The Evidence for Oscillations

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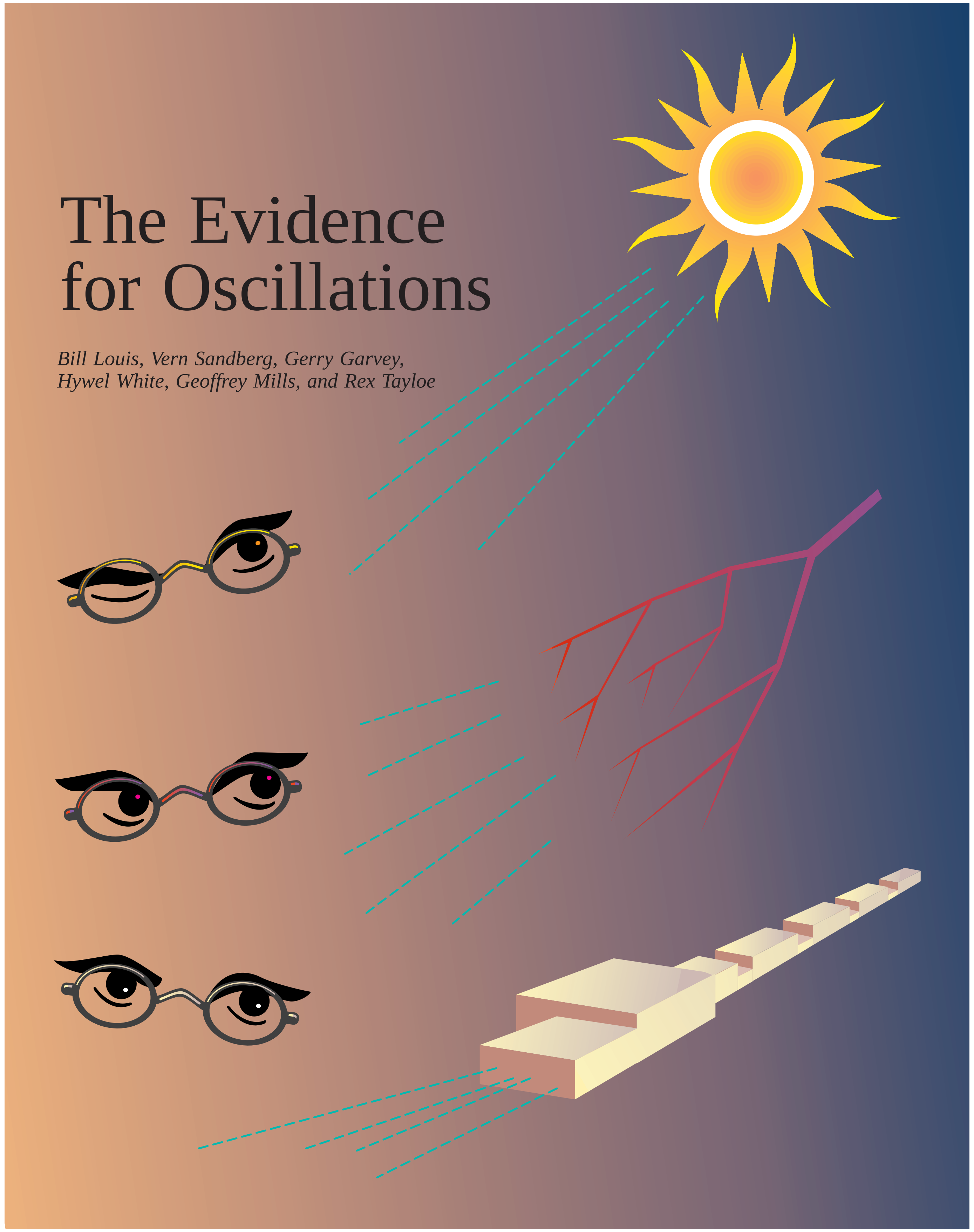


In the Standard Model of particle physics, the masses of quarks and the mixing between quark mass states are well known. For leptons, however, the neutrino masses are unknown (and it is even questionable whether neutrinos have mass). If anything, neutrinos weigh very little—current mass limits are $\nu_e < 10$ electron volts[1] (eV), $\nu_\mu < 170$ kilo-electron-volts (keV), $\nu_\tau < 24$ million electron volts (MeV)—but exactly how little is something physicists would like to determine. A neutrino mass of only a few electron volts, for example, would likely affect cosmology and possibly affect the evolution of the universe.

Neutrino oscillations offer one of the best ways to measure small neutrino masses and mixings. (For the purists, oscillations also represent a beautiful example of quantum mechanics.) Oscillations refer to a periodic changing of one neutrino type into another, a phenomena that can occur only if neutrinos have mass. In that case, neutrinos would be described by three states ν_1, ν_2, and ν_3 with masses m_1, m_2, and m_3, respectively. If our understanding of the quarks is to guide our thinking about the leptons, however, those mass states would be different from the states associated with weak decays (the flavor states ν_e, ν_μ, and ν_τ). The flavor states would most likely be mixtures of the mass states. (Mass and mixing are discussed in detail in the primer, "The Oscillating Neutrino," on page 28.)

Consider, for example, a model in which only two neutrino types mix together (two-generation mixing). The electron neutrino and the muon neutrino are conventionally described as a combination of ν_1 and ν_2:

$$\begin{aligned} \nu_e &= \cos\theta\, \nu_1 + \sin\theta\, \nu_2 \\ \nu_\mu &= -\sin\theta\, \nu_1 + \cos\theta\, \nu_2 \ . \end{aligned}$$

The angle θ is called the *mixing angle*. It is an arbitrary parameter that can be determined only by experiment. Note that if θ is small, there is an approximate one-to-one correspondence between the flavor states and the mass states, that is, $\nu_e \approx \nu_1$ and $\nu_\mu \approx \nu_2$.

