A major purpose of the Technical Information Center is to provide the broadest dissemination possible of information contained in DOE's Research and Development Reports to business, industry, the academic community, and federal, state and local governments.

Although a small portion of this report is not reproducible, it is being made available to expedite the availability of information on the research discussed herein.

1

LA-UR -84-2072

CONF-8405193--14

Los Alamos National Laboratory is operated by the University of California for the United States Department of Energy under contract W-7405-ENG-36

LA-UR--84-2072

DE84 014009

TITLE: THE EMC EFFECT: ASYMPTOTIC FREEDOM WITH NUCLEAR TARGETS

AUTHOR(S): Geoffrey B. West

Invited talk at the Conference on the Intersections Between Particle and Nuclear Physics, Steamboat Springs, Colorado, May 23-30, 1984

MASTER

Los Alamos — Los Alamos National Laboratory, Los Alamos, New Mexico 87545

FORM NO 836 R4
ST. NO 2629 5/81

THE EMC EFFECT: ASYMPTOTIC FREEDOM WITH NUCLEAR TARGETS

Geoffrey B. West
Theoretical Division, Los Alamos National Laboratory
Los Alamos, NM 87545

ABSTRACT

General features of the EMC effect are discussed within the framework of quantum chromodynamics as expressed via the operator product expansion and asymptotic freedom. These techniques are reviewed with emphasis on the target dependence.

INTRODUCTION

The observation that the structure function of a nucleus is not simply A times that of the nucleon even at very large momentum transfers was first reported by the European Muon Collaboration working at CERN[1] and has therefore become known as "the EMC effect." This apparently subtle nuclear effect seen by a very high energy probe is a natural topic for a conference on the "intersections of particle and nuclear physics." I shall discuss it from the high energy physics standpoint and try to emphasize model independent aspects of the analysis. Over ten years ago a general theoretical approach was invented for understanding deep inelastic lepton scattering[2] so it is natural to use it to analyze the EMC effect. Indeed almost everything I have to say in this talk could have been done at that time by an intelligent graduate student. The techniques of the analysis are based upon Wilson's operator product expansion and the renormalization group, neither of which is familiar to most nuclear and medium energy physicists. In view of this I shall spend part of my time reviewing them. I shall explain how they generalize the quark-parton model and incorporate constituent or bag models into the general structure. Before doing so, however, I first want to review definitions and kinematics in order to present the experimental data. I shall assume some familiarity with the classic SLAC-MIT experiments and their interpretation via the "naive" quark-parton model.[3] The main thrust of this talk will be to emphasize those aspects that follow from the general field theoretical framework incorporating quantum chromodynamics (QCD).

KINEMATICS AND DEFINITIONS

Consider inelastic lepton scattering from some target where only the scattered lepton is detected (see Fig. 1.). We shall assume Born approximation in the electroweak coupling so, for example, in the electromagnetic case (one photon exchange, see Fig. 2) the cross section can be expressed as a deviation from the Mott value:

$$\frac{d^2\sigma_A}{dE_f d\Omega_f} = \left(\frac{d\sigma}{d\Omega_f}\right)_{Mott} W_A(q^2,\nu) \tag{1}$$

This deviation, known as the structure function, depends on the two independent Lorentz scalars q^2 and $\nu \equiv p\cdot q/M_A$ where M_A is the target mass (so $p^2 = M_A^2$). In the rest frame of the target ($\underline{p} = \underline{0}$), ν is simply the energy lost by the scattered lepton. Because the exchanged gauge boson has spin 1, W is usually decomposed into further pieces representing electric and magnetic transitions. For ease of presentation we shall ignore any such subtlety due to spin until the very end. Formally

$$W_A(q^2,\nu) = \sum_N |\langle N|j|p,A\rangle|^2 (2\pi)^4 \, \delta^{(4)}(p+q-p_N) \tag{2}$$

representing the sums of squares of transition matrix elements due to the current j supplied by the scattered lepton (as in the lower vertex of Fig. 2). One can use completeness of the final states $|N\rangle$ to rewrite this as a current correlation function:

$$W_A(q^2,\nu) = \int d^4x \; e^{iq\cdot x} \langle p,A|j(x)j(o)|p,A\rangle$$

Being an effective total cross section there is an optical theorem relating W_A to the virtual forward Compton scattering amplitude $T_A(q^2,\nu)$ (as in Fig. 3):

$$W_A(q^2,\nu) = \mathrm{Im}\; T_A(q^2,\nu)$$

$$\equiv \mathrm{Im} \int d^4x \; e^{iq\cdot x} \langle p,A|T[j(x)\; j(o)]|p,A\rangle \tag{3}$$

Eq. (3), and in particular the representation of T_A as a time-ordered product of two currents separated by a space-time distance x_μ, will be the point of departure for our theoretical discussion in Part III below.

EXPERIMENTAL OBSERVATIONS

A. Bjorken Scaling

From its definition W has units of $(\text{energy})^{-1}$; it is therefore natural to form the dimensionless combination: $F_A(q^2,X) \equiv \nu W(q^2,\nu)$ where $X \equiv q^2/2M_A\nu$ is a dimensionless variable first introduced by Bjorken. Notice that $0 \lesssim X \lesssim 1$, with $X = 1$ representing the elastic threshold. For $q^2 \gtrsim$ few $(\text{GeV/c})^2$, $F_A(X,q^2)$ turns out to be almost independent of q^2; Fig. 4 shows early data from SLAC and DESY plotted versus $\omega \equiv 1/X$ for a wide range of q^2. The scaling phenomenon is quite apparent.[4] In the standard quark-parton picture[3,5] the large X (small ω) region is associated with the valence

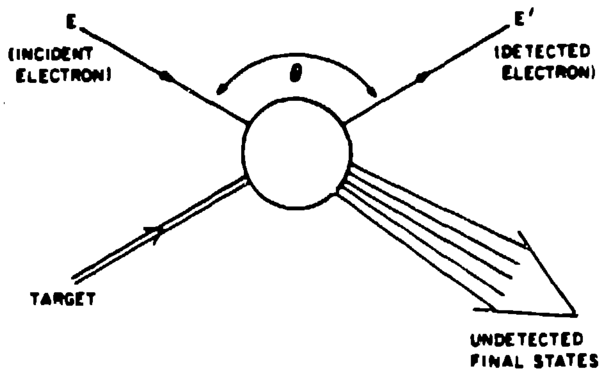


Fig. 1. General graph illustrating inelastic lepton (in this case electron) scattering from an arbitrary target.

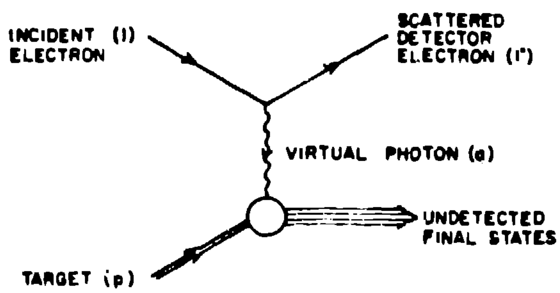


Fig. 2. The one-photon exchange approximation.

