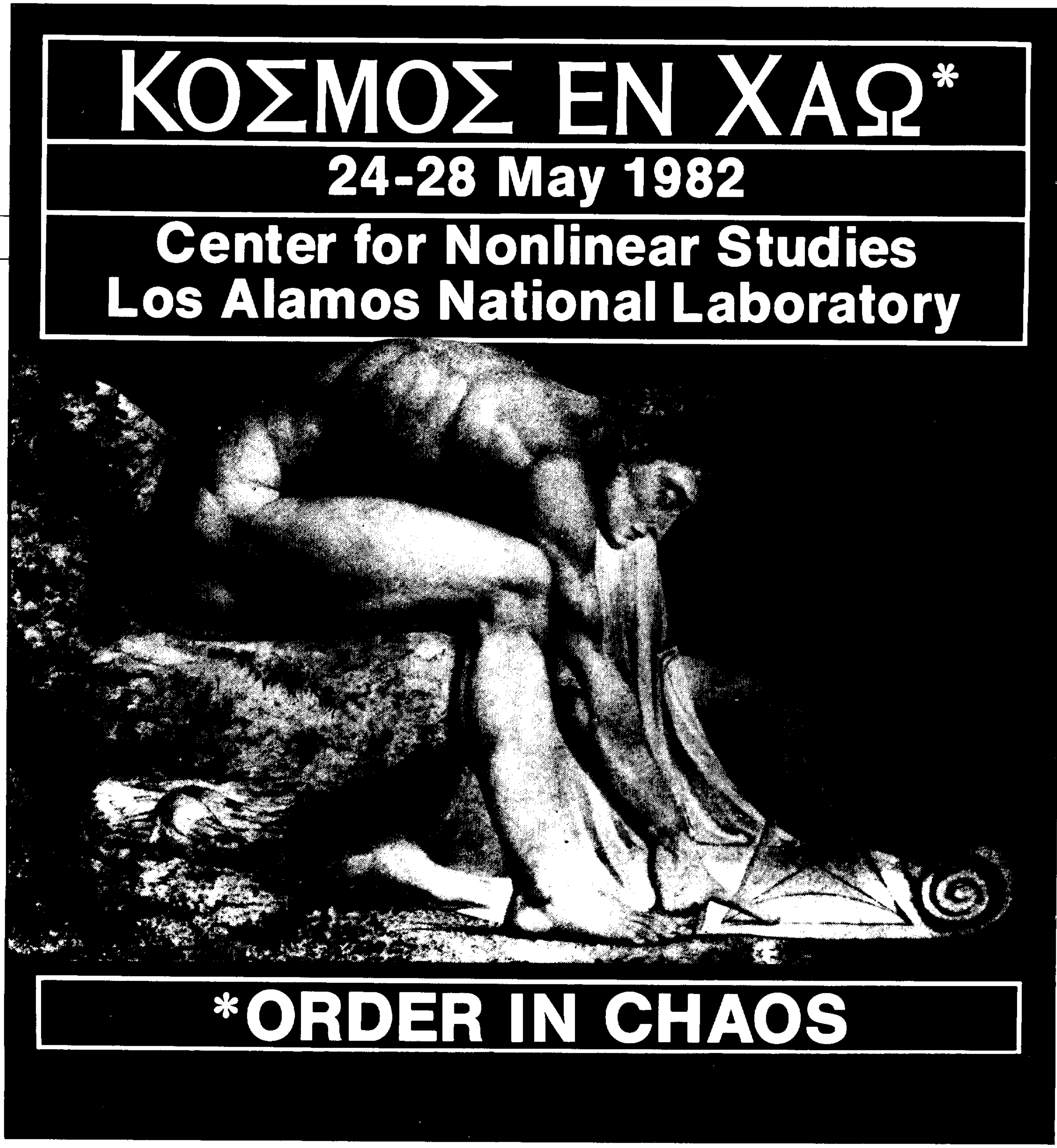


*Conference poster by Gail Flower*

# Review of the CNLS Conference on Chaos in Deterministic Systems

*by David Campbell, Doyne Farmer, and Harvey Rose*

The phrase "order in chaos" seems self-contradictory: chaos is, after all, conventionally viewed as the complete absence of order. Yet precisely this title attracted two hundred and ten scientists from fourteen countries to Los Alamos from May 24-28, 1982, to attend the second annual international conference of the Center for Nonlinear Studies. The purposes of the conference were to survey the recent rapid developments and to anticipate the trends for future research in the area of "chaos in deterministic systems." The breadth of scientific interest in this topic was reflected in the variety of subjects discussed at the meeting. Presentations ranged from abstract mathematics through numerical simulations to experimental studies of fluid mechanics, chemistry, and biology. Even weather prediction made an appearance.

To appreciate the appeal of the conference title-and the importance of the field of research it describes—requires a closer look at the apparently contradictory components. The concepts of "order" and "determinism" in the natural sciences recall the predictability of the motion of simple physical systems obeying Newton's laws: the rigid plane pendulum, a block sliding down an inclined plane, or motion in the field of a central force are all examples familiar from elementary physics. In contrast, the concept of "chaos" recalls the erratic, unpredictable behavior of elements of a turbulent fluid or the "randomness" of Brownian motion as observed through a microscope. For such chaotic motions, knowing the state of the system at a given time does not permit one to predict it for all later times. In place of the determinism of the orderly systems, one has only probabilistic estimates and statistical averages.

Thus, in some sense, the possibility that chaos exists in deterministic systems runs directly counter to our intuition. To understand that this possibility is nonetheless real, we can refer to the deeper insight of Henri Poincare, one of the founders of modern dynamical systems theory. Writing in the pre-quantum era of pure Newtonian determinism, Poincare noted that

