Redistribution of Angular Momentum in Thick Accretion Disks

by Wojciech H. Zurek and Winy Benz

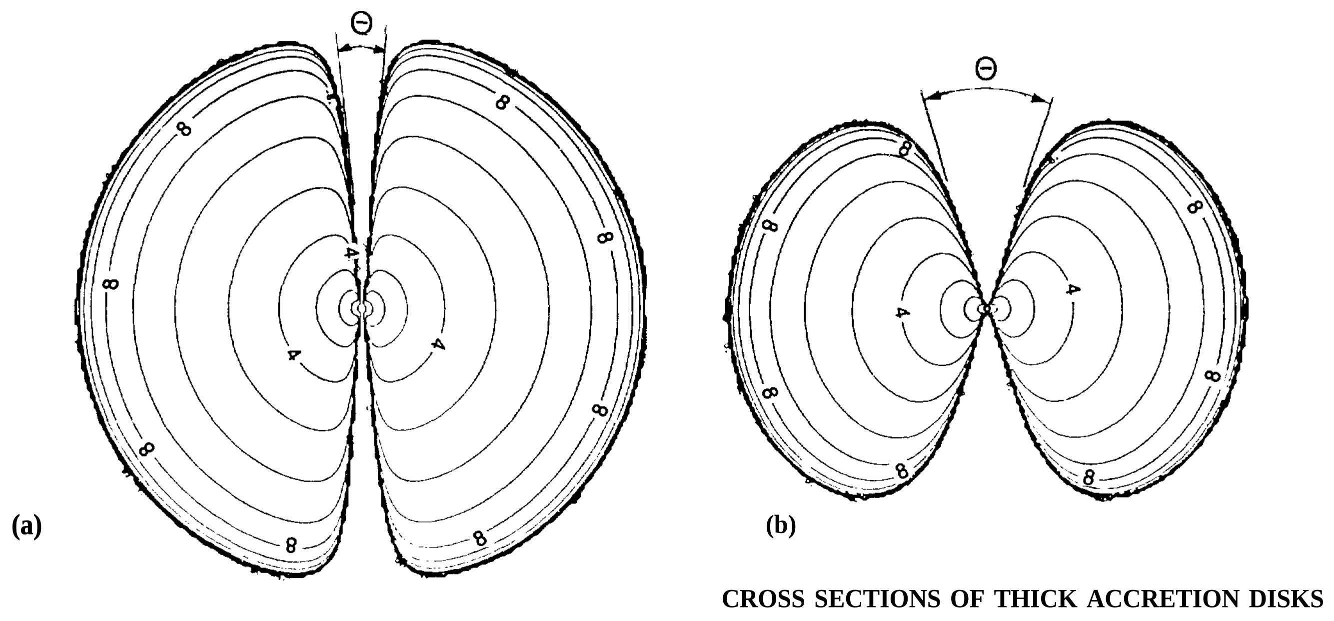

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Fig. 2. Isodensity contours of two tori with approximately the same inner and outer radii, but with different distributions of specific angular momentum: (a) a torus with J_s = constant and (b) a torus with J_s - $r^{0.27}$ Note the change in the funnel opening angle from -10° in (a) to -30° in (b). The density decreases by twelve orders of magnitude from the innermost to the outermost contours.

The importance of the distribution of angular momentum is illustrated in our recent numerical simulations of thick accretion disks. These torus-like disks, in contrast to the flat, pancake-like Keplerian disks, have a height above the equatorial plane comparable to their extent in the radial direction. Until recently the constant specific angular momentum (J_s = constant) variety of such non-Keplerian accretion disks around massive black holes was considered the best model for the central “powerhouse” in quasars and active galactic nuclei. However, doubts about the validity of that hypothesis were raised in 1984 by analyses of the stability of the disks by Papaloizou and Pringle. Our numerical simulations confirm those first suspicions. More important, we were able to demonstrate that growing instabilities in a constant-J, torus rapidly redistribute angular momentum, causing the torus to become thinner and more Keplerian. Hence thick accretion disks with constant J_s cannot be regarded as models for astrophysical objects.

The equilibrium configuration of thick accretion disks, for constant J_s and large pressure forces (sound speeds comparable to rotational velocities), looks like a fat torus, or doughnut (Fig. 1a). The sides of the torus form a funnel, with a small opening angle 6, about the rotation axis. As noticed by Lynden-Bell, this is a perfect shape for a deLaval nozzle,* that is, for accelerating the enormous supersonic jets observed to be emanating from so many active galactic nuclei. Moreover, the steep walls of the funnel allow the luminosity of the torus to exceed the Eddington limit, a property that would be indeed useful in modeling variable quasars with huge power outputs.