The fascinating aspect about mixing is what it implies for the neutrinos. Once born, a muon neutrino has some probability of being detected as an electron neutrino. That probability depends on the distance x that the muon neutrino has traveled and is given by the expression

$$P(\nu_\mu \to \nu_e) = \sin^2 2\theta \sin^2\left(\frac{1.27\Delta m^2 x}{E_\nu}\right) \ ,$$

where $\Delta m^2 = m^2{}_2 - m^2{}_1$ (the difference of the squares of the neutrino masses) in electron volts squared (eV^2), E_ν is the neutrino energy in million electron volts, and x is measured in meters. The expression is essentially that of a sinusoidal wave,

$$P(\nu_\mu \to \nu_e) = A \sin^2\left(\frac{\pi x}{\lambda}\right) \ ,$$

with the amplitude of the wave given by $A = \sin^2 2\theta$, and the wavelength, λ, which is also called the oscillation length, given by $\lambda = \pi E_\nu / 1.27\Delta m^2$. The mass difference Δm^2 can be determined from the oscillation wavelength, while the mixing angle θ is deduced from the wave's amplitude.

There is every reason to believe, however, that each flavor neutrino would be a mixture of all three mass states (three-generation mixing). The formalism for three-state mixing is a little more complex and yields an expression for the oscillation probability that is similarly more complex than the simple expression given above. For example, instead of a single parameter θ characterizing the mixing, there are three independent parameters. Likewise, there are three mass differences, (although only two are independent). Fortunately, if the mass scales are quite

[1]The limit on electron neutrino mass is controversial; see R. M. Barnett et al., *Physical Review D* **54**, 1 (1996) for a complete discussion.

Table I. Solar-Neutrino Data: Contributions to the Detected Signal from Each Solar-Neutrino-Producing Reaction (Expressed as a Fraction of the Total Signal) and the Ratio of the Measured Rate to the Predicted Rate

	SAGE and GALLEX	Chlorine	Kamiokande
Reaction			
pp	0.538		
^{7}Be I	0.009		
^{7}Be II	0.264	0.150	
^{8}B	0.105	0.775	1
pep	0.024	0.025	
CNO	0.060	0.041	
Ratio*	0.62 ± 0.1	0.29 ± 0.03	0.29 ± 0.03

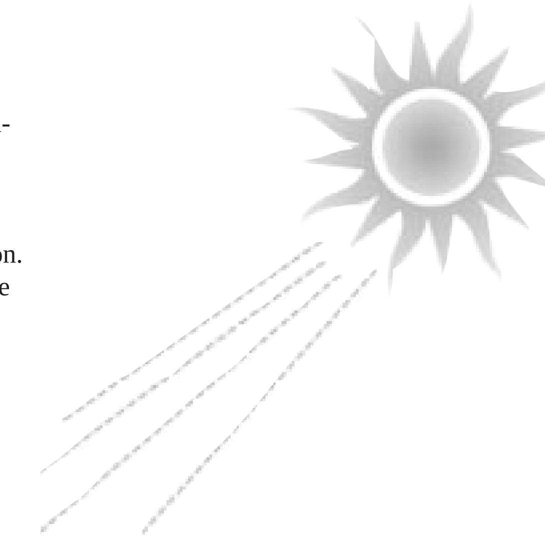
*Predicted rate based on the Bahcall-Pinsonneault standard solar model.

different ($m_3 >> m_2 >> m_1$, for example), then the two-generation mixing model is a good approximation to the three-generation model. Therefore, the data from any one experiment is often analyzed in terms of the two-generation oscillation parameters Δm^2 and $\sin^2 2\theta$. It should be kept in mind, however, that such an analysis is likely an oversimplification of the physics, and different experiments may indicate very different mass differences and mixing angles for the same type of oscillation.

The potential for a given neutrino type to appear or disappear suggests two ways to perform neutrino oscillation experiments: either one looks for a *decrease* in the flux of the original neutrino type (known as a disappearance experiment), or one looks for the presence of a neutrino type other than the original one (an appearance experiment). Ideally, one also measures the neutrino flux as a function of distance, so that periodic variations of the flux become apparent.

Because neutrino detectors are so large, moving one back and forth and from place to place is essentially impossible. Instead, "single-point" measurements are made at a fixed distance from the source, which has the unpleasant consequence of making the search for oscillations a shot in the dark. The ideal distance between a neutrino source and detector would be half of the oscillation length, but that length cannot be calculated in advance because the mass difference Δm^2 is unknown. If the detector happens to be placed well within a quarter of an oscillation length from the source, most neutrinos will be in their original state and no effect will be seen. Likewise, if the detector happens to be many oscillation lengths away, then the unavoidable spread in the momenta of the original neutrinos will wash out any oscillation effects. An experiment is sensitive to only a range of oscillation lengths and will probe only a limited set of possible values of the neutrino mass difference and mixing angles.

Despite these limitations, more than a dozen experiments, spanning three decades of research, have been mounted in an effort to observe the chance metamorphosis of one neutrino type into another. To date, there is no smoking gun in the experimental evidence, only tantalizing hints that such a startling transformation does indeed occur. Without conclusive proof, physicists argue vehemently over whether neutrino oscillations have been observed. The debate is entirely appropriate. On an issue as important as the initial evidence for mixing in the lepton sector and neutrino mass, the scientific community requires convincing proof before accepting any claims. One facet to the debate that should be emphasized is that the evidence for oscillations comes from three different neutrino sources: neutrinos born in the core of the Sun, neutrinos created in the earth's upper atmosphere, and neutrinos that emanate from man-made sources (accelerators and reactors). Each of these sources provides different fluxes, energies, and types of neutrinos. They also have different sources of errors. Disappearance experiments that look at solar or atmospheric neutrinos have all indicated a reduced neutrino flux, but the reduction is only relative to a theoretical prediction. Because the neutrinos being observed are created by natural processes and hence are not completely characterized, physicists cannot rule out that the apparent reduction in flux is merely a theoretical miscalculation. The neutrinos streaming out of man-made sources are better characterized, but the experiments must contend with much greater backgrounds. At present, only one appearance experiment—the accelerator-based liquid scintillator neutrino detector (LSND) experiment—has seen evidence for oscillations.