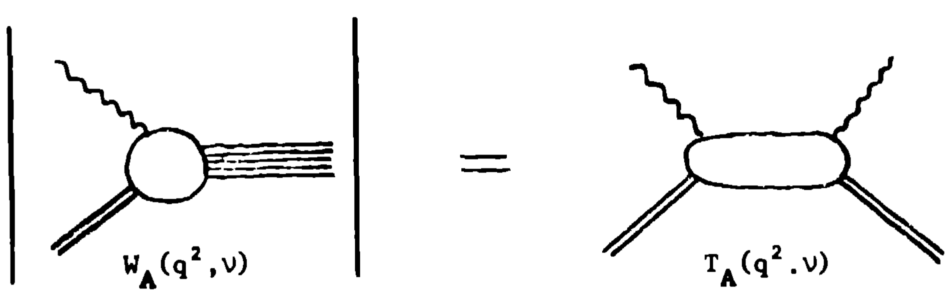


Fig. 3. Symbolic representation of the optical theorem (Eq. 3).

quarks normally identified with the "static" quark content of the target. The small X (or large ω) region, on the other hand, is associated with the sea of virtual $q\bar{q}$ pairs which can loosely be identified with the meson cloud in the "old-fashioned" picture of hadronic structure.

B. Comparison with QCD

The spread in F_A due to the variation in q^2 is consistent with the predictions of QCD. This comparison is usually done in terms of the moments of F_A defined by[2]

$$M_A(q^2,n) \equiv \int_0^1 dX\, X^{n-2}\, F_A(q^2,X) \tag{4}$$

The reason for this, which I will return to below, is that according to the operator product expansion (OPE) these factorize at large q^2 into a target independent piece $c(q^2,n)$ and q^2-independent target matrix elements $T_A(n)$ which will be defined below:

$$M_A(q^2,n) \simeq c(q^2,n)\; T_A(n) \;. \tag{5}$$

The $c(q^2,n)$ are properties of the theory which, in QCD, can be reliably calculated from perturbation theory by virtue of its asymptotically free character. This predicts $c(q^2,n) \sim (\ell n\, q^2)^{-\gamma_n}$ where the γ_n are known. As can be seen from Fig. ? the data are consistent with the predicted mild logarithmic deviation from the naive scaling of the parton model.[6,7] Fig. 5 represents some of the earlier plots of the moments and shows the dramatic approach to scaling for $q^2 \lesssim 3(\text{GeV}/c)^2$; I shall return to this "forgotten" aspect of the data at the end of my talk.

C. The EMC Effect

We are now ready to present data on the EMC effect. The original observation[1] was that for a steel target (A=56) $F_A(q^2,X) \neq AF_N(x,q^2)$; F_N is one-half the deuteron structure function which we shall identify with the average of that of the proton and neutron. Notice incidentally that for the nucleon $x = q^2/2M\nu$, the "usual" x; obviously $x = (M_A/M)\, X \simeq AX$. The data has mostly been presented as a ratio

$$R_A(q^2,x) \equiv \frac{F_A(x,q^2)}{F_N(x,q^2)} \tag{6}$$

which is written σ_A/σ_D in Fig. 6. This figure shows a compilation of data taken at SLAC (as well as the original EMC points) for a variety of targets.[8] Although the characteristic shape is clear, it is worth emphasizing that the original EMC data on iron sticks

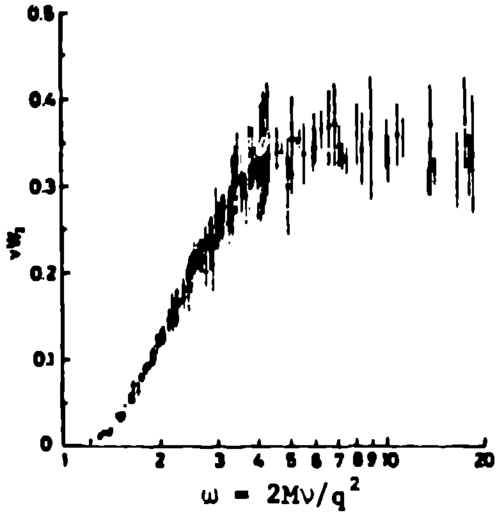


Fig. 4. Early SLAC and DESY data showing the scaling of νW vs. $\omega \equiv 1/x$; see Ref. 4.

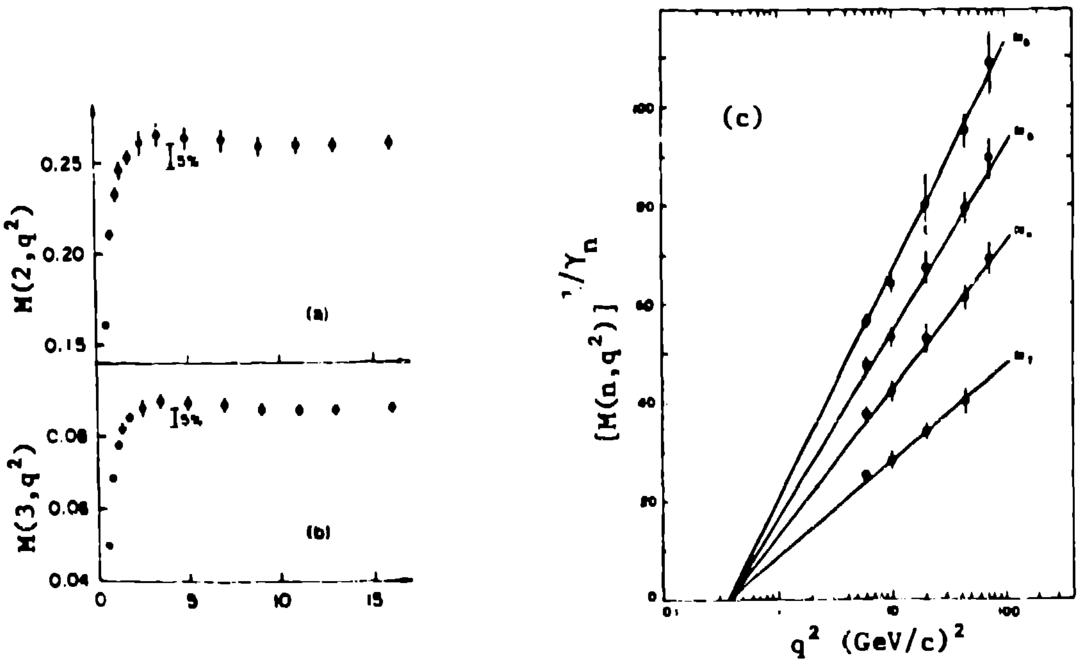


Fig. 5. (a) Early SLAC data showing $M(2,q^2)$ vs. q^2 [see Ref. 4]; (b) similarly for $M(3,q^2)$; (c) $[M(n,q^2)]^{1/\gamma_n}$ vs. $\ln q^2$ showing straight line dependence [Refs. 5,7].

