B. P. Bonner, J. Appl. Phys. **46**, 1427 (1975).

[10] D.L. Tonks, 1986, unpublished.

[11] M.W. Guinan and D.J. Steinberg, J. Phys. Chem. Solids **35**, 1501 (1974).

[12] G. Simmons and H. Wang, ***Single Crystal Elastic Constants and Calculated Aggregate Properties: A HANDBOOK***(M.I.T., Cambridge, 1971).

[13] ***Thermophysical Properties of Matter***, edited by Y.S. Touloukian and C.Y. Ho (Plenum, New York, 1975).

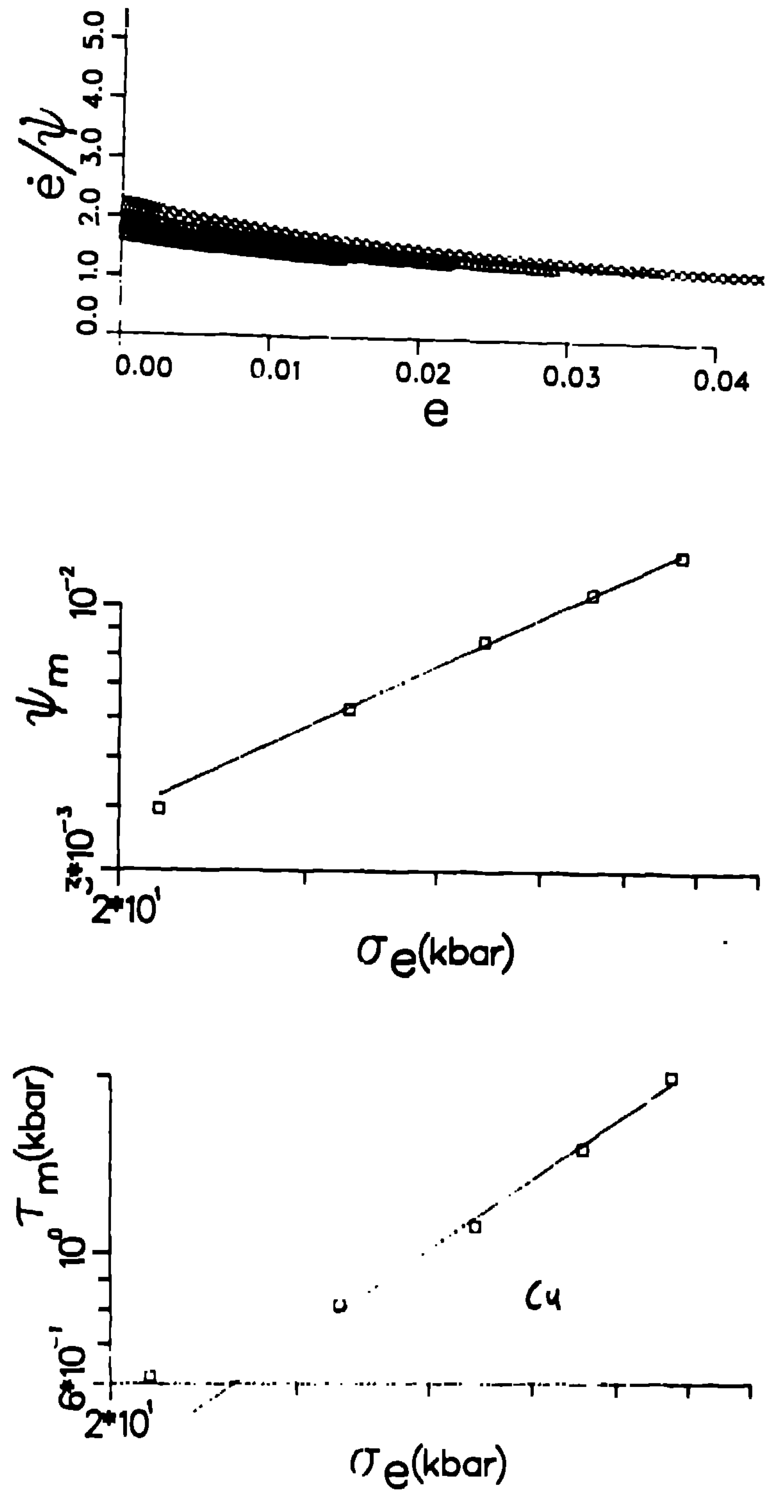
[14]D.E. Munson and L.M. Barker, J. Appl. Phys. **37**, 1652 (1966).

[15]*LASL Shock Hugoniot Data*, edited by S.P. Marsh (University of California, Berkley, 1980), p.148.

[16]D.L. Tonks, *Shock Waves in Condensed Matter*, edited by Y.M. Gupta (Plenum, New York, 1986).