Experimental Evidence

Evidence from Solar Neutrinos. Since the first observation of electron neutrino interactions in a chlorine-laden tank by Ray Davis, Jr., and collaborators, three additional experiments have measured solar-neutrino interactions. Two of those, SAGE and GALLEX, used gallium as a neutrino target, while the third, Kamiokande, used a water-Cerenkov detector in which neutrinos undergo elastic scattering with electrons in the water. All of the experiments have determined that there are fewer neutrinos from the Sun than are expected from the Standard Solar Model (see the article "Exorcising Ghosts" on page 136.)

The four experiments are sensitive to different parts of the solar-neutrino spectrum. The two gallium experiments have the lowest energy threshold (0.233 MeV) and are sensitive to the entire solar-neutrino flux. The chlorine experiment is sensitive to neutrinos with energies greater than 0.8 MeV, while Kamiokande is limited to detecting neutrinos with energies greater than about 7 MeV. The sensitivity of each experiment to the Sun's neutrino-producing reactions and the experimental results are listed in Table I. It is worth emphasizing that all the data presented in the table were gathered over an extended period of time and that each of the experiments has undergone numerous systematic checks. The solar models used to predict the neutrino flux are very much constrained by measured physical parameters, such as the solar luminosity, and like the experiments themselves, have been laboriously tested and refined over the years.

In a 1994 paper, Hata and Langacker (1994) presented a thorough analysis of all of the solar-neutrino data. They considered experimental errors in detail as well as possible variations to the standard solar model. They concluded that the experimental data cannot be explained by variations in solar physics and that neutrino oscillations are strongly favored. Furthermore, the most promising solution is a matter-enhanced, resonant transformation of electron neutrinos to other flavors through the MSW effect (see the article "MSW" on page 156). Given the range of densities in the Sun, the MSW effect could occur if the oscillation length λ is in the range of 10^4 to 10^8 meters. MSW leads to the allowed regions in the Δm^2 and $\sin^2 2\theta$ parameter space shown in Figure 1. The better fit to the data is obtained with a mass difference (Δm^2) of $\approx 10^{-5}$ eV^2 and the smaller mixing angle leading to a $\sin^2 2\theta$ value of $\approx 3 \times 10^{-3}$.

Overall, the experimental evidence that solar neutrinos may undergo oscillations appears firm, although uncertainties in solar dynamics are still a cause for concern.

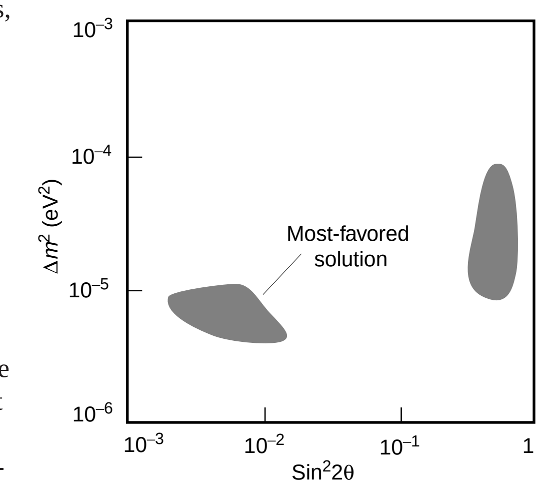


Figure 1. MSW Solutions to the Solar-Neutrino Problem
Given oscillations between only two neutrino mass states (two-generation mixing), a plot can be constructed that shows the values of Δm^2 and $\sin^2 2\theta$ that yield an oscillation probability consistent with the solar-neutrino data. The MSW effect can occur within a continuous range of mass differences and mixing angles, but the four experiments exclude certain values. Two regions (the MSW solutions) are allowed by all four experiments. The region to the left, with the smaller mixing angles, is the one most favored by theorists.