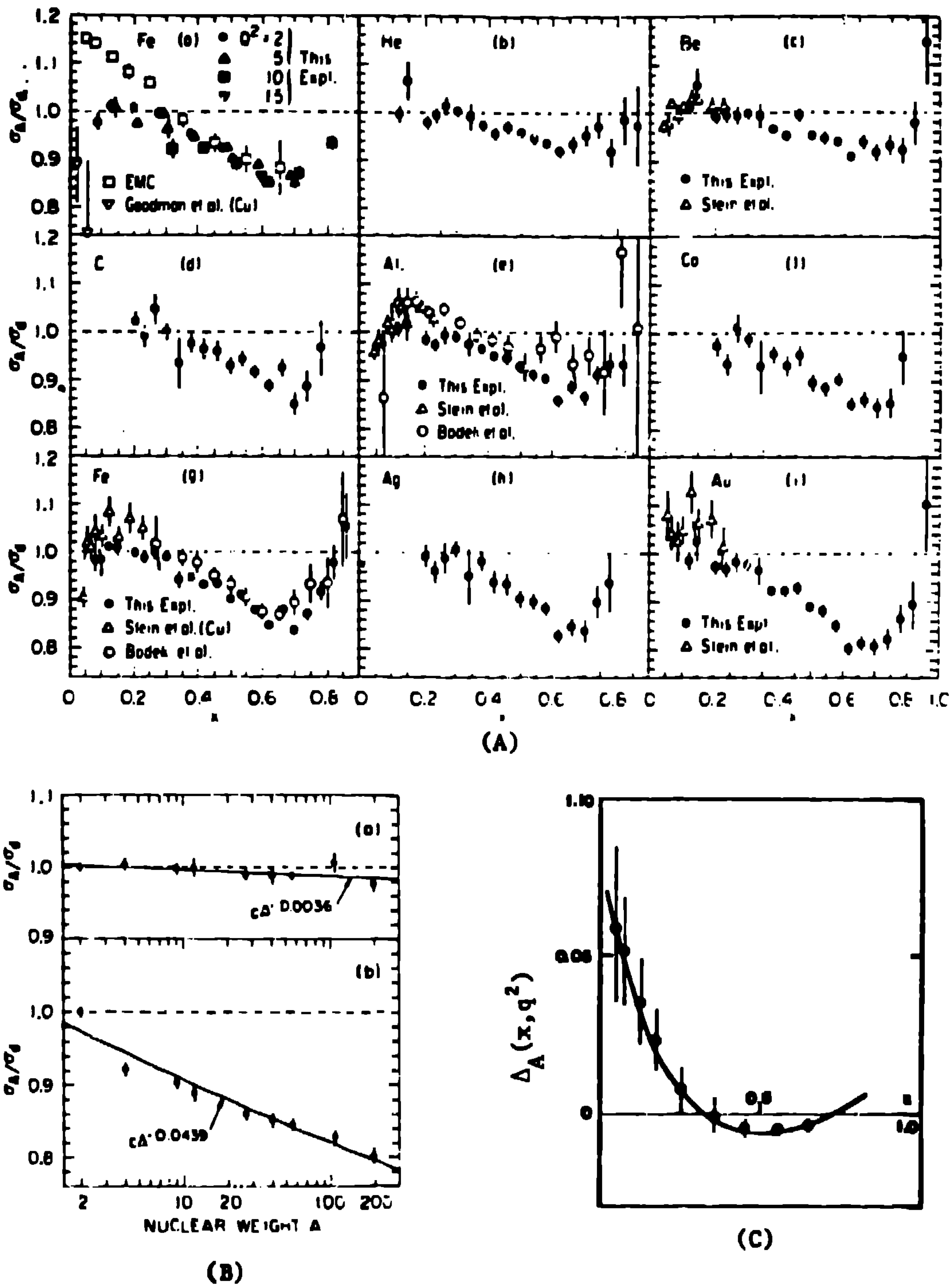


Fig. 6. (A) $R_A(x,q^2)$ vs. x (averaged over q^2) for various targets; (B) $R_A(x,q^2)$ vs. ℓn A for (a) x = 0.3 and (b) x = 0.62; (C) $\Delta_A(x,q^2)$ vs. x (EMC data)--the fit is a quadratic as in Eq. (37).

out like a sore thumb at small x. The sharp rise in R_A as $x \to 1$ is simply due to the fact that one is dividing by $F_N^A(x,q^2)$ which vanishes in this limit. Because of this (and for reasons to be explained below) it is more natural and convenient to represent the data as was originally done by Jaffe,[9] as the difference

$$\Delta_A(q^2,x) \equiv 1/A\ F_A(x,q^2) - F_N(x,q^2) \qquad (7)$$

Also shown in Fig. 6 is the variation of R_A with A at fixed x; the data is consistent with a logarithmic dependence whose shape is strongly x-dependent. Again a large part of this x-dependence is due to using R_A rather than Δ_A. Even though a logarithm obviously gives a good fit, the experimentalists have chosen the form $cA^{\alpha(x)}$ shown in this figure.

THEORETICAL TRUTHS

I now want to give a brief review of our formal theoretical understanding of the structure functions and their relationship to the quark-parton model. This is hardly meant to be complete but rather to give a flavor of the basic ideas that lead up to the use of the moments a the appropriate objects to study.

A. The Light Cone (or Short Distance) Expansion

Recall that the optical theorem allows us to think of the F_A as absorptive parts of the corresponding forward virtual Compton amplitude $T_A(q^2,\nu)$ defined in Eq. (3);

$$T_A(q^2,\nu) = \int d^4x\ e^{iq\cdot x} \langle p,A|T[j(x)\ j(0)|p,A\rangle \qquad (8)$$

We are interested in the limit $q^2 \to \infty$. Without going into details, it is clear that, as in all Fourier integrals, this is sensitive to $x_\mu \simeq 0$, i.e., to the behavior of the current product when the space-time separation becomes very small. Wilson has specified this in a form that is basically the operator generalization of a Taylor series expansion for an ordinary function.[10] For the matrix element needed here this reads

$$\langle p,A|T[j(x)\ j(0)]|p,A\rangle \overset{x\to 0}{\simeq} \sum_n \langle p|O_n|p\rangle (ip\cdot\partial)^n\ C_n(x^2) \qquad (9)$$

where the O_n are local operators bilinear in the quark and gluon fields: e.g., $\bar\psi\lambda_a \not{D} \ldots \not{D}\psi$ or $F_{\mu\alpha} D^\alpha \ldots D^\beta F_{\beta\mu}$, etc. [The D's are the usual non-Abelian covariant derivatives: $D_\mu = \partial_\mu - ig\underline{\tau} \cdot \underline{A}_\mu$]. Using this in Eq. (8) gives

$$T_A(q^2,\nu) \overset{q^2\to\infty}{\simeq} \sum_n c(q^2,n)\ \langle p|O_n|p\rangle\ X^{-n} \qquad (10)$$

where the $c(q^2,n)$ are coefficients defined by

$$c(n,q^2) \equiv (q^2)^n \int d^4x\, e^{iq\cdot x} C_n(x^2) \quad . \tag{11}$$

The factor $(q^2)^n$ is included in order to make the $c(n,q^2)$ dimensionless. The moments arise in projecting out the absorptive part of this equation in order to obtain information on the structure functions. Formally this is accomplished using a Mellin transform to give