> *A very small cause which escapes our notice determines a considerable effect that we cannot fail to see, and then we say that the effect is due to chance. If we knew exactly the laws of nature and the situation of the universe at the initial moment, we could predict exactly the situation of that same universe at a succeeding moment. But even if it were the case that the natural laws had no longer any secret for us, we could still only know the initial situation* approximately. *If that enabled us to predict the succeeding situation* with the same approximation, *that is all we require, and we should say that the phenomenon had been predicted, that it is governed by laws. But it is not always so; it may happen that small differences in the initial conditions produce very great ones in the final phenomena. A small error in the former will produce an enormous error in the latter. Prediction becomes impossible, and we have the fortuitous phenomenon.*

Hence, the crucial ingredient in deterministic chaos is a very sensitive dependence on initial conditions. Motions that start close to each other develop in time in dramatically different ways, and uncertainties in the initial values develop rapidly—exponentially, in fact—in time. Although the motion from instant to instant can be predicted, over macroscopic times it becomes no more predictable than a random sequence.

At first it might appear that the distinction between orderly and chaotic motions is merely one of the complexity of the system involved. In the parlance of dynamical systems theory, the orderly motions described above involve just one "degree of freedom," whereas the chaotic fluid involves many-in conventional hydrodynamics, infinitely many—degrees of freedom. It is thus tempting to associate simple systems with order and complicated ones with chaos.

In fact, this naive association is wrong for several fundamental reasons, some obvious and some subtle. First, everyday experience tells us that complicated systems with many degrees of freedom *can* undergo very orderly motion. For example, a fluid in smooth (laminar) flow moves in a regular, totally predictable manner.

Second, it is less familiar but nonetheless true that very simple physical systems can exhibit chaotic behavior, with all the associated randomness and unpredictability. Numerical experiments show that the motion of a rigid plane pendulum, if damped and driven, becomes truly chaotic. This result illustrates strikingly that a completely deterministic system can produce chaos without the addition of any external random noise. In other words, you don't have to put randomness in to get it out. The existence of deterministic motions that produce chaos is a clear example of order in chaos.

Third, it is now well established that, at least in some cases, the chaos observed in very complicated systems can be understood quantitatively in terms of simple models that involve very few degrees of freedom. This profound result, several examples of which were presented at the conference, is perhaps the most significant manifestation to date of order in chaos.

