Shock Waves versus Sound Waves

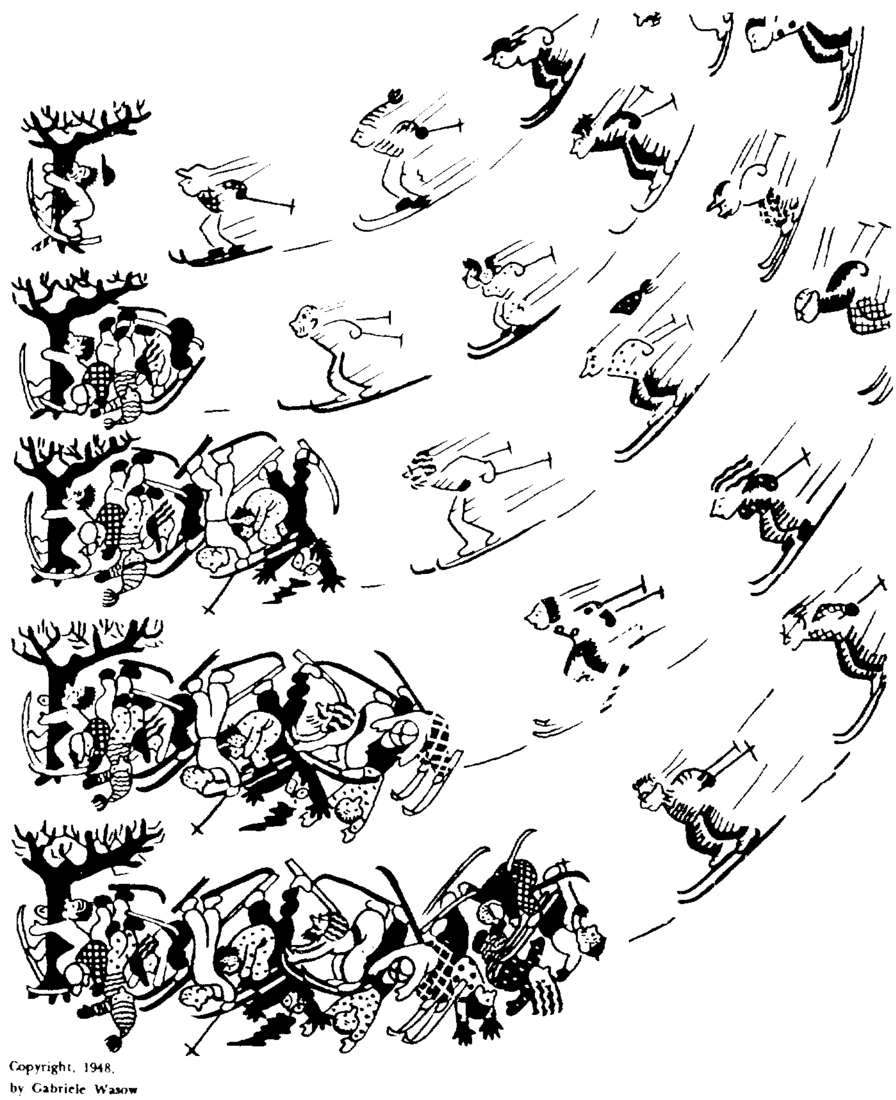
SIDEBAR 1

Explosions, projectiles whizzing by at supersonic speeds, high-speed collisions of solids—what do these phenomena have in common?

They all create very large changes in local pressure over very short times, and these violent pressure changes self-steepen into shock fronts, or shock waves. Unlike a sound wave, which is a small-amplitude compression wave that propagates at the local sound speed and leaves the state of the medium unchanged, a shock front is a nonlinear wave that abruptly changes the state of the supersonically approaching gas. The gas is generally at a higher temperature and pressure after it has passed through the shock (or. equivalently, after the shock has passed through the gas). Moreover, the shock-heated gas moves subsonically with respect to the shock. As we describe below, the narrow region defined as the shock front is a region where thermodynamic processes arc irreversible.

