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ANGLE-AVERAGED COMPTON CROSS SECTIONS

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The scattering of a photon by an individual free electron is characterized by six quantities:

- α = initial photon energy in units of m_0c^2
- α_s = scattered photon energy in units of m_0c^2
- β = initial electron velocity in units of c
- ϕ = angle between photon direction and electron direction in the laboratory frame (LF)
- θ = polar angle change due to Compton scattering, measured in the electron rest frame (ERF)
- τ = azimuthal angle change in the ERF.

We present an analytic expression for the average of the Compton cross section over ϕ, θ, and τ. The lowest order approximation to this equation is reasonably accurate for photons and electrons with energies of many keV. The average over β is not specifically addressed in this work.

Under a transformation to the ERF the photon energy becomes

$$\alpha' = \alpha\gamma(1-\beta\cos\phi)$$
$$\gamma = (1-\beta^2)^{-1/2} \tag{1}$$

By applying the usual formula for Compton scattering by a motionless electron and transforming back to the LF, one obtains an expression for the scattered photon energy:

$$\frac{\alpha_s}{\alpha} = \frac{\gamma^2(1-\beta\cos\varphi) + \gamma^3\beta\cos\theta(\cos\varphi-\beta) - \gamma^2\beta\sin\theta\sin\tau\sin\varphi}{1+\gamma(1-\beta\cos\varphi)(1-\cos\theta)\alpha} \qquad (2)$$

Equation (2) represents a boundary surface in ϕ, θ, τ space which determines the limits in the formal definition of the cumulative distribution function

$$F(\alpha,\alpha_s;\beta) = \int_{\tau_1}^{\tau_2} f(\tau)\,d\tau \int_{\theta_1}^{\theta_2} f(\theta)\,d\theta \int_{\varphi_1}^{\varphi_2} f(\varphi)\,d\varphi \qquad (3)$$

$F(\alpha, \alpha_s;\beta)$ is the probability that photons scatter to an energy α_s or less by interacting with electrons of "speed" β. The limits are determined by eq. (2) and $f(x)$ represents the distribution function of variable x.

The transformation from the variables α, α_s, β to new variables z, ξ, a

$$\begin{aligned} z &= \frac{1}{\gamma^2\beta}\left(\frac{\alpha_s}{\alpha}-1\right) \\ \xi &= \frac{\alpha_s}{\gamma\beta} - \beta \\ a &= \frac{\alpha_s}{\gamma} \end{aligned} \qquad (4)$$

considerably simplifies eq. (2):

$$-z = (1-\cos\theta - a)\cos\varphi + (1-\cos\theta)\xi + \frac{1}{\gamma}\sin\theta\sin\varphi\sin\tau \qquad (2)'$$

The new variable z represents the change in photon energy due to the doppler shift, and is roughly on the range (-2,2) as we shall see. Variable ξ is of the order 10^{-1} for photons and electrons in the kilovolt range, since $\alpha \approx 10^{-2}$ and $\beta \approx 10^{-1}$ in that case. Variable a is then of the order of 10^{-2} and usually affects the final result only slightly. Thus, in addition to simplifying eq. (2), transformation (4) represents an approximate reduction in the number of dependent variables.

The normalized distribution functions for the angles are chosen as follows:

$$f(\tau) = \frac{1}{2\pi} \qquad -\pi \le \tau \le \pi \qquad (5)$$

$$f(\theta) = \frac{1}{A}\,\frac{2 - 2(1-\alpha')(1-\cos\theta) + (1-2\alpha'+\alpha'^2)(1-\cos\theta)^2 + \alpha'(1-\cos\theta)^3}{[1+\alpha'(1-\cos\theta)]^3}$$

$$0 \le \theta \le \pi \qquad (6)$$

$$f(\varphi) = \frac{1}{2}\sin\varphi \qquad 0 \le \varphi \le \pi \qquad (7)$$

Eq. (5) represents the assumption that photon polarization can be ignored, eq. (6) is the usual Klein-Nishima formula expressed in the ERF, and eq. (7) implies that the photon-electrons in the LF are distributed isotropically. The normalization factor A in eq. (6) depends on ϕ because of the appearance of α' in the θ-distribution. Letting

$$r = 1 - \cos\theta \tag{8}$$

for notational convenience gives

$$A = \int_0^2 dr \int_0^1 \frac{1}{2}\, d(\cos\varphi)\; \frac{2-(1-\alpha')r+(1-2\alpha'+\alpha'^2)r^2+\alpha' r^3}{(1+\alpha' r)^3} \tag{9}$$