From this general motivation of the theme "order in chaos," we turn to a discussion of the specific results described at the conference. (The accompanying table lists the authors and titles of the talks presented.) Very roughly, the presentations divided into two major areas. First, there were attempts to identify the qualitative and quantitative essential features of deterministic chaos to describe and model it more accurately. Second, there were discussions of the transition from regular motion to chaos. Here the focus was on identifying various possible routes and establishing whether they had "universal" properties that were independent of the details of the mathematical model or physical system being studied.

SHORT SUBJECT

## 1982 CNLS CONFERENCE TALKS[a]

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| **Review** | |
| M. Feigenbaum<br>Los Alamos National Laboratory | An Overview of Order in Chaos |
| H. Swinney<br>University of Texas, Austin | Observations of Chaos |
| **Theoretical** | |
| S. Aubry<br>Laboratoire Leon Brillouin and<br>Los Alamos National Laboratory | Devil's Staircase and Order Without Periodicity in Condensed Matter Physics |
| J. D. Farmer<br>Los Alamos National Laboratory | Dimension, Fractal Measures, and Chaotic Dynamics |
| J. Ford<br>Georgia Institute of Technology | Classical and Quantum Billiards: New Insights into Chaos |
| V. Franceschini<br>University of Modena | Bifurcation Phenomena in Truncated Navier-Stokes Equations on a Two-Dimensional Torus |
| J. Guckenheimer<br>University of California, Santa Cruz | Overview of Dynamical Systems Theory |
| M. Gutzwiller<br>International Business Machines Corporation | Stochastic Behavior in Electron Scattering |
| E. Heller<br>Los Alamos National Laboratory | Quantum Manifestations of Classical Chaos |
| P. Holmes<br>Cornell University | Periodically Forced Nonlinear Oscillations of Dissipative Systems: Some Answers and Questions |
| P. Huerre<br>University of Southern California | Long-Time Solutions to the Ginzburg-Landau Equation: A Numerical Study |
| C. Leith<br>National Center for Atmospheric Research | Chaos and Order in Weather Prediction |
| R. MacKay<br>Princeton University | A Renormalization Method for Orbits with Generalized Golden Ratio Rotation Number |
| B. Mandelbrot<br>International Business Machines Corporation | Quadratic Chaos, Scaling, and Fractals |
| J. Marsden<br>University of California Berkeley | Fluids, Vortices, and Coadjoint Orbits |
| E. Ott<br>University of Maryland | Strange Attractors in Crisis |

A significant aspect of the conference was that in each of these areas there were important new developments both in theoretical modeling and prediction and in experimental observation,

These new developments were woven into the previous results in the conference's two introductory reviews surveying the field. In his "Observations of Chaos" Harry Swinney of the University of Texas, Austin, described experimental observations in electrical oscillators, chemical reactions (the Belousov-Zhabotinskii reaction), and fluid flows (Rayleigh-Benard convection and circular Couette flow) that established the existence of deterministic chaos. He reviewed ex-

**SHORT SUBJECT**

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| N. Packard<br>University of California, Santa Cruz | Measures of Chaos in the Presence of Noise |
| D. Ruelle<br>Institut des Hautes Etudes Scientifiques | Unconventional Turbulent Structures |
| S. Shenker<br>University of Chicago | Scaling Behavior in Maps of the Circle |
| E. Siggia<br>Cornell University | A Universal Transition from Quasi-Periodicity to Chaos in Dissipative Systems |
| J. Yorke<br>University of Maryland | The Dimension of Strange Attractors |
| A. Zisook<br>University of Chicago | Universal Effects of Dissipation in Two-Dimensional Mappings |