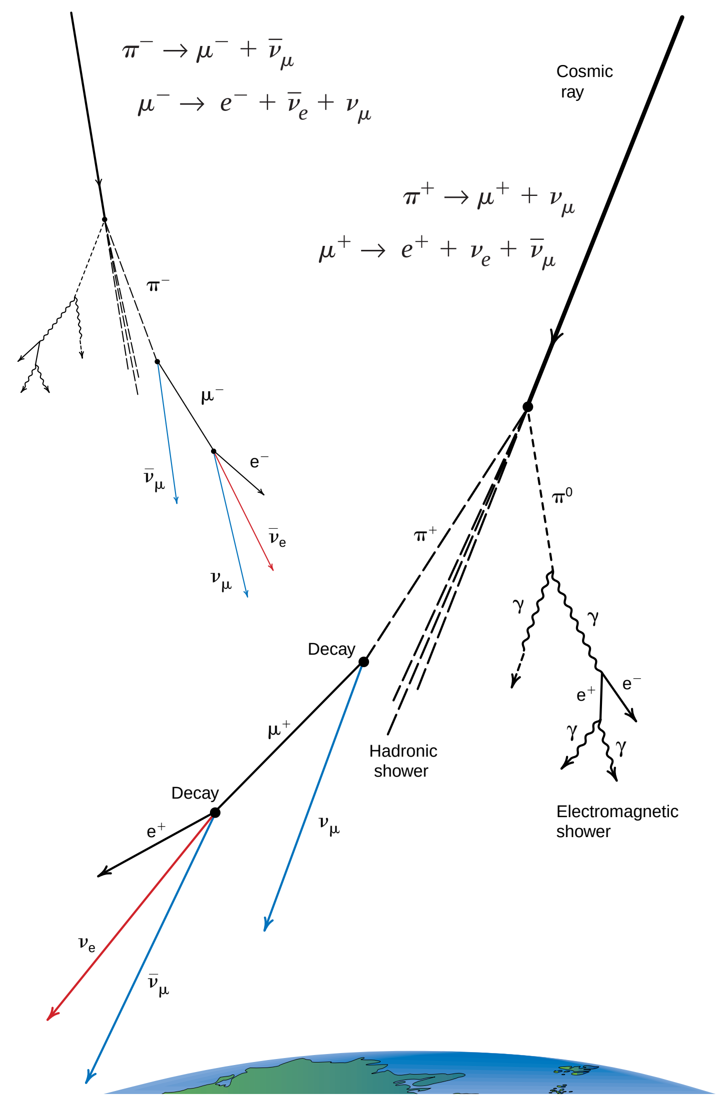


Figure 2. The Atmospheric-Neutrino Source
Collisions between cosmic rays and nuclei in the upper atmosphere can create high-energy pions (π). In the collision shown on the right, a π^+, π^0, and other heavy particles (the hadronic shower) are created. The π^0 decays and produces gamma rays and leptons the electromagnetic shower) but no neutrinos. The π^+ produces two muon neutrinos (blue) and an electron neutrino (red). The collision shown on he left produces a π^-, leading to the production of two muon neutrinos and an electron antineutrino.

The result of the Kamiokande experiment will be tested in the near future by super-Kamiokande, which will have significantly better statistical precision. Also, the neutrino oscillation hypothesis and the MSW solution will be tested by the Sudbury Neutrino Observatory (SNO) experiment, which will measure both charged- and neutral-current solar-neutrino interactions.

Evidence from Atmospheric Neutrinos. Upon reaching the earth, high-energy cosmic rays collide violently with nuclei present in the rarefied gas of the earth's upper atmosphere. As a result, a large number of pions—π^-, π^0, and π^+—are produced (see Figure 2). These particles eventually decay into either electrons or positrons and various types of neutrinos and antineutrinos. (A large number of kaons are also produced by cosmic rays in the upper atmosphere, and these particles also eventually decay into various leptons.) As seen in Figure 2, the decay of either positive or negative pions results in the eventual production of two muon neutrinos (ν_μ and $\bar{\nu}_\mu$) but only one electron neutrino (either ν_e or $\bar{\nu}_e$). Experimenters, therefore, expect to measure two muon neutrinos for each electron neutrino.

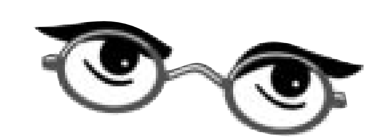
Atmospheric neutrinos are orders of magnitude less abundant than solar neutrinos, but can be readily detected because they have very high energies.

Table II. Results from the Atmospheric Neutrino Experiments

Experiment	Exposure (kiloton-year)	*R*
IMB I	3.8	0.68 ± 0.08
Kamiokande Ring	7.7	0.60 ± 0.06
Kamiokande Decay	–	0.69 ± 0.06
IMB III Ring	7.7	0.54 ± 0.05
IMB III Decay	–	0.64 ± 0.07
Frejus Contained	2.0	0.87 ± 0.13
Soudan	1.0	0.64 ± 0.19
NUSEX	0.5	0.99 ± 0.29

(The neutrino interaction cross sections, and hence the neutrino detection probability, increases dramatically with energy.) Depending on the energy of the incident cosmic ray and how its energy is shared among the fragments of the initial reaction, neutrino energies can range from hundreds of millions of electron volts to about 100 giga-electron-volts (GeV). (In comparison, the highest-energy solar neutrino comes from the ^{8}B reaction, with a maximum energy of about 15 MeV.)

Muon neutrinos produce muons in the detector, and electron neutrinos produce electrons, so that the detector signals can be analyzed to distinguish muon events from electron events. Because the sensitivity of the detectors to electrons and muons varies over the observed energy range, the experiments depend on a Monte Carlo simulation to determine the relative detection efficiencies. Experimental results, therefore, are reported as a "ratio of ratios"—the ratio of observed muon neutrino to electron neutrino events divided by the ratio of muon neutrino to electron neutrino events as derived from a simulation:



$$R = \frac{(\nu_\mu/\nu_e)_{\text{observed}}}{(\nu_\mu/\nu_e)_{\text{simulation}}}$$

If the measured results agree with the theoretical predictions, $R = 1$.

A recent summary of the experimental data is given by Gaisser and Goodman (1994) and shown in Table II. For most of the experiments, R is significantly less than 1: the mean value is about 0.65. (In the table, the Kamiokande and IMB III experiments identify muons in two ways. The first involves identification of the Cerenkov ring, which is significantly different for electrons and muons. The second involves searching for the energetic electron that is the signature for muons that have stopped in the water detector and decayed. A consistent value of R is obtained using either method.) Despite lingering questions concerning the simulations and some systematic effects, the experimenters and many other physicists believe that the observed values for R are suppressed by about 35 percent.