At this point, the integrand $f(\tau)f(\theta)f(\phi)$ is independent of the azimuthal angle change τ, but the boundary surface (2)' depends on τ in a manner that makes explicit representation of the integration limits difficult. This situation can be reversed by transforming the variables ϕ' and τ':

$$\begin{aligned} \cos\phi &= \cos\phi'\cos\delta - \sin\phi'\sin\tau'\sin\delta \\ \sin\phi\sin\tau &= \cos\phi'\sin\delta + \sin\phi'\sin\tau'\cos\delta \end{aligned} \tag{10}$$

This represents a rotation by the angle δ about the axis $\phi = \pi/2$, $\tau = 0$. By choosing

$$\begin{aligned} \sin\delta &= \frac{1}{\gamma}\,\frac{\sqrt{2r-r^2}}{\sqrt{\beta^2 r^2+2r(1-\beta^2-a)+a^2}} \\ \cos\delta &= \frac{r-a}{\sqrt{\beta^2 r^2+2r(1-\beta^2-a)+a^2}} \end{aligned} \tag{11}$$

The boundary equation reduces even further:

$$-z = \sqrt{\beta^2 r^2 + 2r(1-\beta^2-a) + a^2}\,\cos\varphi' + r\xi \qquad (2)''$$

Of course, the integrand $f(\tau')f(\theta)f(\theta')$ is now more complicated. However, odd powers of $\sin\tau'$ and $\cos\tau'$ vanish under τ-integration, which simplifies explicit calculations considerably.

We now define the value of r at the intersection of the surface (2)'' and the $\phi = 0$ (or π) axis:

$$r_{min} = \frac{1}{2}\,\frac{z^2-a^2}{1-a-\beta^2-z\beta} \qquad \text{when } \xi^2 = \beta^2 \qquad (12)$$

or

$$r_{min} = \frac{1-a-\beta^2-z\xi}{\xi^2-\beta^2}\left\{1-\sqrt{1-\frac{(\xi^2-\beta^2)(z^2-a^2)}{1-a-\beta^2-z\xi}}\right\}$$

$$\text{when } \xi^2 \neq \beta^2 \qquad (13)$$

The conditions for the extremes of scattered energy α_s is that $r_{min} = 2$; substituting for (z, ξ, a) in terms of $(\alpha, \alpha_s, \beta)$ gives

$$\alpha_{\substack{smax \\ (smin)}} = \alpha\,\frac{2\beta^2(1+\gamma\alpha)+\frac{1}{\gamma^2}(1+2\gamma\alpha)}{\left(\frac{1}{\gamma}+2\alpha\right)^2-\beta^2\alpha^2}\left\{1 \underset{(-)}{+} \sqrt{1-\frac{\frac{1}{\gamma^2}\left[\left(\frac{1}{\gamma}+2\alpha\right)^2-\beta^2\alpha^2\right]}{\left[2\beta^2(1+\gamma\alpha)+\frac{1}{\gamma^2}(1+2\gamma\alpha)\right]^2}}\right\} \qquad (14)$$

We can finally express the cumulative distribution function: For α_s on the interval $(\alpha_{smin}, \alpha_{smax})$ given by (14),

$$F(z,\xi,a) = \int_0^{2\pi} \frac{d\tau'}{2\pi} \int_{r_{min}}^{2} \frac{dr}{A} \int_{\varphi_1'}^{\varphi_2'} \frac{1}{2}\, d(\cos\varphi')\, \frac{2-2(1-\alpha')r+(1-2\alpha'+\alpha'^2)r^2+\alpha' r^3}{(1+\alpha' r)^3} \tag{15}$$

The quantities A and r_{min} are given by eqs. (9), (12) and (13), and the the transformed ERF photon energy is

$$\alpha' = \gamma\alpha\{1 - \beta[\cos\delta\cos\phi' - \sin\delta\sin\phi'\sin\tau']\} \tag{16}$$

where $\cos\delta$ and $\sin\delta$ are given by equations (11). When z is less than zero, the ϕ' limits are given by:

$$\cos\varphi_1' = \frac{(z+\xi r)}{\sqrt{\beta^2 r^2 + 2r(1-a-\beta^2)+a^2}} \tag{17}$$

$$\cos\phi_2' = 1.$$

For $z \geq 0$, the corresponding limits become:

$$\cos\phi_1' = -1$$

$$\cos\varphi_2' = \frac{-(\zeta+\xi r)}{\sqrt{\beta^2 r^2 + 2r(1-a-\beta^2) + a^2}} \tag{18}$$