**Experimental**

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|---|---|
| I. Epstein<br>Brandeis University | Oscillations and Chaos in Chemical Systems |
| L. Glass<br>McGill University | Chaos in a Petri Dish: Nonlinear Dynamics of a Cardiac Oscillator |
| H. Haucke and Y. Maeno<br>University of California, San Diego, and Los Alamos National Laboratory | Time-Dependent Convection in $^3$He/Superfluid $^4$He Solution |
| R. Keolian<br>University of California, Los Angeles | Generation of Subharmonic and Chaotic Behavior in High-Amplitude, Shallow-Water Waves |
| O. Lanford, 111<br>University of California, Berkeley | Period Doubling in One and Several Dimensions |
| A. Libchaber<br>Group de Physique des Solides de l'Ecole Normale Superieure | Mercury in a Magnetic Field, A Rayleigh-Benard Study |
| J. D. Roux<br>University de Bordeaux | Chaos in the Belousov-Zhabotinskii Reaction |
| R. Shaw<br>University of California, Santa Cruz | The Dripping Faucet as a Model Chaotic System |
| C. W. Smith<br>University of Maine | Bifurcation Universality for First-Sound Subharmonic Generation in Superfluid $^4$He |

[a]Proceedings of the conference will be published by North-Holland, as a special issue of *Physica D* and also as a hardbound volume. The Center gratefully acknowledges support for the conference from the Applied Mathematical Sciences Program in the U.S. Department of Energy's Office of Basic Energy Sciences.

perimental data showing there are at least seven well-defined routes leading from smooth, regular motion to chaos. In his "Overview of Order in Chaos" Mitchell Feigenbaum of Los Alamos reviewed the theoretical description of deterministic chaos, recalling some of the essential ideas and methods and introducing simple model systems to illustrate these results. These introductory surveys set the stage for over a score of additional presentations, in which theory and experiment, abstraction and observation, were mingled in an appropriately chaotic manner.

To explain both the experimental and the theoretical results in more detail, it is necessary to introduce some concepts from dynamical systems theory, which is the formal discipline underlying the study of all types of motion. These general concepts were discussed, in slightly differing contexts, in the survey by Feigenbaum and in the talks of Ford, Guckenheimer, Holmes, and Marsden. Dynamical systems can be divided into two broad categories—conservative and dissipative—depending on whether or not the energy is conserved. The Navier-Stokes equations of fluid mechanics are an important (infinite degrees of freedom) dissipative dynamical system, since the viscosity converts the energy of fluid motion into heat. Most of the research presented at the conference dealt with dissipative systems, where the long-time behavior is controlled by various kinds of "attractors." That is, different initial conditions evolve in time "toward" an attractor, and after initial transients die out the motion reduces, in a well-defined sense, to motion "on" the attractor,

A simple model description of a dissipative dynamical system, used in Feigenbaum's talk and familiar to the readers of *Los Alamos Science,** is the discrete "logistic map," in which one point in the interval [0, 1] is transformed to another according to

$$x_{n+1} = \lambda x_n (1 - x_n) \quad . \tag{1}$$

Since $x_{n+1}$ follows uniquely from $x_n$, the map is deterministic. To view this map as a dynamical system, we need only think of the number of iterations of the map as "time" and the sequence of points $x_1$, $x_2$, $x_3$, ... as the "motion." As a function of the parameter λ, the map has a variety of attractors. First, for $0 < \lambda < 1$, this map has a "fixed point" attractor at x = O. As a simple exercise on a pocket calculator will demonstrate, for this range of λ initial points anywhere in the interval are eventually attracted, after many iterations (long time), toward the point $x$ = O. In real physical systems this type of attractor corresponds to motion that does not change in time. Thus, for example, when a pot is filled with water and placed on a flat surface, the initial sloshing dies out and the fluid comes to rest. For $1 < \lambda < 3$ the fixed point at the origin is unstable, and a new stable fixed point at x = 1 – l/L appears. Analogous behavior is seen when a pot of water is heated and *steady* convection rolls form. Even though the fluid is moving, because the flow pattern is constant in time, the attractor is a fixed point.

A second type of attractor found in Eq. 1 is a periodic limit cycle, in which the sequence of values of $x_n$ repeats itself periodically. As λ is increased in the range $3 < \lambda < 3.59...$ , there is the famous sequence of periodic cycles with periods $2^n$, $n$ = 1, 2, 3, . . . . In our analogy to a heated pot of water, a limit cycle corresponds to *oscillatory* convection rolls in which the flow pattern changes periodically in time.