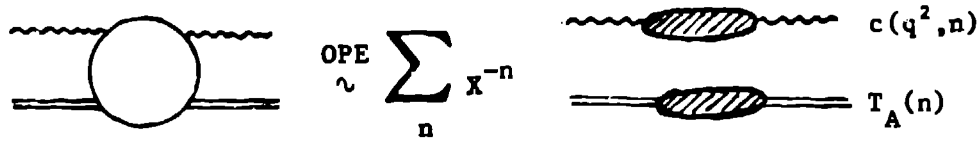
$$M_A(q^2,n) \simeq c(q^2,n)\; T_A(n) \tag{5}$$

where

$$T_A(n) \equiv \langle p|O_n|p\rangle \quad . \tag{12}$$

B. The Renormalization Group and Asymptotic Freedom

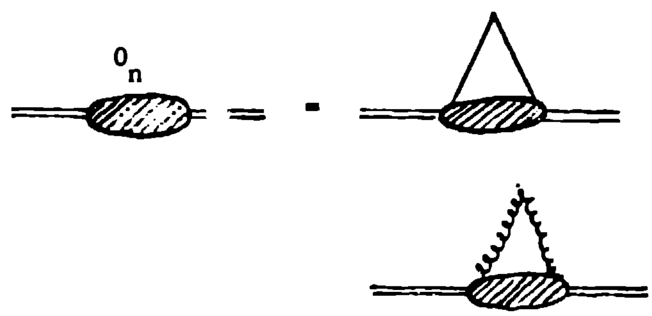
Equation (5) represents a general statement that the underlying dynamics is described by a field theory. Up to now we have not needed to specify what the field theory actually is. This enters via the dimensionless coefficients $c(q^2,n)$ which satisfy certain scaling constraints due to the renormalizability of the theory. Just as in ordinary dimensional analysis where the invariance of the theory to the choice of units requires variables that appear independent to be inextricably linked, so renormalizability requires momenta and coupling constants to be intimately connected. Renormalization forces the introduction of some arbitrary mass scale (μ, say) in order to define the physical renormalized coupling constant. The renormalization group simply expresses the invariance of the physics to this choice of scale; it can therefore be thought of as the generalization of ordinary dimensional analysis to field theory. Roughly speaking, one typically finds that a dimensionless quantity like $c(q^2,n)$ must be expressible in a form involving the combination of $(q^2/\mu^2)\exp(1/2\, bg^2 + \ldots\ldots)$ where b is calculable. Thus if $b > 0$ the UV behavior (at fixed coupling) is equivalent to taking $g^2 \to 0$ at fixed q^2, whereas if $b < 0$ it is the IR that is equivalent to this limit. It turns out that in QED $b < 0$ thereby justifying the use of perturbation theory ($g^2 \to 0$) for calculating the low energy regime. On the other hand in QCD, $b > 0$ so that perturbation theory can be effectively used to calculate the large q^2 behavior. This is basically the statement of asymptotic freedom. At sufficiently large q^2 the coupling is effectively small so that the target behaves as if it were composed of quasi-free quarks and gluons. Even though the effective coupling inside the target may be large, the renormalization group allows us to transform the scattering situation to an equivalent small coupling problem if we probe it with a large q^2 current. As already remarked the above argument leads to the conclusion[2] that in QCD $c(q^2,n) \sim (\log q^2)^{-\gamma_n}$ for large q^2 with the γ_n calculable from perturbation theory. Symbolically the results of this analysis can be summarized diagramatically as follows:



(13)

In QCD (for large q^2)

$c(q^2,n) \equiv$  QUARK LOOP (+ RADIATIVE GLUON CORRECTIONS) (14)

and $T_A(n) \equiv$ 

QUARK "CONSTITUENT"

GLUON "CONSTITUENT"

(15)

C. Equivalence to the Quark-Parton Model

The above analysis strongly suggests taking the quasi-free scattering picture seriously; this can be made even more suggestive by introducing the inverse Mellin transforms of $c(q^2,n)$ and $T_A(n)$:

$$c(q^2,n) \equiv \int_0^1 dX\, X^{n-2}\, F_q(q^2,X) \tag{16}$$

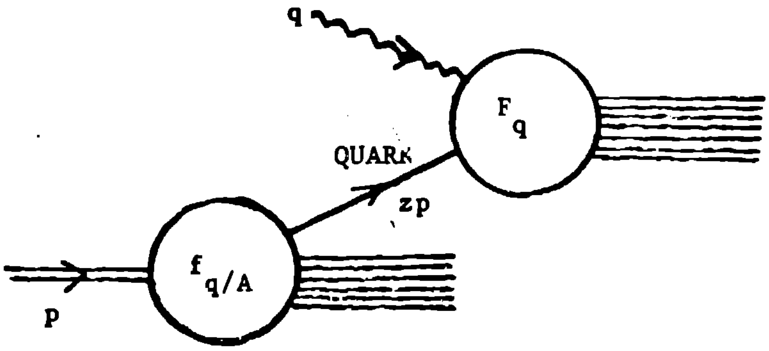
and

$$T_A(n) \equiv \int_0^1 dz\, z^{n-1}\, f_{q/A}(z) \tag{17}$$

[That the integrals cut off at 1 can be justified from the original definitions.][11] Substituting these representations into the moment equation (5) gives, upon inversion

$$F_A(q^2,X) = \int_0^1 dz\, f_{q/A}(z)\, F_q(q^2,\tfrac{X}{z}) \tag{18}$$

This equation has a natural interpretation in terms of scattering from quarks: $f_{q/A}(z)$ is the probability that a quark carries a fraction z of the total momentum (p) and $F_q(q^2,x)$ is its stucture function:



$c(q^2,n)$

$T_A(n)$

[Note that at the top vertex $X' \equiv q^2/2zp\cdot q = X/z$ as it should.] From Eq. (16) we can interpret $c(q^2,n)$ as the moments of the quark structure functions. The situation is actually slightly more complicated than this because there are gluon components to the $f(z)$ as indicated in Eq. (15); nevertheless the general structure and interpretation stay essentially intact.

In the original naive parton model[3,5] the scattering from the quarks was further assumed to be quasi-elastic so that $F_q(q^2,X) = Q^2\delta(1-X)$ where Q is the quark charge leading to $F_A(q^2,X) = Q^2 f_{q/A}(X)$ (summed over quark types). This leads to the identification of F_A with quark (and gluon) distribution functions, an interpretation which has remained, even though the more natural nomenclature is to use the $f_{q/A}(z)$ as the distribution functions. The f's have the advantage that they are properties of the target like a wave function and are not q^2-dependent.

