

# Shock Waves, Rarefactions, and

SIDEBAR 2

## Contact Discontinuities

The nonlinear equations of ideal gas dynamics support three types of nonlinear “waves”: shock fronts, rarefactions, and contact discontinuities.

Contact discontinuities are surfaces that separate zones of different density and temperature. By definition such a surface is in pressure equilibrium, and no gas flows across it. When, as often happens, the tangential velocity of the gas on one side of the surface differs considerably from that of the gas on the other side, the surface is called a slip discontinuity. The boundary between a

supersonic jet and the ambient gas is an example of a slip discontinuity.

The other two types of nonlinear waves arise from abrupt changes in pressure. Shock fronts accompany compression of the medium, and rarefactions accompany expansion of the medium. As a simple example of how these waves arise, consider the one-dimensional flow of gas in a small-diameter cylindrical tube, which is fitted with a piston at one end ( $x = 0$ ) and closed at the other end. Figure A shows the time-dependent flow that results when the piston is suddenly pushed

into the cylinder and how a shock front forms as sonic disturbances generated by the piston converge. These sound waves travel along trajectories called characteristic paths, or simply characteristics. In general, the characteristics of a set of hyperbolic partial differential equations are the paths along which certain variables are conserved and are thus the paths along which information travels. The characteristics for the equations of ideal gas dynamics are (1) the streamlines along which matter flows and entropy is conserved and (2) the plus and minus

Fig A. Compression in a Shock Tube

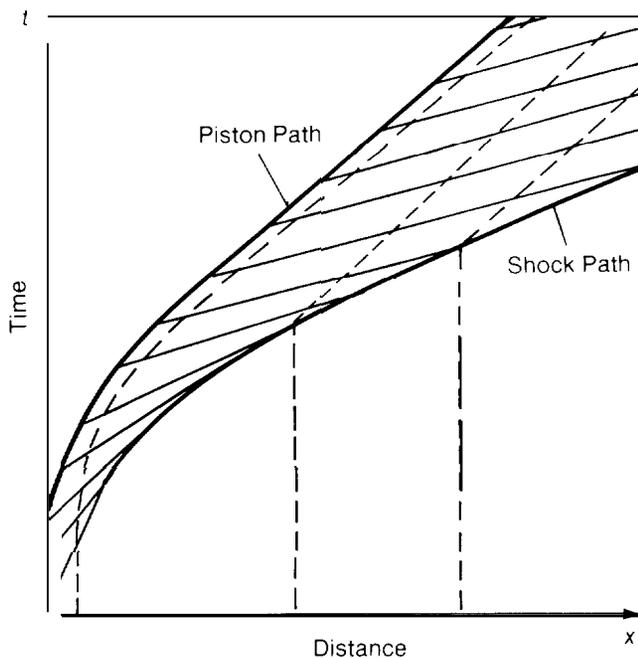
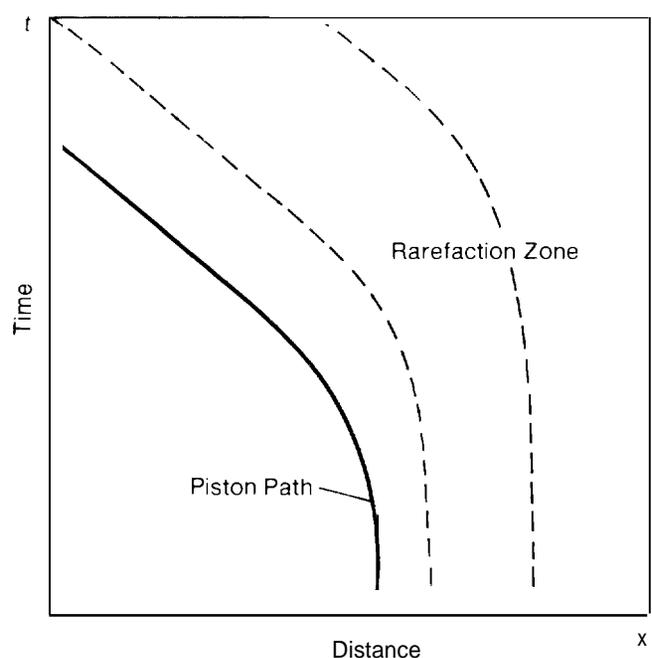


Fig B. Expansion in a Shock Tube



— Characteristic Paths  
— Particle Paths

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characteristics along which information travels from one point to another by sound waves. The so-called Riemann invariants, which are linear combinations of the fluid velocity and sound speed, are conserved along the plus and minus characteristics. In general, characteristics form a curvilinear coordinate system that reduces a complicated set of nonlinear partial differential equations to a coupled set of ordinary differential equations.