The third type of attractor in Eq. 1 is much less familiar and, in fact, is called a strange attractor, a term first introduced by David Ruelle. These strange attractors (also called chaotic attractors) occur in Eq. 1 for certain values of $\lambda > 3.59...$ and describe chaotic "motion" in the map in the sense that the sequence of points {x. } is random. For some time, it has been thought that these strange attractors underlie the chaos observed in more complicated dynamical systems. Thus, for example, the turbulence seen in a pot of boiling water can be described by a strange attractor. One of the most exciting aspects of the conference, which we shall discuss in detail later, was the conclusive evidence from a variety of experiments that the chaos in several real systems can be described by low-dimensional strange attractors.

Chaotic or strange attractors are elegant incarnations of "order in chaos." Since many initial conditions collapse onto the attractor, the number of degrees of freedom "actively" participating in the chaos can be many fewer than in the full system. On the other hand, the chaos is real, because nearby points on the attractor separate initially at an exponential rate (determined by the positive Lyapunov exponents), causing small errors to amplify very fast and producing sensitive dependence on initial conditions.

Strange attractors are like bakers. Thinking of the whole space of possible initial conditions as dough, a strange attractor grabs the dough, stretches it, and then folds it back onto itself. Just as a small drop of vanilla will quickly get mixed throughout the dough by this process, a strange attractor rapidly mixes together all the initial conditions that it attracts, creating chaos.

The dimension of an attractor is, roughly speaking, the number of "active modes" that are left once all the transients have died out. It turns out, though, that the folding described above creates a complicated structure, something like the filo dough of Greek pastries. This structure, dubbed fractal by Mandelbrot and described in detail in his talk, makes dimension a difficult concept to define; in particular, the dimension need *not* bean integer. Relations among the dimension of a chaotic attractor and its other properties-such as the values of the Lyapunov exponents—were discussed in the presentations of Farmer and Yorke.

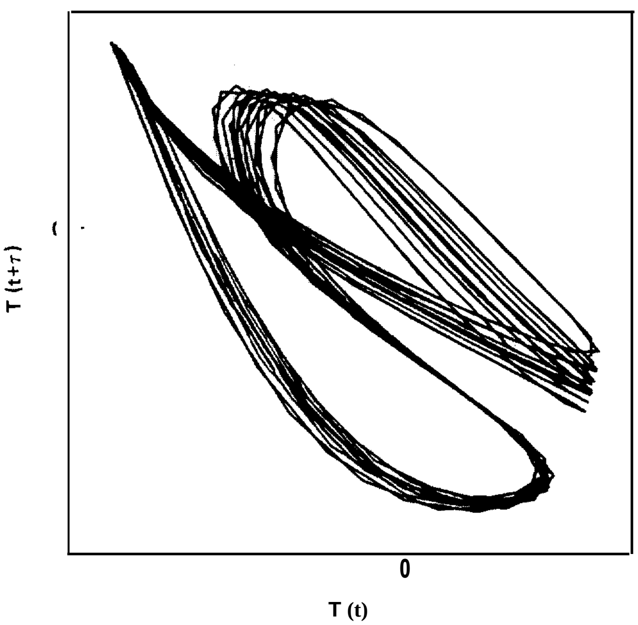
Given a chaotic dynamical system, one of the central problems is to ferret out the strange attractor—assuming it is present—and to estimate its dimension and other properties. This and related problems were analyzed in theoretical models in the talks by Farmer, Holmes, Huerre, Marsden, and Ott. In particular, Farmer's and Huerre's presentations underscored by example the possibility, mentioned in many other talks, that even in a dissipative dynamical system with infinitely many degrees of freedom, the chaotic attractor

● *Mitchell J. Feigenbaum, "Univeral Behavior in Nonlinear systems, "* Los Alamos Science, *Vol. 1, No. 1, 4-27 (1980).*