The first question to ask is how such non-Keplerian accretion disks can exist, since matter at small radial distances *r* has "too much" angular momentum (more than the Keplerian value) and matter at large *r* has "too little" angular momentum. The pressure in the disk is responsible for maintaining this distribution of angular momentum: the matter at large *r* is being prevented from moving inward by the pressure, and the matter at small *r* is being kept from moving outward by the pressure-mediated weight of the outer parts of the disk.

Proponents of J_s = constant accretion disks assume that the effective turbulent viscosity of such a disk is very small, so that it is all but impossible to transport angular momentum outward as matter accretes toward the massive body at the center of the disk. However, if the central massive body is a black hole (as it is almost certain to be for quasars and active galactic nuclei, including the nucleus at the center of our own Milky Way), then it is possible to get rid of the angular momentum of accreting matter by pushing it into the black hole. The necessary push can be provided by applying pressure from far away and "force-feeding" the black hole with gas.

Using an idealized model, Papaloizou and Pringle challenged the foundations of the thick accretion disk theory by showing that constant- J_s tori are violently unstable against nonaxisymmetric shear-driven perturbations. The instabilities revealed by their linear analysis are Helmholtz instabilities and in some ways are analogous to "fire-hose" instabilities. However, the gas does not stream out in random directions, as would water from a hose left unattended. Instead the gas deflected from its equilibrium orbit by the instability is bound by the gravitational potential and so produces density inhomogeneities, pressure gradients, and sound waves, which, in turn, produce more deflections, which lead to more sound waves.

In a second paper Papaloizou and Pringle extended the stability analysis to include very thin tori (like slender bicycle tires) with J_s varying as r^q. For $q < 2 - \sqrt{3} \cong 0.2679$, the tori were found to be unstable. For greater values of q a large class of unstable modes is stabilized. Their linear analysis did not, however, reveal the ultimate fate of the original configuration.