Figure captions

FIG. 1. Copper results. The upper panel shows the ratio of $\dot{\epsilon}$, the normal total strain rate, to $\dot{\epsilon}^p$, the plastic strain rate, as a function of normal strain for five shocks. The middle panel shows, for the same five shocks, ϵ^p_m, the plastic strain at the point of maximum deviatoric stress, versis σ_r, the normal stress at the shock end. The squares are the calculation; the line is a least squares fit. The lower panel shows the same except that τ_m, the maximum deviatoric stress, is plotted instead of ϵ^p_m.
FIG. 2. Uranium results displayed in the same way as FIG. 1.



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2*10¹
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τm(kbar)
10⁰
6*10⁻¹
Cu
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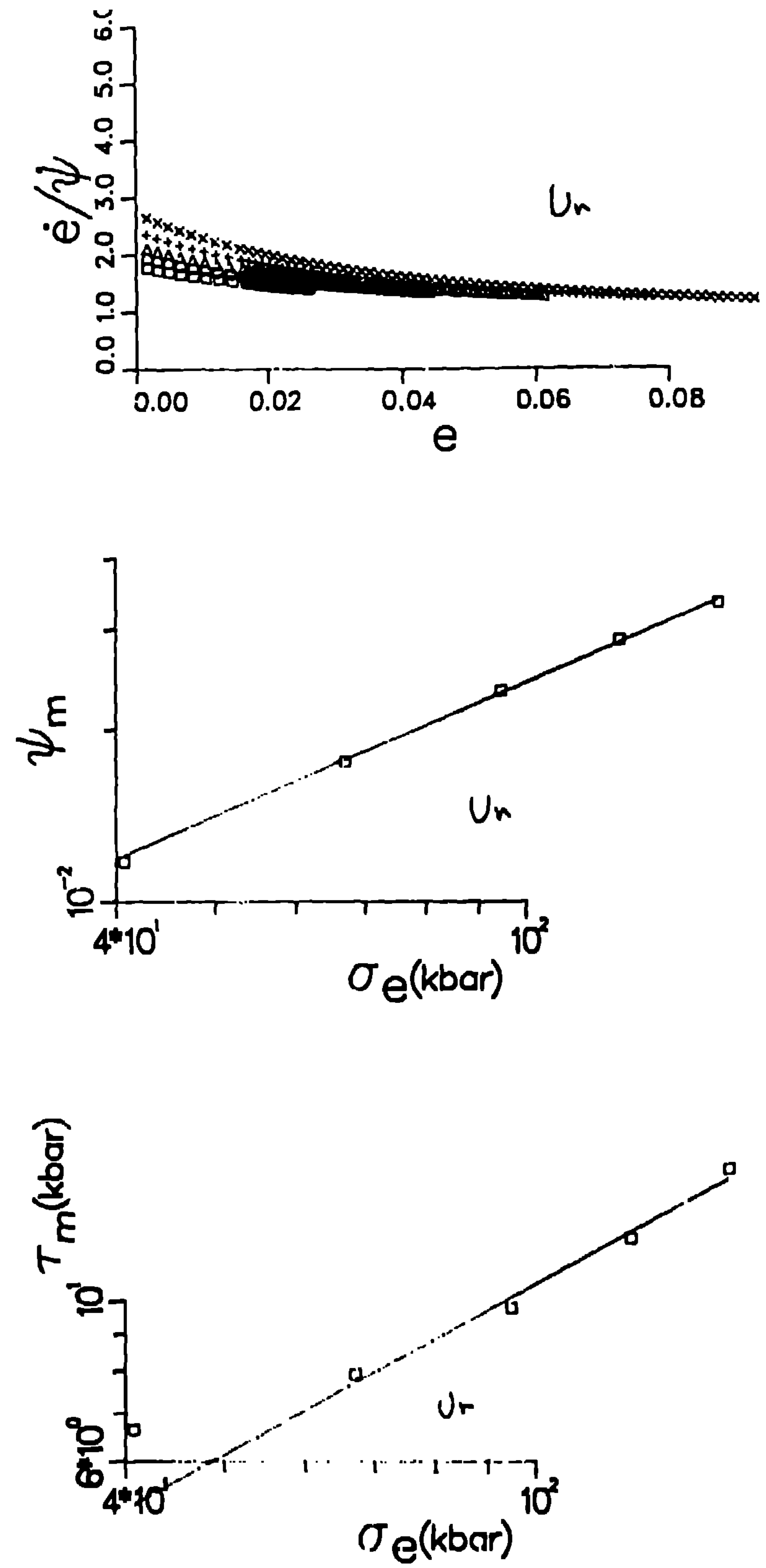
Figure 1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4*10^1
10^2
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Figure 2