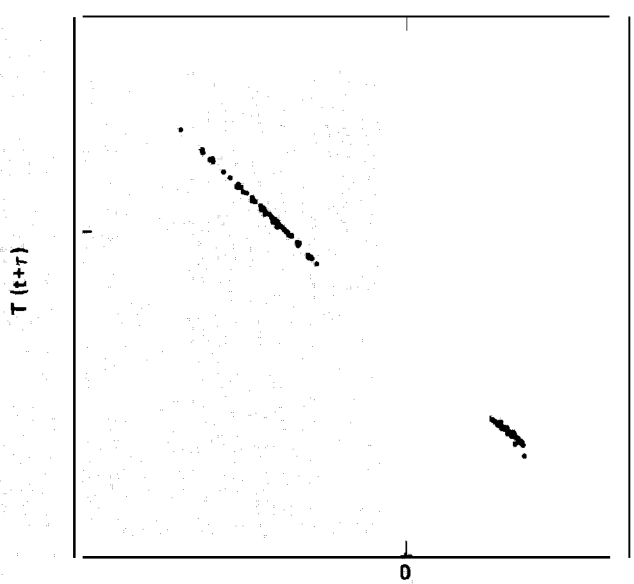
## SHORT SUBJECT

may be low dimensional, involving perhaps only two or three active modes. In some of the most exciting reports to the meeting, this possibility was confirmed or at least supported in a variety of experimental systems. The experiments described included fluid flows (Haucke and Maeno, Keolian, and Libchaber), surface tension (Shaw), chemical reactions (Epstein and Roux), and physiological studies of heart beat irregularities (Glass). The accompanying figure, which is from the presentation of Haucke and Maeno, contains a portrait of the slightly more than two-dimensional strange attractor that evolved from convective flow in a Rayleigh-Benard cell containing a mixture of helium-3 and superfluid helium-4 at 4 degrees kelvin.

Other experimental examples of chaos were much closer—some amusingly. some frighteningly—to our day-to-day reality, Have you ever been kept awake at night by a dripping faucet? If so. you might have noticed that some faucets drip periodically, while others drip in an unpredictable, apparently chaotic manner. In his talk Shaw analyzed a dripping faucet, and showed that, in some cases, the time intervals between drips are determined by a nearly two-dimensional chaotic attractor. Your frustration with the television weatherman might be moderated by considering some of the points raised by Leith. The extreme sensitivity of weather to initial conditions has long been recognized. Indeed, Leith recalled that one of the first simple dynamical systems known to lead to deterministic chaos was developed by meteorologist Edward Lorenz in the context of weather prediction. On the basis of model experiments, turbulence calculations. and studies of the differing evolution of similar atmospheric states, meteorologists estimate that the average doubling time for errors (which is related to the Lyapunov exponents mentioned earlier) is two and a half days. Yet another example, one with which none of us would want to be familiar, was discussed by Glass. The normal "motion" of a heart is a regular oscillation, which can be modeled by a periodic attractor. Glass presented experimental evidence that under certain stimuli the underlying attractor can




(a)




(b)

***Illustrations from the presentation of Hauke and Maeno showing the (nearly) two-dimensional strange attractor underlying chaotic flow in a mixture of helium-3 and superfluid helium-4. (a) A "phase portrait" formed by plotting the temperature*** T ***at time*** t ***vs the temperature at a delayed time*** t + $\tau$ ***as measured on a probe. Since the attractor is nearly two-dimensional, it can be pictured as a ribbon (perhaps with folds and twists), and the plot is a projection of this ribbon onto a two-dimensional surface. (b) A "slice" through the attractor formed by making what is technically called a Poincare section. The fact that each of the two disjoint parts of this slice looks one-dimensional (that is, like a line) demonstrates that the attractor is approximately two-dimensional.***

change from a limit cycle to a strange attractor, with the resulting chaos possibly being related to a heart attack.