D. Sum Rules

Certain combinations of the O_n represent conserved quantities such as the baryon number, charge, etc., which are invariant to renormalization. They are therefore "dimensionless" as far as the renormalisation group is concerned and so the corresponding γ_n vanish leading to sum rules.[2,3] If we remain within purely electromagnetic scattering then there is only one conserved O_n namely the energy-momentum tensor whose corresponding γ_2 vanishes. For a purely singlet combination (such as F_N), one thereby obtains the sum rule

$$M_A(q^2,2) \equiv \int_0^1 dx\; F_A(x,q^2) = \frac{\langle Q^2\rangle}{1 + 16/3N_f} \tag{19}$$

Here N_f is the number of flavors and $\langle Q^2\rangle$ the average charge squared of the quarks in the theory. For four flavors this predicts $M_A(q^2,2) = 5/42 \simeq 0.119$ whereas for six $5/34 \simeq 0.147$. The data on deuterium gives $\simeq$ 0.15 in remarkably good agreement with six quarks.[7] Further sum rules can be derived from other conserved quantities; however, they lead to sum rules that relate the EMC to the weak structure functions[2] an example of which is discussed briefly below.

Several points should be emphasized about this sum rule:
a) the right-hand side is independent of q^2 as would be the case in the naive parton model;
b) its value is independent of the target;
c) for the non-singlet combination (such as the difference between proton and neutron), all $\gamma_n > 0$, so all corresponding moments eventually vanish;
d) the factor $(1+16/3n_f)^{-1}$ in (19) simply represents the fraction of energy-momentum carried by the quark degrees of freedom, the rest being carried by the (electrically neutral) gluons: thus $N_q/(N_q + N_G) = 3n_f/(3n_f + 2x8) = (1 + 16/3n_f)^{-1}$ assuming an SU(3) color symmetry.

E. Summary

i) The pattern of scale breaking is determined by the target independent $c(q^2,n)$ which in QCD behave like $(\ell n\ q^2)^{-\gamma_n}$. [Thus any target can be used to "test" QCD.] They can be thought of as the moments of the quark structure functions.

ii) The shape of $F_A(x,q^2)$ reflects the quark/gluon distribution inside the target and is determined by the matrix elements $T_A(n) \equiv \langle p|O_n|p\rangle$.

APPLICATION TO NUCLEI

Thus far we have reviewed the standard framework for understanding the structure function data within the context of QCD. Its crucial property of asymptotic freedom allows a quark-parton quasi-free scattering interpretation to be made. We now turn our attention to the EMC effect and focus on the target dependence of the analysis. Within the conventional quark-parton model, the data says that in a nucleus the sea is enhanced over its contribution in the nucleon. The energy momentum sum rule, Eq. (19), requires that such an enhancement at small x be compensated for at large x so there must be a depletion at larger x. Many models have been invoked to explain this phenomena,[12] most replacing the quark-gluon degrees of freedom by effective ones such as pions, nucleons, heavy baryons such as six-quark configurations, and so on. These are presumably not unreasonable especially since there is good evidence that nuclei can be well described by nucleons and mesons. There are, of course, inevitable problems of how to match such a description with the fundamental ideas of QCD and, in particular, how to correctly describe the scattering process. An alternative view is to stay within a quark-gluon picture and use the fact that a quark can travel further in a nucleus than in a nucleon. Below I shall show that this picture emerges naturally from the OPE QCD analysis applied to the nucleus. However, rather than discussing particular models, I first want to concentrate on what features of the data can be considered as <u>model independent</u>.[13] All of the analysis that I am going to present could have been carried out over ten years ago by the proverbial "intelligent graduate student." Had he been asked by his wise professor to look at the target dependence of the OPE analysis, I believe he would have predicted an EMC effect. Whether he would have become famous or whether the EMC effect would have remained an amusing curiosity is left to the reader to decide.

A. Model Independent Features

i) Basic Formula Relating $F_A(q^2,x)$ to $F_N(q^2,x)$

We have seen that the OPE leads to the moment equations

$$M_A(q^2,n) = c(q^2,n)\ T_A(n) \tag{5}$$

These are valid for any A and, in particular, they are valid for the nucleon:

$$M_N(q^2,n) = c(q^2,n)\ T_N(n) \tag{20}$$

In the usual analysis[2] one eliminates the target dependent matrix elements by writing the equation at another value of q^2 (q_0^2 say): $M_A(q_0^2,n) = c(q_0^2,n)\ T_A(n)$ thereby deriving an "evolution equation" for a particular target relating its behavior at one value of q^2 to another:[14]

$$M_A(q^2,n) = \frac{c(q^2,n)}{c(q_0^2,n)}\ M_A(q_0^2,n)$$

Here, however, we want to relate one target to another;[15] this can readily be accomplished by dividing (5) by (20) to obtain

$$M_A(q^2,n) = t_{N/A}(n)\ M_N(q^2,n) \tag{21}$$

where $t_{N/A}(n) \equiv T_A(n)/T_N(n)$. As before when dealing with the quarks it is natural to introduce the inverse Mellin transform of $t_{N/A}$

$$t_{N/A}(n) = \int_0^1 dz\ z^{n-1}\ f_{N/A}(z) \tag{22}$$

in order to invert (21). With this definition one straightforwardly obtains

$$F_A(q^2,X) = \int_0^1 dz\ f_{N/A}(z)\ F_N(q^2,\frac{X}{z}) \tag{23}$$

This is a remarkable equation because its structure is identical to Eq. (18) and it is tempting to interpret it in a similar fashion, namely, that the scattering can be represented as if it were quasi-free from extended physical nucleons with a momentum distribution given by the q^2-independent functions $f_{N/A}(z)$. Obviously some care must be taken in making this a strict interpretation and I shall return to this question later. At the moment I simply want to call attention to the fact that F_A can be represented either this way or by Eq. (18) which is based upon the fundamental degrees of freedom; this suggests that there are probably several equivalent model ways of interpreting the EMC effect.

ii) Digression on the Sum Rule - A Problem

Recall that the second moment (n = 2) is (at least for the singlet piece) target independent, so

$$M_A(q^2,2) = M_N(q^2,2) \tag{24}$$

leading to $t_{N/A}(2) = 1$. This reinforces one's temptation to make a physical interpretation of (23) since it is equivalent to

$$\int_0^1 dz\, z\, f_{N/A}(z) = 1 \tag{25}$$

which represents momentum conservation.

A more important consequence of (24), however, is that

$$\int_0^A dx\, \left[\frac{F_A(q^2,x)}{A} - F_N(q^2,x)\right] = 0 \tag{26}$$

or, using the definition, Eq. (7)

$$\int_0^A dx\quad \Delta(q^2,x) = 0 \tag{27}$$

Since $F_N(q^2,x) = 0$ for $x > 1$, $\Delta > 0$ in this region, so it must have at least one zero in the range $0 \leq x \leq 1$. Furthermore, if it starts out positive (as in the original EMC data) it must have at least two zeros for $0 < x < 1$, as indeed it does.