The lowest order approximation to (15) is found by simply ignoring α' in the integrand and letting $a = 0$ in the equation for α_{min}. This gives the very simple result:

$$F_0(\zeta,\xi) = \frac{1}{2} \pm h_0(r_{min}) + \zeta g_0(r_{min}) + \xi g_1(r_{min}) \tag{19}$$

where h_0, g_0, g_1 are polynomials in $r_{min}^{1/2}$:

$$h_0 = \frac{3}{8} r_{min} - \frac{3}{16} r_{min}^2 + \frac{1}{16} r_{min}^3 \tag{20}$$

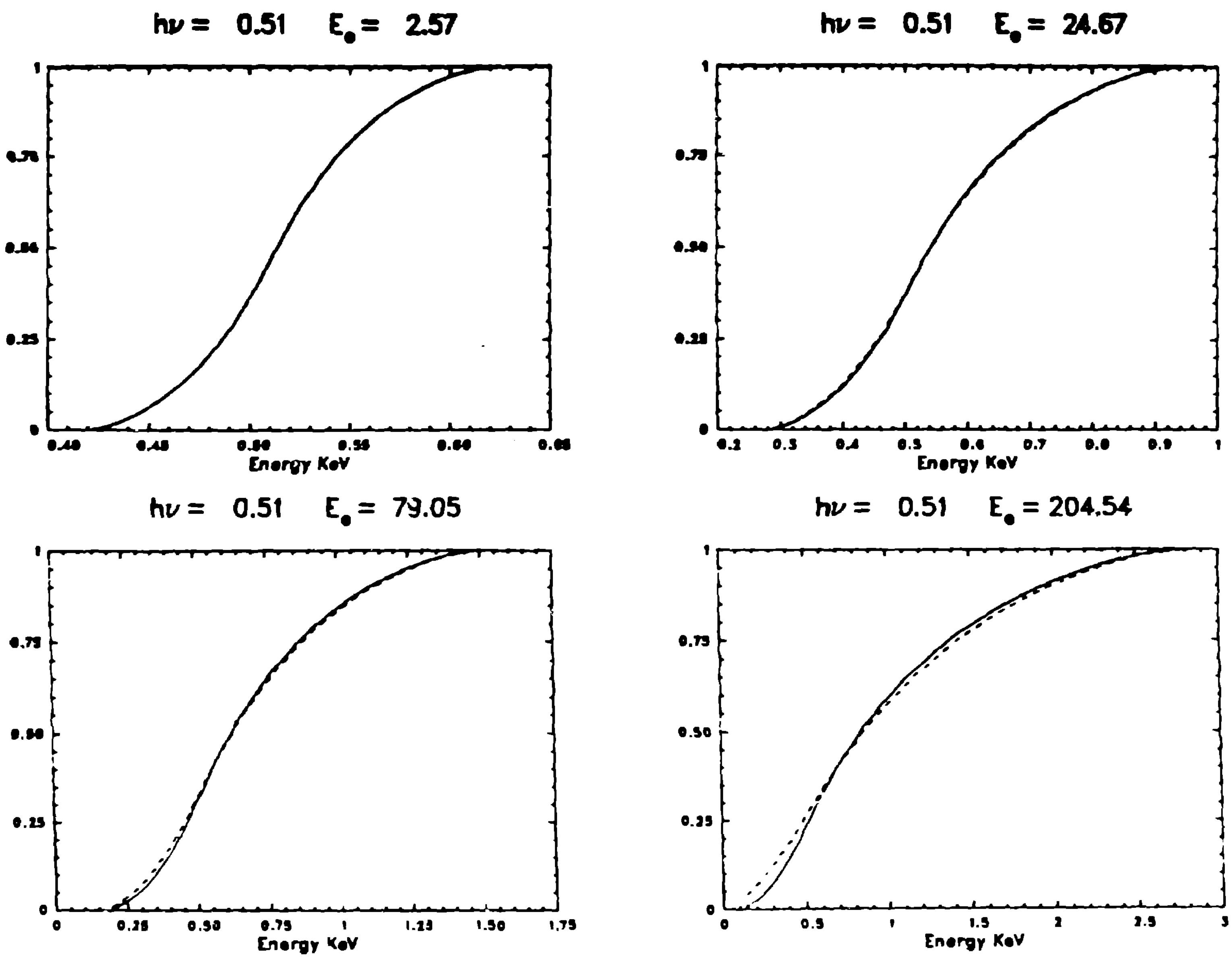
$$g_0 = \frac{11}{20} - \frac{3}{4\sqrt{2}} r_{min}^{1/2} + \frac{1}{4\sqrt{2}} r_{min}^{3/2} - \frac{3}{40\sqrt{2}} r_{min}^{5/2} \tag{21}$$

$$g_1 = \frac{23}{70} - \frac{1}{4\sqrt{2}} r_{min}^{3/2} + \frac{3}{20\sqrt{2}} r_{min}^{5/2} - \frac{3}{56} r_{min}^{7/2} \tag{22}$$