Thus far we have focused on the *nature* of deterministic chaos, which was the first of the two major areas discussed at the conference. The second major area, routes to chaos, was also well represented both theoretically and experimentally. In his survey Swinney identified and gave experimental evidence for seven distinct routes to chaos, thus stressing the point that no single scenario exists to describe the transition to chaos in deterministic systems. On the theoretical side, the talk of Franceschini revealed the variety of routes to chaos in finite mode truncations of the Navier-Stokes equations. Since in this brief review we cannot possibly discuss all these routes, we shall focus on the two that arose most frequently in the presentations at the conference: the period doubling transition to chaos and the transition from periodic to quasi-periodic motion (which involves two or more frequencies that are not rational multiples of each other) to chaos.

The period doubling route to chaos has already been obliquely mentioned in our discussion of the logistic map. In fact, this route is known to exist in a wide class of dynamical systems, and, when it is observed, its essential properties are "universal" in the sense that they do not depend on the details of the specific system. This quantitative universality was first discovered and then extensively analyzed by Feigenbaum. Examples were discussed in his survey, in a presentation of two-dimensional maps by Zisook, and in a more general and abstract setting by Lanford. Experimentally, this route was observed in chemical reactions (Swinney and Roux), Rayleigh-Benard convection experiments in mercury (Libchaber), first sound generation in superfluid helium-4 (Smith), heart beat irregularities (Glass), dripping faucets (Shaw), and water waves (Keolian).

One underlying mechanism for the periodic-quasi-periodic-chaotic transition to chaos was originally suggested by Ruelle and Takens in a general abstract mathematical framework. More recent work on a somewhat different, more explicit mechanism was discussed in the talks of Shenker and Siggia, and related calculations were described by MacKay. Experimental evidence for this route to chaos was discussed in chemical systems (Roux) and in Rayleigh-Benard convection experiments in mercury (Libchaber). In several of the experimental talks, observations of some of the other routes to chaos mentioned by Swinney were also discussed.

Although most of the conference presentations fell into one of the two main areas already discussed, a number of talks addressed other topics related to chaos in deterministic systems. The nature of chaos in conservative systems, in which there cannot be attractors, was mentioned briefly in several talks and discussed more extensively by MacKay and Ford, Chaos in conservative systems has its historical roots in the fundamental questions of statistical mechanics. Why should a gas of interacting particles be described by the well-known statistical ensembles of Gibbs? Although we expect a large, isolated collection of interacting particles to be in thermal equilibrium, there is no generally applicable mathematical theorem that corroborates this expectation. In the specific context of the billiard ball problem, Ford discussed the possibility that the chaotic dynamics might lead to a state resembling thermal equilibrium.

Among the other topics discussed, several appeared to point the way to significant problems of the future. In emphasizing deterministic chaos, we have thus far explicitly excluded external noise or thermal fluctuations, which could add a separate, nondeterministic source of randomness to a dynamical system. Since in any experiment some level of noise can be anticipated, the response of chaotic deterministic systems to noise is a very important question. In particular, does external noise destroy the order in deterministic chaos? In his presentation Packard discussed this point and the scaling properties of information production rates for chaotic systems with external noise.

The possible role of chaos not in the time evolution of a dynamical system but in the spatial structure of condensed matter systems was discussed in the talks of Aubry and RueIle.

Finally, there were presentations concerned with the manifestations of chaos in quantum mechanical systems. Gutzwiller displayed an example where the solutions to a particular Schrodinger equation depended sensitively on initial conditions. (Note that the sensitivity to initial conditions displayed in this example and in classical dynamics is quite different from the indeterminism in any measurement embodied by Heisenberg's uncertainty principle.) Heller illustrated the relations between the structure of quantum mechanical states and the orbits of the corresponding chaotic classical system.

In a very real sense, the Center for Nonlinear Studies' conference represented the "end of the beginning" of the field of deterministic chaos. Many of the fundamentals of low-dimensional chaos are theoretically modeled and experimentally verified, and a variety of intriguing questions seem ripe for answering. Given the panoply of models and the range of observed phenomena, it was no surprise that by the end of the conference most of the participants appeared ready to agree with the American poet Wallace Stevens, who, in his poem "Connoisseur of Chaos," asserted that

*The squirming facts*
*exceed the squamous mind.* ■