Now, however, we come to a problem: from what we have just said

$$\int_0^1 dx\, \Delta(q^2,x) < 0 \tag{28}$$

whereas the EMC data taken with an iron target and exhibited in Fig. 6(C) appears to give the opposite sign! Several possibilities come to mind for a way out of this apparent violation.

a) $F_A(q^2,x)$ is anomalously small for $0.7 \lesssim x < 1$.

b) There are large finite q^2 scaling violations due to non-single components.

c) The normalization of the data is incorrect.

The first seems unlikely. The second is certainly possible but, to us, also seems unlikely. Our reasoning is as follows: iron is predominantly an equal number of protons and neutrons and is, therefore, to a good approximation, in a singlet state just as the deuteron is. As remarked earlier, the energy-momentum sum rule for the deuteron is in good agreement with the data provided there are six flavors; there is, of course, the question as to whether all six quarks are "operable" since the presumed top quark may have a very large mass[16] and the q^2's are at most ~100 $(GeV/c)^2$. Taking this at face value, however, suggests that finite q^2 corrections to the sum rule are small. We might, therefore, expect corrections to (27) also to be small. Furthermore, purely non-singlet contributions tend to be small near $x = 0$ and large near $x = 1$, as evidenced from the difference[17] $F_N - F_p$, whereas the effect in Δ is the other way round: namely large near $x = 0$ and small near $x = 1$. The logarithmic corrections to the sum rule can actually be eliminated using neutrino data. One simply replaces[18] $F_A(q^2,x)$ by $[F_A(q,x) - 1/6\, F_A^{\nu}(q^2,x)]$ so if there is a large finite q^2 correction to F_A there must be a similar one in the corresponding neutrino function. Obviously the analogous sum rule for Δ should be examined when more accurate ν data is forthcoming.

Our own feeling is that the violation of the sum rule is due to some systematic normalization problem with the EMC data. This view appears to be supported by the data from SLAC which shows a much smaller effect in the small x region. Indeed a casual survey of the SLAC plots [Fig. 6] indicates that they do not violate the sum rule. It is of course quite possible that if all the data were carefully corrected for finite q^2 and σ_L/σ_T effects, the situation would be quite different. Clearly a careful analysis is called for. One final point worth noting is that since scaling violations are larger at small q^2 where the SLAC data is taken, this ought to show a greater violation of the sum rule than the EMC if this is indeed due to such effects.

iii) Formula for Calculating $f_{q/A}(n)$

We have just seen that in order to calculate the difference Δ, we need the distribution function $f_{N/A}(z)$ whose moments are the $t_{N/A}(n)$ - see Eqs. (22) and (23). The $t_{N/A}(n)$ are defined as the ratio $T_A(n)/T_N(n)$ each $T(n)$ being the target matrix element of the quark and gluon bilinears pictured in Eq. (15). As already mentioned when discussing the quark-parton model [see Eq. (17)], the triangle graph shown there can be expressed in the form

$$T_A(n) = \int_0^1 dz\, z^{n-1} f_{q/A}(z) \qquad (29)$$

In the target rest-frame $z = k_-/M_A$ where k is the quark momentum and $k_- = k_0 - k_3$, the direction 3 being identified with that of the photon. The quark (or gluon) distribution function $f_{q/A}(z)$ is given by[3,11,20]

$$f_{q/A}(z) = z \int_{\omega_0^2}^{\infty} d\omega^2 \int_{k_0^2}^{\infty} dk^2\, D_q^2(k^2)\, m_{q/A}(k^2,\omega) \qquad (30)$$

where $k_0^2 \equiv \frac{z(z+\alpha)M_A^2}{1-z}$. $D_q(k^2)$ is the quark (or gluon) propagator and $m_{q/A}$ the absorptive part of its forward scattering amplitude from the target. The invariant mass of the "spectator" state will be denoted by ω. Also in (30) $\alpha \equiv \omega^2/M^2_A - 1$. The combination $D_q^2(k^2)m_{q/A}(k^2,\omega)$ can be thought of as the square of the relativistic target wave function and will therefore be denoted by $|\psi_{q/A}(k)|^2$.

B. A Little Model Dependence

i) Calculation of $f_{q/A}(z)$

Up to now we have made no explicit use of the fact that the target itself is a bound state of quarks and gluons. Technically this produces a so-called anamolous threshold[21] in ω at a value ω_0 determined by the "binding energy" (B). If we change variables to $\varepsilon \equiv (\omega + m - M_A)/M$, where m is the effective quark (or gluon) mass, then the bound state nature of the target can be imposed by constraining $\varepsilon \ll A$ with a threshold beginning at B/M.[22] This constrains the mass of the virtual spectator states to be reasonably close to M_A; put slight differently, the bound state nature of the "wave function" effectively cuts-off the k-integration. We can impose the bound-state physics on (30) simply by using ε and restricting it to be much less than A; if, at the same time, we rescale z to z/A we obtain

$$f_{q/A}(z) = 2M^2 zA \int_{B/M} d\varepsilon \int_{k_0^2}^{\infty} dk^2\, D_q^2(k^2)\, m_{q/A}(k^2,\varepsilon)\ , \qquad (31)$$

where now $k_0^2 \simeq \frac{M^2 z(z+2\varepsilon-2m/M)}{1-z/A}$ and z runs from 0 to A. We shall not enter here into a discussion of the precise physical meaning of the effective quark mass m nor of its effective binding energy B.[22]

For the present purposes they can simply be thought of as parameters characterizing the system that enter either as threshold values or positions of singularities.

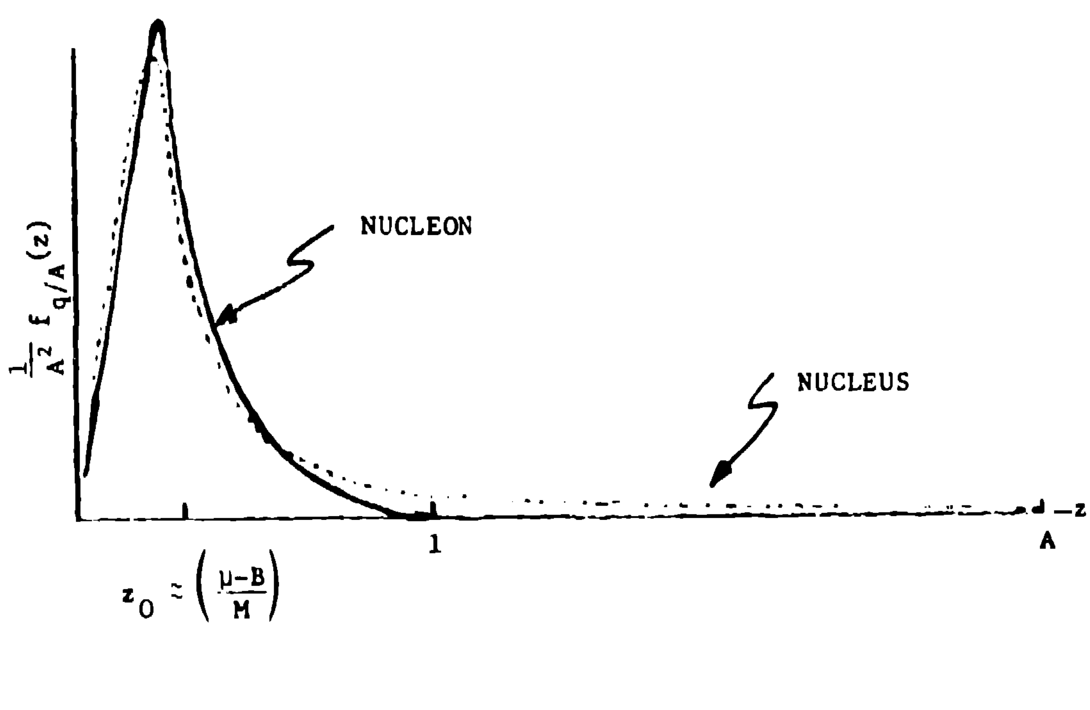
Now suppose all nuclei are essentially alike as far as their single quark and gluon binding energies and wave functions are concerned (except possibly for a normalization factor $\sim A^2$). The extra factor A comes from the assumption that $m_{q/A}(k^2,\varepsilon) \simeq A\, m_{q/N}(k^2,\varepsilon)$. Then the dominant difference in their distribution functions is due to the factor $(1-z/A)^{-1}$ in the lower limit of the k^2-integration. This factor is sensitive to the large k^2 behavior of the wave function and is basically only important when $z \to A$.