The cartoon from Courant and Friedrichs' classic book on supersonic flow illustrates the formation of a steep front in a discontinuous medium, namely, a train of skiers. The skiers, barreling single file down the narrow run. pile up in a heap as first one skier gets wrapped around a tree, and then, before he can warn the skier behind him to slow down, the next skier crashes into him, and so on. The pileup of human wreckage creates a steep front moving up the slope away from the tree analogous to a receding shock front. As in a continuous medium. formation of the front depends critically on the fact that the flow of skiers is "supersonic" in the sense that it is faster than the speed with which the medium, in this case the skiers, can respond to new boundary conditions. The high "pressure" of the skiers behind this front is analogous to the change of state experienced by shock-processed media.

About a hundred years ago Stokes, Earnshaw, Riemann, Rankine, Hugoniot, and Lord Rayleigh deduced the conditions that prevail at shock fronts in gases. The framework they used to analyze this compressible flow are the equations of ideal gas dynamics. These nonlinear equations describe conservation of mass, momentum, and energy in an ideal fluid, a fluid in which all changes in kinetic and internal energy are due to pressure forces. Heat conduction and viscous stress are ignored and entropy is constant, so all thermodynamic changes are adiabatic and reversible. It was known that, within this context, infinitesimal pressure changes generate linear compression waves, better known as sound waves, that travel with the local sound speed c ($c^2 = (\partial P/\partial \rho)_S$). But what happens when the amplitude of a compression wave is finite?



An example of a receding shock wave. From **Supersonic Flow and Shock Waves** *by R. Courant and K. O. Friedrichs (New York:Interscience Publishers, Inc., 1948),*

Figure A illustrates what happens. At time t_1 we have a finite-amplitude sound wave such that the pressure variation δP is of order P. Variations in pressure imply variations in sound speed, since for an ideal gas $P \propto c^2$. Thus each point on the waveform propagates with its local sound speed, which is greater at the peaks than in the troughs. With time the waveform steepens (as shown at t_2) and eventually breaks (as shown at t_3) to produce multiple values for the state variables of the gas, P, p, and T. Of course this prediction is wrong. Instead nature inserts a shock front (dashed line) just before the wave breaks, and the flow variables remain single-valued.

How does this happen? On a microscopic level large gradients in temperature and velocity at the front of the steepening wave cause the irreversible processes of heat conduction and viscous stress to dominate in a region with a width equal to a few collision mean free paths and to counteract the self-steepening process so that a single-valued shock front forms. The net effect on a macroscopic level is that mass, momentum, and energy are conserved across the shock front, but entropy is not; it increases as relative kinetic energy is dissipated into heat through atomic or molecular collisions.

In 1864 Riemann was the first to analyze wave-steepening within the context of ideal gas dynamics. He mistakenly assumed that entropy was conserved (in other words, that all processes were adiabatic) across shock fronts as it is for finite-amplitude sound waves. Later, Rankine, Rayleigh, and Hugoniot showed that an adiabatic shock front would violate conservation of energy, and therefore shock fronts must be non-adiabatic and irreversible. Figure B shows the irreversible changes caused by the passage of a shock front in contrast to the reversible changes produced by a sound wave. To model these irreversible effects, the form of the Euler equations of gas dynamics we have adopted must be modified as described in Fig. 2 of the main text.