Subsequent polynomials g_n are useful in higher order approximations to (15).

$$g_n(r_{min}) = \frac{3}{16\sqrt{2}} \int_{r_{min}}^{2} (2-2r+r^2)\, r^{n-\frac{1}{2}}\, dr \tag{23}$$

Comparisons of F_0 with the numerically determined "exact" result are given in figures (1) through (3). One can see that the agreement is surprisingly close for photon and electron energies of many 10's of kilovolts. Maxwellian averages, moments of the angular distribution and corrections beyond this lowest order will be discussed in subsequent publications.



hν = 0.51 E_e = 2.57
hν = 0.51 E_e = 24.67
hν = 0.51 E_e = 79.05
hν = 0.51 E_e = 204.54
Energy KeV
Energy KeV
Energy KeV
Energy KeV

Figure 1

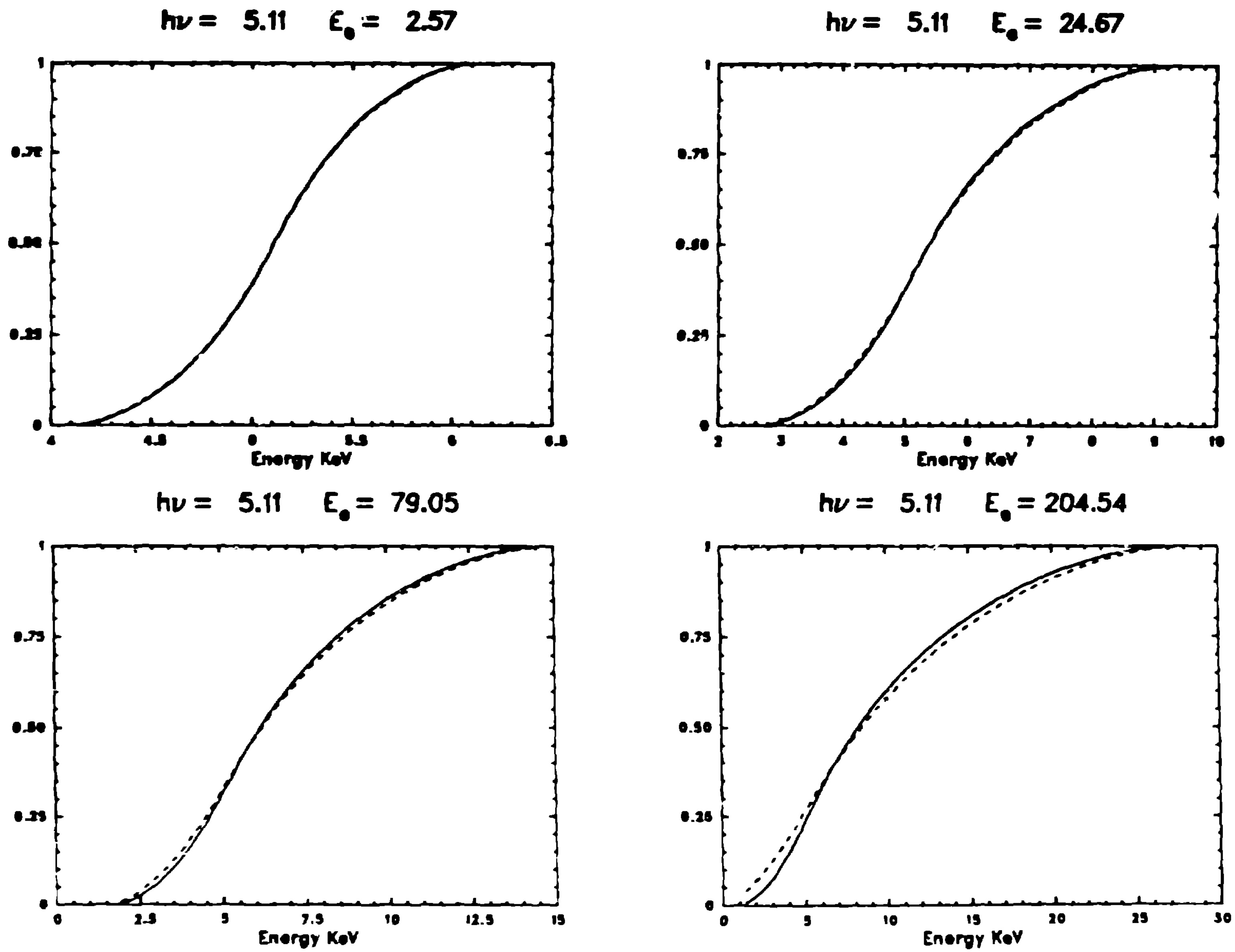

hν = 5.11 E_e = 2.57
hν = 5.11 E_e = 24.67
hν = 5.11 E_e = 79.05
hν = 5.11 E_e = 204.54
Energy KeV
Energy KeV
Energy KeV
Energy KeV