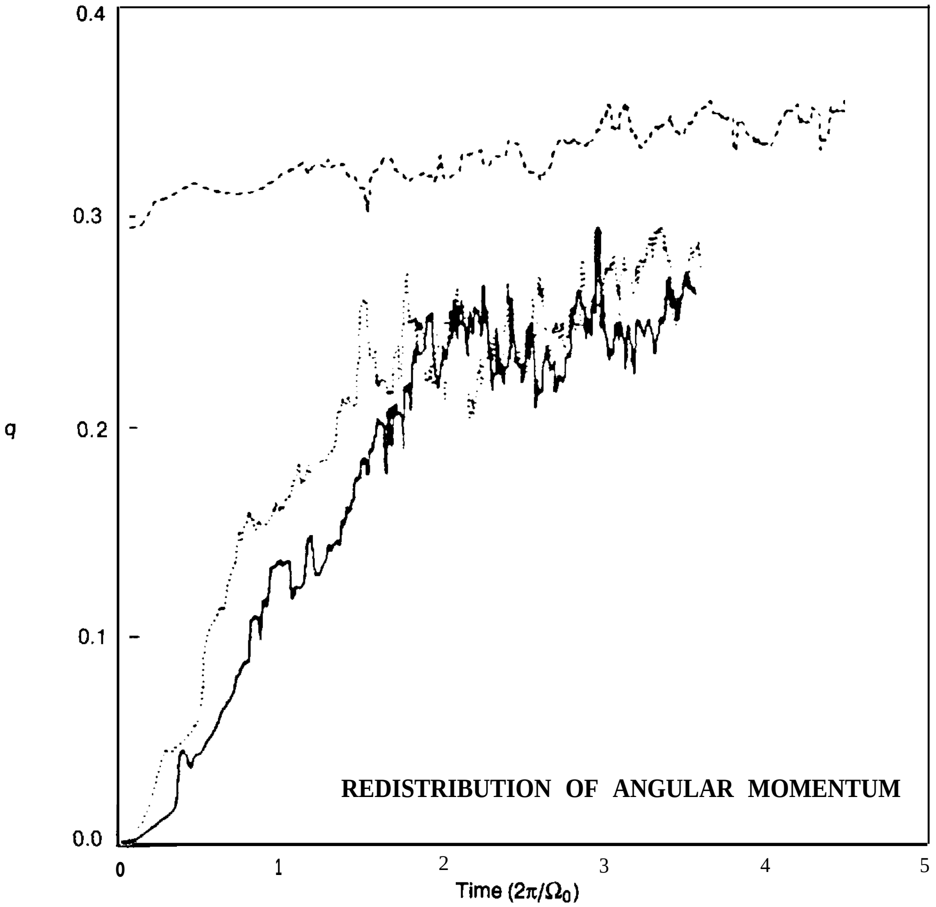


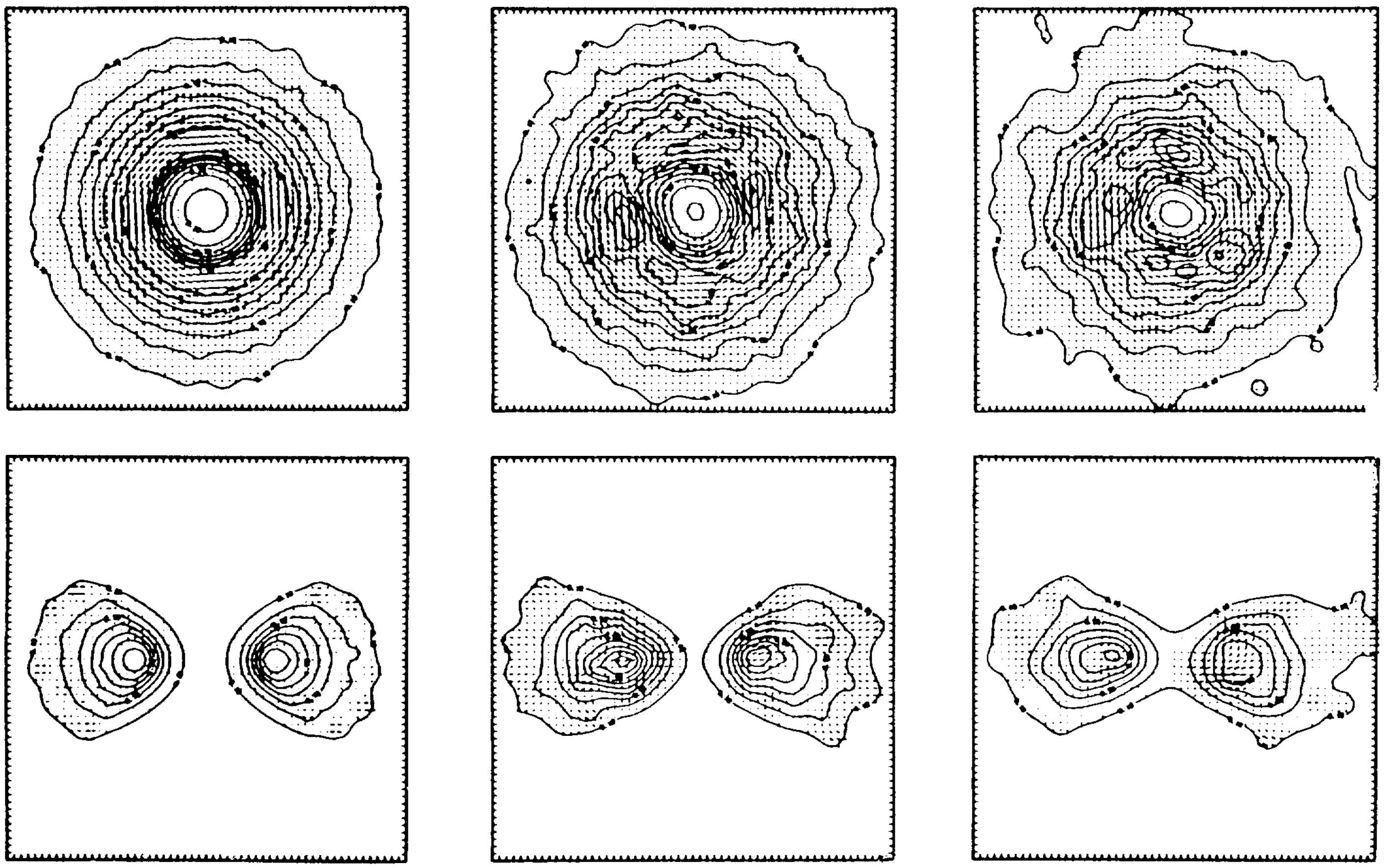
Fig. 2, The exponent q as a function of time, where q is calculated from a power-law fit ($J_s \sim r^q$) to the specific angular momentum distribution obtained from the numerical simulation. The results are shown for three simulations. Note that, for the two disks with initially constant specific angular momentum, q increases rapidly from O to about 0.27 within about two rotation periods. After the critical q = q_c-0.27 is reached, the redistribution of angular momentum slows down to the rate observed in a disk with initial J_s-$r^{0.3}$, It is not yet known whether this slow rate of angular momentum transport is caused in part by nonaxisymmetric instabilities or is totally explained by a numerical viscosity that is an unavoidable artifact of such calculations. We are planning to study this problem further.