The Kamiokande group has also reported what is known as a zenith-angle dependence to the apparent atmospheric-neutrino deficit. Restricting the data to neutrinos that come from directly over the detector (a zenith angle of 0 degrees and a distance of about 30 kilometers) yields $R < 1.3$ (that is, more muon to electron neutrino events are observed than predicted by theory). Neutrinos that are born closer to the horizon (a zenith angle of 90 degrees) and have to travel a greater distance to reach the detector result in $R < 0.5$. Finally, neutrinos that have to travel through the earth to reach the detector (roughly 12,000 kilometers) result in an even lower value for R. The apparent

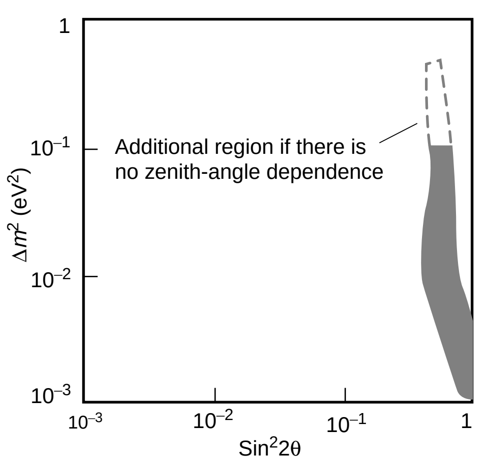


Figure 3. Allowed Δm^2 and Sin$^2 2\theta$ Region for Atmospheric Neutrinos

he data for the atmospheric-neutrino xperiments is best explained with arge mixing angles, and to be onsistent with other experiments, the arameter space is limited to the egion shown in gray. The disputed enith-angle dependence of the data mits the values of Δm^2 to less than .1 eV^2. If the zenith-angle dependence an be discounted, then mass differ-nces as high as 0.5 eV^2 are allowed. ew data from the next generation of tmospheric-neutrino experiments hould settle the question of zenith-ngle dependence.

zenith-angle dependence shows up only for neutrinos with energies greater than 1.3 GeV.

This single piece of evidence has had a significant impact on the allowed region of Δm^2 and $\sin^2 2\theta$ (see Figure 3). The fact that little disappearance effect is observed for a zenith angle of ~0 degrees means that the oscillation length is much greater than 30 kilometers, so that

$$\frac{\pi E_\nu}{1.27\,\Delta m^2} \gg 30 \quad .$$

With $E_\nu \approx 6$ GeV, one finds that $\Delta m^2 \ll 0.5$ eV^2. Given this small value for Δm^2, neutrinos emerging from some high-energy accelerators would have oscillation lengths on the order of hundreds of kilometers. A number of proposals have suggested placing huge neutrino detectors at comparable distances from an accelerator in an effort to investigate $\nu_\mu \rightarrow \nu_e$ oscillations.

However, the statistical significance of the reported zenith-angle dependence is not large. Moreover, a preprint from the Irvine-Michigan-Brookhaven (IMB) collaboration reports no such dependence, and early data from the experiment that has succeeded Kamiokande—super-Kamiokande—is consistent with only a slight zenith-angle dependence. If the zenith-angle dependence disappears, then the atmospheric data is consistent with $\Delta m^2 > 0.15$ eV^2 and an oscillation length on the order of 20 kilometers. This value of Δm^2 is compatible with the LSND observation discussed below.

Evidence from Accelerator-Produced Neutrinos. To date, LSND at the Los Alamos Neutron Science Center (LANSCE) is the only accelerator experiment to have evidence for neutrino oscillations. The experiment uses a detector that contains 167 metric tons of dilute liquid scintillator placed 30 meters from the beam stop for the LANSCE proton beam. Neutrinos are produced from the decay of positive pions that come to rest in the beam stop:

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \quad \text{and} \quad \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \quad .$$

No electron antineutrinos are produced in this reaction chain. Thus, LSND seeks evidence for $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations by looking for electron antineutrino interactions in the detector. The charged-current weak interaction of electron antineutrinos with protons results in the creation of a positron and a free neutron:

$$\bar{\nu}_e + p \rightarrow e^+ + n \; .$$

The positron instantly streaks through the detector and produces both Cerenkov and scintillation light. The neutron, after a mean lifetime of 186 microseconds, is captured by a proton to form deuterium, D, and a 2.2-MeV gamma ray is produced:

$$n + p \rightarrow \mathrm{D} + \gamma \; .$$

The gamma ray also creates light in the detector. The signature for the electron antineutrino event is the correlation of the positron's Cerenkov and scintillation light with scintillation light produced by the 2.2-MeV gamma ray. Because of the low energy of the LANSCE beam (800 MeV), the neutrino backgrounds in LSND are quite small and well understood. The largest background is from electron antineutrinos that are produced when negative muons decay at rest in the beam stop. This decay channel, however, is suppressed by a factor of 7×10^{-4}