If one assumes that the fall-off in k^2 is governed by the quark mass and that both ε and $m \ll M$ then $f_{q/A}(z)$ peaks sharply at $z = z_0 \simeq (m - B)/M$. As a simple illustrative example consider the analogue to the zero range approximation: $m(k^2,\varepsilon) \sim \delta(\varepsilon - B/M)$ and $D_q(k^2) \sim (k^2 - m^2)^{-1}$, then

$$f_{q/A}(z) \simeq \frac{N\beta A^2 z(1 - z/A)}{(z-z_0)^2 + \beta^2} \tag{32}$$

where $\beta^2 = B(2m - B)/M^2$.

The difference in distributions between the nucleon and nucleus can therefore be pictured as follows: (assuming, for definiteness, that $m > B$).

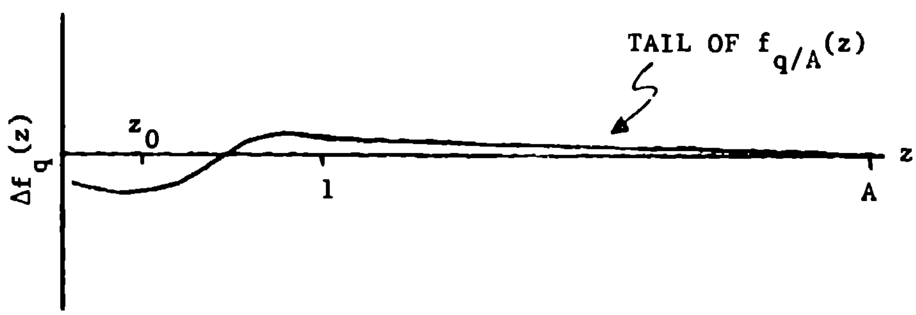


ii) Estimate of Δ

From its definition via Eq. (21), we can express $t_{N/A}(n)$ in the form:

$$t_{N/A}(n) = A^{2-n} + A^{2-n} \frac{\int_0^A dz\, z^{n-1} \Delta f_q(z)}{\int_0^1 dz\, z^{n-1}\, f_{q/N}(z)} \tag{33}$$

where $\Delta f_q(z) \equiv A^{-2}\, f_{q/A}(z) - f_{q/N}(z)$ is the difference in quark distributions in the nucleus over that in the nucleon. This is a convenient form because Δ receives all its contribution from the second term which, as expected, is dependent on $\Delta f_q(z)$. From what has been said above $\Delta f_q(z)$ looks as follows:



Notice that it consists of two pieces: the long positive slowly varying tail of $f_{q/A}(z)$ from z = 1 to z = A [recall that $f_{q/N}(z)$ vanishes for z > 1] and a piece from z = 0 to z = 1 which changes sign and is predominantly negative minimizing at $z \simeq z_0$.

This picture can be used to estimate $f_{N/A}(z)$ and thence Δ. Because $f_{q/N}(z)$ is strongly peaked at $z = z_0$, Δ can be well approximated by

$$\Delta(q^2,x) \simeq z_0 \int_x^A dz\, F_N(\tfrac{x}{z})\, \Delta F_q(zz_0) \tag{34}$$

$$= \int_{xz_0}^{Az_0} dz\, F_N(\frac{xz_0}{z})\, \Delta F_q(z) \tag{35}$$

showing its explicit dependence on the difference in quark distributions. Because of the above mentioned properties of ΔF_q, this expression gives the following structure for Δ:

$$\Delta(x,q^2) \simeq a + [b_0 + b_1 \ln (A/x)]\, x + [c_0 + c_1 \ln (A/x)]\, x^2 + \ldots \quad (36)$$

If F_N and Δf are finite polynominals then so is Δ. The coefficients are q^2-dependent and of magnitude β. Ignoring the logs for the moment one can obtain an excellent fit to the EMC data with the quadratic

$$\Delta(q^2,x) \simeq a + bx + cx^2 \quad (37)$$

with $a \simeq 0.075$, $b \simeq -0.3$ and $c \simeq 0.245$. The sum rule constrains these to satisfy $a + b/2 + c/3 \simeq 0$. Although this appears to be satisfied by the fit, this is misleading since the sum rule is violated only by an amount $\sim 10^{-2}$ which is much larger than a, b or c!

The signs of the b_i are directly related to the slope of F_N which is predominantly negative which agrees with the fit. The presence of the logarithm is only sensitive to the variation with target and also agrees nicely with the log A dependence of the data. Indeed, in terms of the ratio we predict

$$R_A(q^2,x) \simeq 1 + \frac{b_1 x \ln A}{F_N(q^2,x)} \quad (38)$$

with $b_1 < 0$, i.e., R should decrease with $\ln A$ as it does. Note that Eq. (38) also says that $\partial R_A/\partial \ln A \propto x/F_N(q^2,x)$ which is in good agreement with the data.

Thus the standard framework provides a description of the general features of the data. More specific statements can be made concerning the size of the coefficients if a more specific form is chosen for the wave function.

SUMMARY AND CONCLUSIONS

1. The OPE and asymptotic freedom show that the scattering can be described as if it were either incoherent scattering from quarks or incoherent scattering from extended effective constituents such as the nucleon.
2. The formalism suggests that the difference Δ_A should be parameterized by a polynomial in x, an excellent fit to the EMC data is obtained with a quadratic.
3. The energy-momentum sum rule requires that the area under Δ_A should vanish. This is violated by the EMC data though not by that from SLAC. Arguments were given that this was probably due to a systematic normalization problem though it is quite conceivable that finite q^2 effects could be the origin.
4. The log A dependence of the effect comes out naturally with a predicted slope for R_A, $\propto x/F_N(x)$.