Since sonic disturbances travel in the gas at the local sound speed  $c$ , and since the gas immediately ahead of the piston is moving with the piston speed  $v_{piston}(x)$ , the sound waves emanating from each point along the piston path travel at a speed of  $v_{piston}(x) + c$ . Hence at each point  $x$  the characteristic makes the same angle with the piston trajectory. As a result, the characteristics (red) emanating from the curved portion of the piston path (the acceleration phase) converge to form a shock front. (The bow shock of a supersonic projectile also forms where sonic disturbances generated along the trajectory converge, except that in this case the sonic disturbances move through a stationary rather than a moving medium and therefore all travel at speed  $c$ .) The shock front ahead of the piston travels along the tube at supersonic speeds. Gas particles farther away from the tube remain stationary until the shock front passes, and then they travel at a speed equal to the final constant speed of the piston. The closer spacing of the particle paths (dashed lines) gives graphic evidence of shock compression.

Figure B shows the time-dependent flow that results when the piston is suddenly withdrawn from some position within the cylinder. As in Fig. A, characteristics drawn at a constant angle to the piston path represent the path of sonic disturbances generated in the gas immediately adjacent to the piston. In this case the characteristics (blue) emanating from the curved portion of the piston path diverge to form a rarefaction zone (gray). The greater separation of the particle paths in the rarefaction zone indicates that the gas is expanding. ■

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Other data on supersonic flows are available from studies of jet and rocket propulsion. Since these studies were aimed at designing more efficient, higher thrust engines, virtually all of them have focused on the steady-state, or time-averaged, properties of the jet flow, as has our discussion so far.

### Time-Dependent Jets in Nature

The time-dependent evolution of jets becomes important when we consider supersonic "jets" in nature. One example is the supersonic jet produced by the eruption of a gas-rich volcano, such as the eruptions of Krakatoa and Mount St. Helens. The blast wave that accompanies the formation of such a volcanic jet produces widespread devastation. Experiments have shown that these transient supersonic jets have structural properties resembling those of steady-state jets.

Other examples are the numerous jets emanating from astrophysical objects. In our galaxy protostars, old binary star systems associated with supernova remnants, and planetary nebulae produce jets of ionized gases. A number of these stellar jets have knots of bright emission that remind one of the bright spots in the exhaust jet shown in Fig. 1. Spectral lines from highly excited atoms in the knots indicate that these bright spots are regions of high temperature, probably produced by shock heating. The Doppler shifts of the spectral lines demonstrate that these jets are still evolving with time.

Our final example of time-dependent jets, and the ones that exhibit the most remarkable and mysterious stability, are the enormous jets emanating from quasars and other galaxies. These jets, known as radio jets because most of them can be observed only with radio telescopes, range from thousands to millions of light years in length, or about fifty to one hundred jet diameters. Many of them exhibit bright knots of radio emission in the beam. A radio jet typically ends in a "hot spot"—a small region of intense radio

emission—embedded in a larger amorphous lobe of fainter emission (the opening illustration is an exception).

The emission spectra of radio jets, which are continuous and polarized, are produced by synchrotron emission from a nonthermal population of relativistic electrons gyrating in a magnetic field. This nonthermal component probably coexists with a diffuse thermal plasma in some uncertain mixture to form the "stuff" of extragalactic radio sources. We know little else about the composition of the jets and the lobes and, in the absence of spectral lines, have no direct measure of jet speeds from Doppler shifts.

In a landmark paper, written in 1974 when only the radio lobes, and not the jets themselves, had been observed, Roger Blandford and Martin Rees interpreted the hot spots in the lobes as the point of impact of a supersonic gas jet impinging on the ambient medium as the jet advances. [In other words, the radio lobes are powered by fluid-like beams, or jets, originating at the centers of radio galaxies. They suggested that the main structural features of the jets could be explained by ideal gas dynamics, that is, by the nonlinear differential equations of compressible fluid flow. Magnetic fields and complex plasma dynamics that might affect local regions could be ignored when considering the overall structure of a jet,

The fluid-beam hypothesis of Blandford and Rees provides a simple physical explanation for radio observations and a framework for detailed model building, but within this framework the issue of stability becomes paramount. We know from laboratory experiments that Kelvin-Helmholtz shear instabilities grow rapidly on the boundary of supersonic gas jets and effectively decelerate and disrupt the flow. How do extragalactic jets propagate such great distances without suffering the same fate? Are they or are they not, governed primarily by the equations of gas dynamics? If so, what are the critical parameters determining the stability and the observed morphology of the flow? We undertook to answer these questions by

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