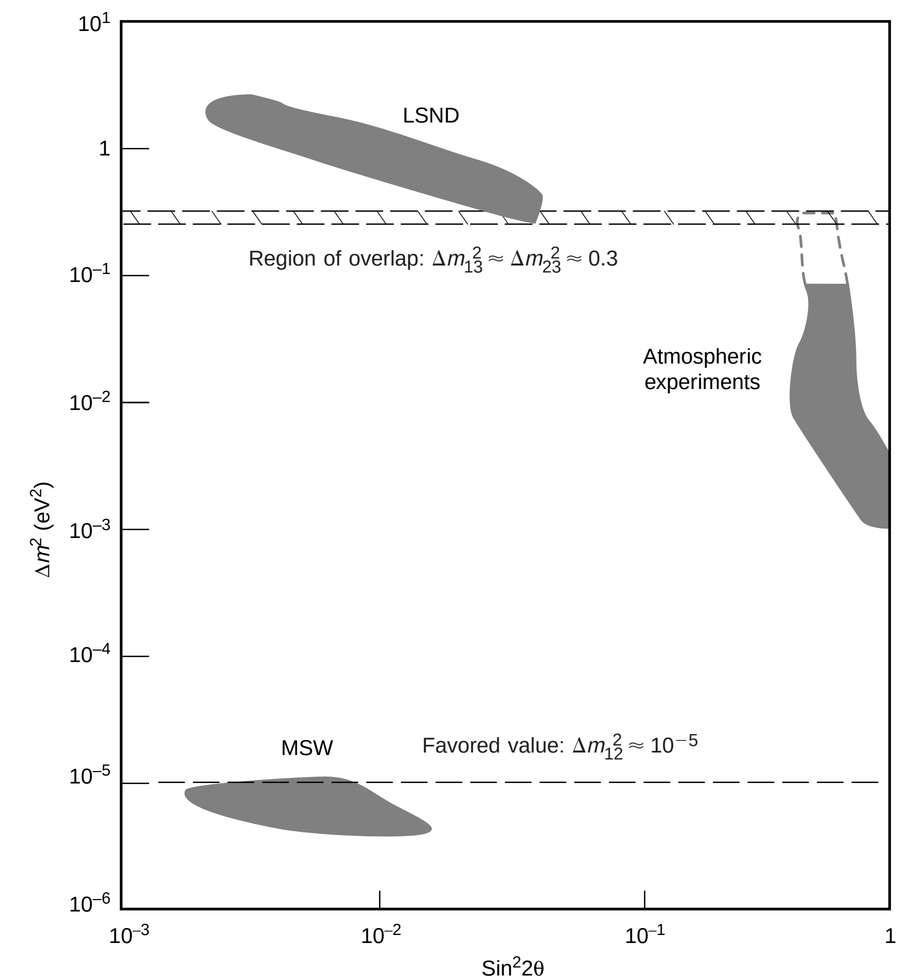


Figure 4. Allowed Regions for All Three Neutrino Sources

There are a minimum of two mass difference scales: one associated with the MSW solution to the solar-neutrino experiments ($\Delta m^2 \approx 10^{-5}$ eV^2) and the other associated with the atmospheric and LSND experiments ($\Delta m^2 \approx 10^{-1}$ eV^2), assuming no zenith-angle dependence. A three-generation mixing model is needed to explain all the data.

relative to the positive muons that decay at rest. The complete story of the LSND experiment is told in the article "A Thousand Eyes" on page 92.

The experiment has reported evidence for $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations by observing an excess of 22 electron antineutrino events above background. This number corresponds to an oscillation probability of $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \approx 0.3$ percent, with the Δm^2 and $\sin^2 2\theta$ parameter region shown in Figure 4. For comparison, the figure also shows the regions of Δm^2 and $\sin^2 2\theta$ allowed by the solar- and atmospheric-neutrino experiments.

The evidence for neutrino oscillations from LSND is strengthened by results from a complementary $\nu_\mu \rightarrow \nu_e$ oscillation search using the same detector and source. In the search, experimenters have observed an excess of 19 electron neutrino events above background. This "second" experiment (the two neutrino searches are actually performed simultaneously) has completely different systematic errors and backgrounds from those of the $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillation search. The second set of neutrinos, which come from pions that decay in flight, have higher energies than those produced by muons that decay at rest. Thus, it is interesting that the decay-in-flight analysis shows a signal, although of lesser significance, that indicates the same favored regions of Δm^2 and $\sin^2 2\theta$ as the decay-at-rest analysis. A comprehensive analysis of the decay-in-flight and decay-at-rest data is in progress.

Theoretical Interpretation of the Data

If one ignores the zenith-angle dependence of the atmospheric neutrinos, there appear to be two distinct mass differences implied by the data. As seen in Figure 4, one is a "small" mass difference associated with the solar neutrinos ($\Delta m^2 \approx 10^{-5}$ eV^2); the other is a "large" difference associated with the atmospheric and LSND experiments ($\Delta m^2 \approx 10^{-1}$ eV^2). The data is particularly puzzling with regard to the solar and LSND results. Both experiments presume to be observing oscillations between electron and muon neutrinos, but it takes very disparate mass differences to explain their respective data sets. It is therefore natural to ask whether any consistent picture can be made of all the experimental results. (If the zenith-angle dependence is shown to be valid, then there are three distinct mass differences and the answer is no: the data cannot be explained by any consistent oscillation formalism involving only three neutrinos.)

As stated at the beginning of this article, analysis of the data in terms of a two-generation mixing model may be an oversimplification of the physics. In that model, only two neutrino types are considered. Rewriting the equations for electron neutrinos and muon neutrinos in matrix form yields

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} \\ -\sin\theta_{12} & \cos\theta_{12} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} .$$

The 2×2 mixing matrix contains only sines and cosines and depends on a single parameter, θ_{12}. (The subscripts have been added to emphasize that θ_{12} characterizes the mixing between states ν_1 and ν_2.)