The dissipative nature of a shock front implies that it can maintain itself only in the presence of a driving force. A simple example of a driven shock front is given in Sidebar 2. ■

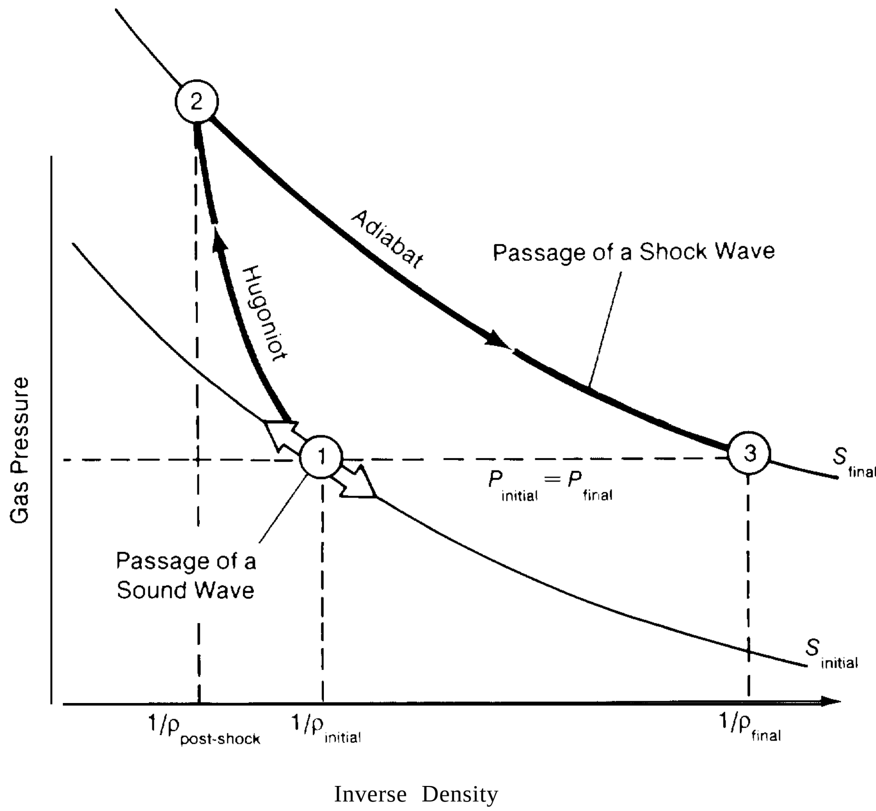


Fig. B, Effects of the passage of a sound wave and of a shock wave. As a sound wave passes through a gas, the pressure and density of the gas oscillates back and forth along an adiabat (a line of constant entropy], which is a reversible path. In contrast, the passage of a shock front causes the state of the gas to jump along an irreversible path from point 1 to point 2, that is, to a higher pressure, density, and entropy. The curve connecting these two points is called a Hugoniot, for it was Hugoniot (and simultaneously Rankine) who derived, from the conservation laws, the jump conditions for the state variables across a shock front, After passage of the shock, the gas relaxes back to point 3 along an adiabat, returning to its original pressure but to a higher temperature and entropy and a lower density. The shock has caused an irreversible change in the gas.

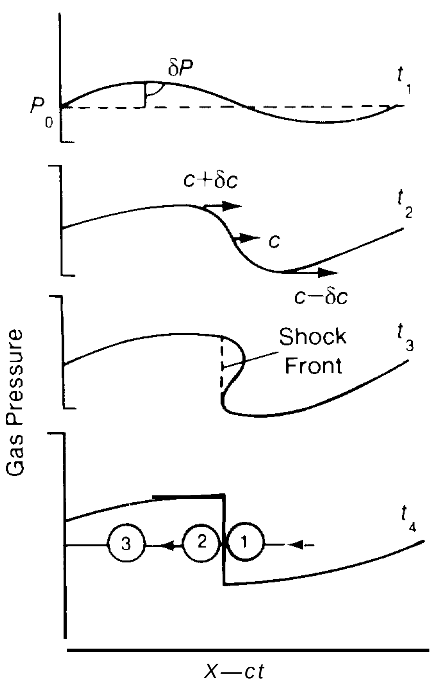


Fig. A. Self-steepening of a finite-amplitude sound wave. In the region where the state variables of the wave (here, pressure) would become multi-valued, irreversible processes dominate to create a steep, single-valued shock front (vertical dashed line).