**The action of a deLaval nozzle is described by M. L. Norman and K.-H. A. Winkler in "Supersonic Jets," Los Alamos Science Number 12, 1985.*

We have extended such stability analyses to the nonlinear regime by adding a small random density perturbation (of the order of 1 percent) to an initial equilibrium configuration with constant J_s (see Fig. 3, t = 0). These numerical experiments not only confirm that such disks are unstable but also show that a fat accretion torus is forced to undergo a "crash diet": instabilities redistribute the angular momentum very quickly (on the time scale of about a rotation period J_s) from J_s = constant to J_s - r^{q_c}, where q_c turns out invariably to be 0.27 (Fig. 2). Note that this value for the

Fig. 3. A computer-generated time sequence showing the three-dimensional evolution of the central region of a thick accretion disk with initially constant specific angular momentum. The upper panels show isodensity contours in the equatorial plane of the central region; the lower panels show isodensity contours in a plane parallel to the rotation axis. The density decreases by one to two orders of magnitude over the region shown. Time is expressed in rotation periods of the density maximum. The velocity field is indicated by means of arrows whose lengths are normalized to the maximum value of the velocity in each frame. Following the introduction of a small nonaxisymmetric perturbation, the growth of instabilities causes a rapid redistribution of angular momentum that, in turn, flattens the disk and fills in the central "hole." This simulation was made with a three-dimensional hydrodynamics code that uses the so-called smoothed particle hydrodynamics method (Lucy 1977). This free Lagrangian approach to solving the usual equations of hydrodynamics replaces the continuum by a finite set of spatially extended particles. Thus no mesh is required, and the usual problems as-

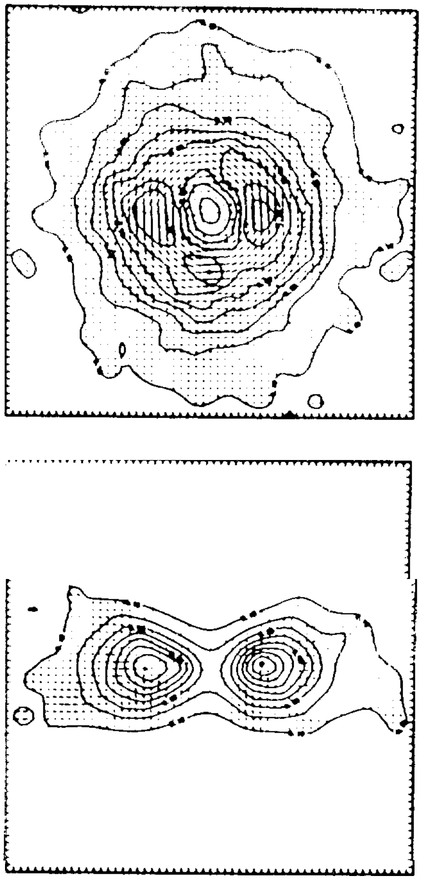
TIME EVOLUTION OF A THICK DISK



$t = o$ $t = 1$ $t = 2$

exponent is about the same as that obtained by Papaloizou and Pringle for the stabilization of "bicycle tire" tori. Figure 3 shows the time evolution of the disk.

sociated with its rezoning are bypassed. The simulation is made by computing the trajectories of 1000 extended particles that interact through pressure forces in a central gravitational potential. Since the particles are allowed to move without any constraints in all three spatial directions and since a mesh is not needed, this method is particularly suited for the simulation of highly distorted flows.



$t = 3$

As the angular momentum is redistributed, the fat torus becomes much thinner and much more Keplerian in appearance. Moreover, the narrow funnel invoked to explain the formation and collimation of relativistic jets becomes much wider (Fig. 1 b) and therefore less effective in producing collimated jets and super-Eddington luminosities.

Regarding the question of angular momentum transport discussed in the main text, our calculations show that, at least for $q < q_c$, shear-driven instabilities provide a powerful source of the "rub," that is, of *a*, the turbulent viscosity. The next obvious question—not addressed properly by the calculations performed to date—is whether the shear-driven instabilities will provide a mechanism for *a* when $q > q_c$. Can these instabilities generate wave-like excitations and "interesting" *a* values in disks that are "barely" stable ($J_s = r^{q_c}$) or almost Keplerian ($J_s = r^{05}$)? We are now exploring this question with one of the new three-dimensional hydrodynamics codes developed at Los Alamos. ■

Further Reading

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