Mixing between three neutrino generations means that not only are there three mass differences, Δm^2_{12}, Δm^2_{13}, and Δm^2_{23}, where $\Delta m^2_{ij} = m^2_j - m^2_i$, but there are also three independent mixing parameters. A simple 3×3 mixing matrix U can be constructed by taking the product of three unitary matrices.[2]

$$U = U_{12} U_{13} U_{23} ,$$

where

$$U_{12} = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} ,$$

$$U_{13} = \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13} \\ 0 & 1 & 0 \\ -\sin\theta_{13} & 0 & \cos\theta_{13} \end{pmatrix} , \text{ and}$$

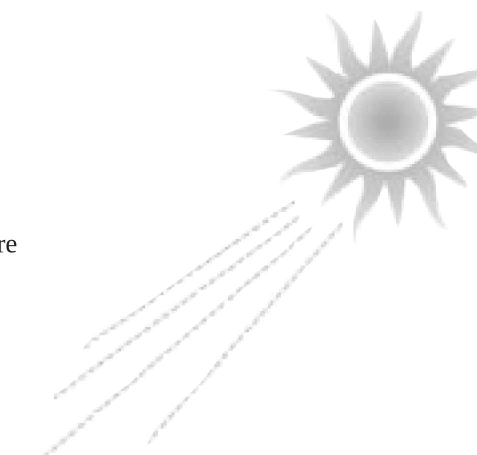
$$U_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} .$$

For convenience, the matrix U will be written as

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} .$$

The elements $U_{\alpha i}$, where $\alpha = e, \mu, \tau$ and $i = 1, 2, 3$, depend only on the products of the sines and cosines of the mixing angles θ_{12}, θ_{13}, and θ_{23}. Mixing between the three neutrino generations takes the form

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} .$$



This formalism is analogous to the quark sector, where strong and weak states are not identical and the resultant mixing is described conventionally by a unitary mixing matrix (the Cabbibo-Kobayashi-Maskawa matrix).

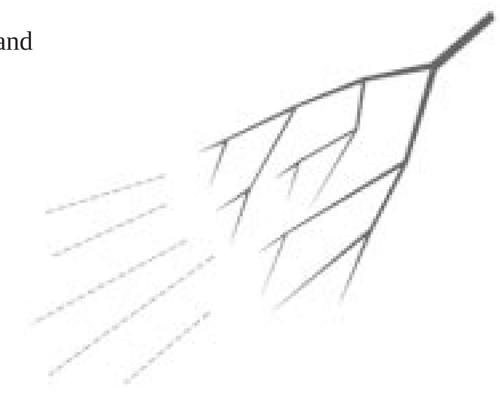
Given the arbitrary mixing matrix above, the oscillation probability is

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{j>i} U_{\alpha i}\, U_{\beta i}\, U^*_{\alpha j}\, U^*_{\beta j} \sin^2\!\left(\frac{\pi x}{\lambda_{ij}}\right) .$$

where

$$\lambda_{ij} = \frac{\pi E_\nu}{1.27 \Delta m^2_{ij}}$$

Just as in the two-generation case, the oscillation length depends upon the mass difference (in electron volts squared), the length x from the source (in meters), and the neutrino energy E_ν (in million electron volts). The oscillation amplitude depends upon the $U_{\alpha i}$.

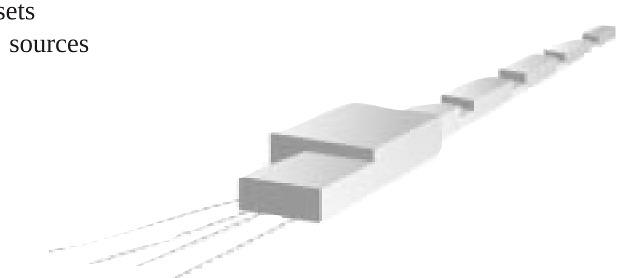


In a three-generation formalism, an oscillation between two flavor neutrinos occurs through all three mass states. To be explicit, the oscillation probability between electron neutrinos and muon neutrinos is given by

$$\begin{aligned} P(\nu_e \to \nu_\mu) = \;& 4U_{e1}U_{\mu 1}U_{e2}U_{\mu 2} \sin^2\!\left(\frac{\pi x}{\lambda_{12}}\right) \\ &+ 4U_{e1}U_{\mu 1}U_{e3}U_{\mu 3} \sin^2\!\left(\frac{\pi x}{\lambda_{13}}\right) \\ &+ 4U_{e2}U_{\mu 2}U_{e3}U_{\mu 3} \sin^2\!\left(\frac{\pi x}{\lambda_{23}}\right) . \end{aligned}$$

The first term in this expression (through λ_{12}) depends on the mass difference Δm^2_{12}. The second term depends on Δm^2_{13}, whereas the last depends on Δm^2_{23}. The coefficients in front of the sinusoidal terms involve all three mixing angles. Because there are multiple terms in the oscillation probability, $\nu_e \to \nu_\mu$ oscillations could appear to occur with different mass scales. An experiment could be sensitive to one or more oscillation lengths, depending on the specific source-to-detector distance x.

An example of a three-generation mixing model is the one put forth by Cardall and Fuller (1996). Their model ignores the zenith-angle dependence and sets $m_1 \approx m_2 << m_3$. All of the data from each of the three types of neutrino sources is then explained by the following mass differences and mixing matrix:



$$\begin{aligned} \Delta m^2_{12} &\approx 10^{-5} \text{ eV}^2, \\ \Delta m^2_{13} &\approx 0.3 \text{ eV}^2, \\ \Delta m^2_{23} &\approx 0.3 \text{ eV}^2, \text{ and} \end{aligned}$$

$$U_{\alpha i} \approx \begin{pmatrix} 0.99 & 0.03 & 0.03 \\ -0.03 & 0.71 & 0.71 \\ -0.03 & -0.71 & 0.71 \end{pmatrix} .$$

In the model used by Cardall and Fuller, electron neutrinos consist almost entirely

f there are more neutrino states than are lowed by the Standard Model (for example, there are right-handed neutrinos), then the ixing matrix could be much larger than 3×3 and could contain more than three arameters. The simple matrix presented ere assumes no right-handed neutrinos and o CP-violating phase.

of the mass state ν_1, whereas muon and tau neutrinos are nearly identical particles that are mostly equal mixtures of the mass states ν_2 and ν_3.