5. Obviously more accurate data are needed, especially near $x = 0$ and $x \gtrsim 1$ where nuclear effects dominate.
6. One area not discussed here, that could be fruitful, is the question of the ratio σ_L/σ_T as a function of A. This is difficult to measure and probably difficult to calculate since it vanishes in leading order.
7. It is not inconceivable that new tests of QCD could be proposed based on the possibility of varying the target. The fact that the EMC data violates the energy-momentum sum-rule points in this direction. Actually this sum-rule does not depend explicity on QCD but only upon the OPE; thus not only is it target independent but also theory independent!
8. Finally, I think it should be mentioned that if the interest in these experiments is to learn about nuclei (rather than QCD), then it is quite conceivable that the very low q^2 ($\lesssim$ few GeV/c^2) might be more relevant. After all, the approach to scaling is governed by correlations in the system: one expects for the second moment (n = 2)

$$M(2, q^2) \simeq M(2, \infty) \, [1 - C(q^2)]$$

Roughly speaking $C(q^2) \sim G^2(q^2)$ the square of the elastic form factor of the target. Fig. 7 shows $M(2, q^2)$ vs. q^2 for the nucleon: the smooth approach to scaling can be well fitted by the factor $1 - G^2(q^2)$. Also shown is the same plot for thermal neutron scattering from atomic argon. The oscillatory approach is in nice agreement with these ideas since the elastic form factor of almost every system except the nucleon (!) is oscillatory reflecting the typical edge to the system.[3] Indeed in such nonrelativistic systems it is this lower q^2 region that is often of more interest! It may, therefore, be that when all the fuss over the EMC effect has died down, physicists will turn their attention to a potentially more interesting region!

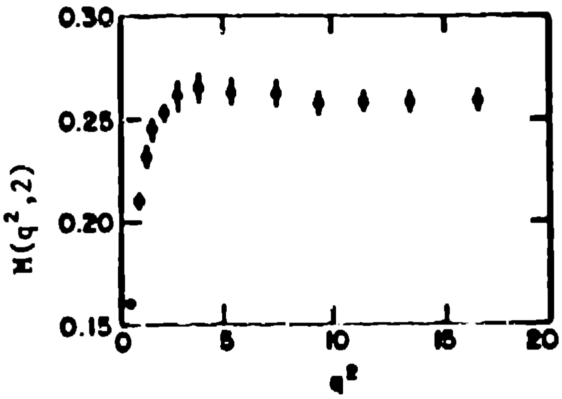


Fig. 7(a) Plot of $M(q^2,2)$ vs. q^2 (SLAC data)[4] showing smoothness of the approach to scaling, reflecting the smoothness of the elastic form factor.

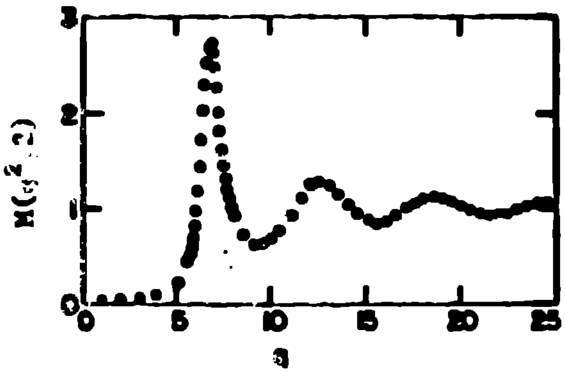


Fig. 7(b) $M(q^2,2)$ vs. q for thermal neutron scattering from argon, showing an oscillatory approach to scaling, reflecting an edge of the wave function.

REFERENCES

1. J. J. Aubert et al., Phys. Lett. 123B, 275 (1983).
2. For a general review of the operator product expansion and asymptotic freedom see, for example, H. D. Politzer, Phys. Rep. 14C, 130 (1974).
3. For a general review of the original deep inelastic experiments with emphasis on their similarity with nonrelativistic atomic and nuclear physics, see G. B. West, Phys. Rep. 18C, 364 (1975).
4. E. D. Bloom, Proc. of the 6th International Symp. on Electron and Photon Interactions at High Energies (North-Holland, Amsterdam, 1974).
5. J. D. Bjorken and E. Paschos, Phys. Rev. 185, 1975 (1969).
6. E. Reya, in "Probing Hadrons with Leptons" (eds: G. Preparata and J. J. Aubert), Plenum, N.Y. 1980.
7. H. Anderson, ibid.
8. R. G. Arnold et al., Phys. Rev. Lett. 52, 72 (1984) and references contained therein.
9. R. L. Jaffe, Phys. Rev. Lett. 50, 228 (1983).
10. K. G. Wilson, Phys. Rev. 179, 1499 (1969).
11. A. DeRújula and F. Martin, Phys. Rev. 22, 1787 (1980).
12. Recent reviews include: R. L. Jaffe, "The EMC Effect: Looking at Quarks in the Nucleus", MIT preprint CTP#1120 and C. H. Llewellyn Smith, "Nuclear Effects in Deep Inelastic Scattering", Oxford Univ. preprint 37/83.
13. Part of this is based upon G. B. West, Los Alamos preprint LA-UR-84-241
14. G. Altarelli and G. Parisi, Nucl. Phys. B126, 298 (1977).
15. A rather different approach has been suggested by F. E. Close, R. G. Roberts and G. G. Ross, Phys. Lett. 129B, 346 (1983) who use a rescaling in q^2 to relate one target to another and interpret this in terms of a change in bag radius. See also

F. E. Close, R. L. Jaffe, R. G. Roberts and G. G. Ross, Rutherford preprint RL-83-106.

16. The question as to when a particular flavor contributes to asymptotic freedom is a subtle one. In the time-like region where real states can be produced the threshold is expected to be at $4m_q^2$ (m_q being the quark mass) and indeed this seems to be borne out in e^+e^- experiments. In the space-like region, of concern here, there is no explicit threshold; this region is, of course, related to the time-like by an analytic continuation which can be quantified by a dispersion relation. This would suggest at the most naïve level that for a very heavy flavor, like the top, the contribution would be down by q^2/m_q^2. For a mass of ~50 GeV this would mean that the effect of the sixth quark might be quite small even at $q^2 \simeq$ 100 $(GeV/c)^2$. Some caution should therefore be exercised in concluding that the sum rule data agrees with the prediction of six flavors.

17. See, e.g., A. Bodek et al., Phys. Rev. Lett. 30, 1087 (1973).

18. With this new combination, $\langle Q^2 \rangle$ on the right-hand-side of (19) is replaced by $[\langle Q^2 \rangle - 1/6]$ which, of course, is still target independent.

19. A. Bodek and R. G. Arnold, private communication.

20. See also G. B. West, Los Alamos preprint LA-UR-83-2504.

21. For a discussion of thresholds in bound state problems see, e.g., G. Barton, "Dispersion Techniques in Field Theory" (W. A. Benjamin, N.Y., 1965)

22. Since quarks are confined there is no reason for B to necessarily be positive.