Figure 2

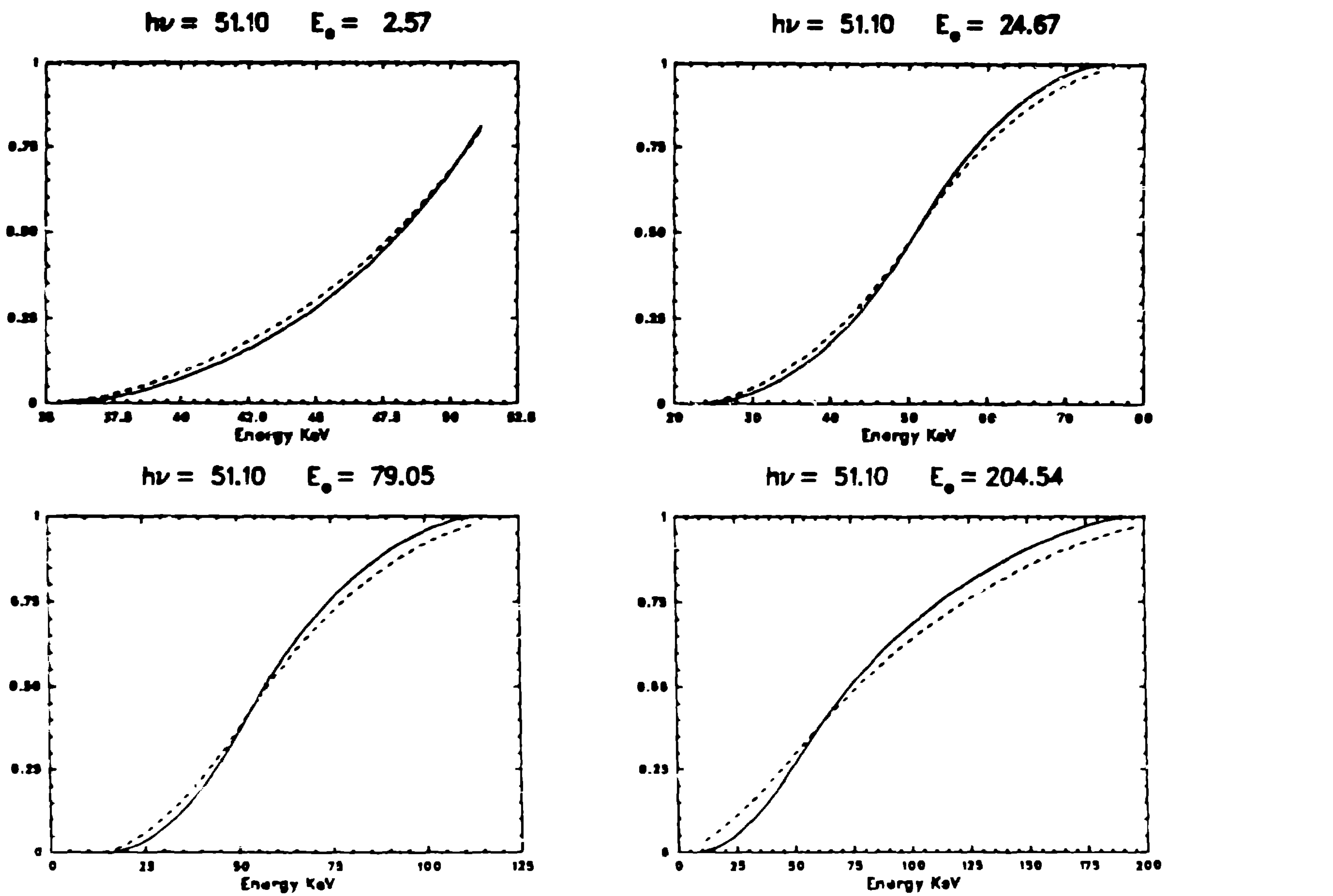

hν = 51.10 E_e = 2.57
Energy KeV
hν = 51.10 E_e = 24.67
Energy KeV
hν = 51.10 E_e = 79.05
Energy KeV
hν = 51.10 E_e = 204.54
Energy KeV

Figure 5