The data from both LSND and the solar-neutrino experiments are explained as evidence for an oscillation between electron neutrinos and muon neutrinos, with an oscillation probability governed by $P(\nu_e \rightarrow \nu_\mu)$. Substituting the matrix elements $U_{\alpha i}$ into the formal equation for the oscillation probability yields

$$P(\nu_e \rightarrow \nu_\mu) \approx \underset{\substack{\uparrow \\ \text{solar}}}{0.0025 \sin^2\left(\frac{\pi x}{\lambda_{12}}\right)} + \underset{\substack{\uparrow \\ \text{LSND}}}{0.0025 \sin^2\left(\frac{\pi x}{\lambda_{13}}\right)} + \underset{\substack{\uparrow \\ \text{LSND}}}{0.0018 \sin^2\left(\frac{\pi x}{\lambda_{23}}\right)} .$$

For LSND, the distance between the detector and source, x, is approximately 30 meters. Given the mass differences and the neutrino energies, the probability is dominated by the last two terms, which means that LSND is observing "indirect" oscillations between muon neutrinos and electron neutrinos. Although those two neutrinos are most closely associated with m_1 and m_2, the oscillation occurs because of the mass difference between m_1 and m_3 (that is, Δm^2_{13}) and between m_2 and m_3 (Δm^2_{23}). (A "direct" oscillation would depend only on Δm^2_{12}.) This indirect oscillation has a negligible effect on the neutrinos coming from the Sun ($x \approx$ 140 million kilometers). The density of matter in the Sun, however, is such that the MSW effect can resonantly enhance oscillations between two neutrinos with a very small mass difference. Electron neutrinos oscillate into muon neutrinos as they travel from the core of the Sun to its surface (a distance on the order of 100,000 kilometers). The first term in the probability expression—the one that depends on Δm^2_{12} through the term $\sin^2(\pi x/\lambda_{12})$—dominates in this case.

The atmospheric data is explained simply by having muon neutrinos oscillate into tau neutrinos, with a probability that is dominated by the last term in the expression for $P(\nu_\mu \rightarrow \nu_\tau)$:

$$P(\nu_\mu \rightarrow \nu_\tau) \approx 0.0018 \sin^2\left(\frac{\pi x}{\lambda_{12}}\right) + 0.0018 \sin^2\left(\frac{\pi x}{\lambda_{13}}\right) + \underset{\substack{\uparrow \\ \text{atmospheric}}}{1.016 \sin^2\left(\frac{\pi x}{\lambda_{23}}\right)} .$$

Cardall and Fuller readily admit that their solution is "rather fragile," in that small adjustments to the allowed parameter space for any one of the experiments may not permit a global fit. Still, their solution currently explains all the data and sets up a "natural" framework for viewing the apparent disparity among experimental results. (It is important to emphasize that there are many possibilities other than the Cardall-Fuller solution. Some of these involve sterile neutrinos, inverted mass hierarchies, and new particles.)

Future Experiments

The possible solution by Cardall and Fuller will be tested in the near future by several ongoing and proposed experiments. Super-Kamiokande (located in the Kamioka Mine in Japan) and BOREXINO (located in the Gran Sasso tunnel in Italy) will test the results of the solar-neutrino experiments. SNO, which is located in the Creighton Nickel Mine in Canada, has the capability to measure neutral- and charged-current neutrino interactions. The experiment should directly test the solar-neutrino oscillation hypothesis and MSW solution and could possibly test for the existence of sterile neutrinos.

Super-Kamiokande will also continue to take data on atmospheric neutrinos. Establishing the statistical significance of any zenith-angle dependence is one of its major goals. CHORUS and NOMAD, two experiments at the European Center for Nuclear Research (CERN), are looking directly for muon neutrino to tau neutrino oscillations. There are also proposals to look for the appearance of tau neutrinos in a beam of muon neutrinos produced by a distant accelerator. MINOS would be located in the Soudan Mine in Minnesota and would use Fermilab in Illinois as its neutrino source. ICARUS, situated in the Gran Sasso tunnel, would be 732 kilometers from its neutrino source at CERN. ICARUS could also be sensitive to solar and atmospheric neutrinos.

KARMEN, located at the ISIS pulsed-neutron spallation source in Great Britain, and BOONE, to be located near Fermilab, will test the LSND solution. Together, these current and proposed experiments should be able to prove whether neutrino oscillations are indeed responsible for the discrepancies between theory and data. In the future, however, we look forward to the day when neutrino oscillation experiments move from the "discovery" of neutrino oscillations to the measurement of oscillation parameters, neutrino masses, and lepton-sector mixing angles. ■

Left to right: Rex Tayloe, Geoffrey Mills, Hywel White, Vern Sandberg, Bill Louis, and Gerry Garvey. For biographies of Hywel White, Vern Sandberg, and Bill Louis, see "A Thousand Eyes." For a biography of Gerry Garvey, see "The Oscillating Neutrino."

Rex Tayloe was born and grew up in central Indiana. After earning his B.S. in physics from Purdue University, Tayloe attended the University of Illinois, where he received his Ph.D. for studies of hyperon physics at the Low Energy Antiproton Ring at CERN. He then joined the Laboratory and the LSND experiment as a postdoctoral staff member in 1995. He currently lives in Santa Fe and, in his spare time, enjoys mountain biking and working on his house.

Geoffrey Mills was born in Ithaca, New York, and grew up in Madison, Wisconsin. He received his B.S. in physics and mathematics from the University of Colorado. In 1985, Mills received his Ph.D. in experimental physics from the California Institute of Technology for research on the decay properties of the tau lepton. He then spent five years at the European Center for Nuclear Research studying the Z^0 boson at the Large Electron Positron collider. Since that time, Mills has been at the Laboratory pursuing the study of elementary particles while also skiing, hiking, playing the piano, and raising two boys with his wife, Ellen.

Further Reading

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