Capsule Storage and Density-Analog Techniques

by

H. C. Paxton
CAPSULE STORAGE AND DENSITY-ANALOG TECHNIQUES

by

H. C. Paxton

ABSTRACT

Density-analog schemes for describing critical arrays of fissile units have a long history. They originated as methods for generalizing results of subcritical measurements on weapon capsules. Such measurements were needed to establish reasonably efficient rules for capsule storage. Although specific density-analog models have been improved throughout the years, they are now largely replaced by comprehensive tabulations of critical-lattice parameters. Certain simplified forms are still useful as convenient formulas for extrapolation or for gross sorting of safety features.

INTRODUCTION

In the 1940's the need to store massive weapon capsules led to density-analog schemes for representing arrays of fissile units. To keep the scale of storage facilities under control, it was necessary to generalize the results of subcritical "vault tests" that were undertaken at the Los Alamos Scientific Laboratory (LASL) in 1947. Recent declassification of information about these tests and about subsequent measurements involving weapon components allows us to review the history of density-analog techniques. In addition to explaining the origin and applications of these techniques, we point out the extent to which they have been displaced by precise representations of critical arrays, and suggest ways in which they can still be useful.

TESTS WITH WEAPON COMPONENTS

Vault Tests. The name "vault test" was derived from the arrangement shown in Fig. 1. A lightweight framework provided locations for as many as 27 capsules in a cubic array, which could be surrounded completely by concrete. The framework was in two parts and was mounted on tracks so that the two portions, with part of the concrete, could be withdrawn in opposite directions. A scram signal automatically actuated the withdrawal, and it was possible to reassemble remotely. Concrete walls and roof were 1 ft thick and the floor was of 6-in.-thick reinforced concrete. The interior dimension of the cubic vault could be adjusted to 3, 4, or 5 ft. Spacings provided by the framework could be adjusted to correspond.

Basic components, called "fissile units," were U(93)-Pu combinations, each equivalent in reactivity to a sphere of about 20 kg U(93). An entire capsule, called a "tamped unit," was a fissile unit surrounded by natural uranium varying in thickness from ½ to 2½ in. Neutron-multiplication values were measured as adjacent lattice positions were filled progressively by fissile units, then by tamped units, and finally by fissile units opened into halves.
Sets of curves for the fissile units appear in Fig. 2, along with results of later measurements on smaller vaults. All reciprocal-multiplication curves were extrapolated to criticality, although results for the larger vaults were highly fictitious. The cross-multiplication curves \(M_x\), which are normalized to unity for a single unit, are reasonable representations of interaction. An arbitrary rule, adopted for all storage arrays, is that the overall value of cross multiplication should not exceed two.

To provide a basis for generalization, extrapolated critical data from these tests were represented as in Fig. 3. These results apply to fully reflected arrays, and similar curves were obtained when vault walls and top were removed. The apparent power relationships between critical number and lattice density,

\[ N_c = \text{const} \rho^{-s}, \]

suggested the relation for the critical mass of a single unit as density is changed uniformly

\[ m_c = m_{\text{co}} (\rho/\rho_0)^{-s}, \]

where \(m_{\text{co}}\) is the critical mass at full density \(\rho_0\). Because of this similarity, the term "density analog" was applied to relation (1).

Tests with Larger Capsules. Some years after the vault tests, other subcritical measurements were made on arrays of larger cadmium-plated U(93)-Pu capsules, each equivalent in reactivity to a sphere of about 32 kg U(93). Results for large arrays of such units in 16-in.-square by 25-in.-high containers were reported by Schuske of Dow Chemical Company's Rocky Flats Plant.2,3

Although tests were directed more toward practical storage arrangements than idealized lattices, some extrapolated data were deduced for cubic arrays. Figure 4 gives typical results for planar (two-dimensional) arrays of these units on a concrete floor. With a little imagination these data could be extended to give the critical spacing for an infinite planar array. The corresponding surface density, along with similar results for large low-density units,2 suggested the surface-density correlation of Fig. 5. (It should be noted that general validity of the "suggested storage limit" has been disputed.)

As expected, reciprocal cross-multiplication curves for linear (one-dimensional) arrays of the large capsules level out at smaller numbers than do the corresponding curves for planar arrays. Figure 6 gives such a comparison for 16-in. spacings.

Out of these measurements and density-analog extrapolations grew general rules for criticality control in the storage of capsules and enriched-uranium components of gun-type weapons. These rules persist, despite the fact that they could be refined in terms of recent information, such as that in "Guide for Criticality Control in the Storage of Fissile Materials."

The storage limits follow.

<table>
<thead>
<tr>
<th>Type of Array</th>
<th>Minimum Center-to-Center Spacing</th>
<th>Allowable Number of Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>linear</td>
<td>16 in. (40.6 cm)</td>
<td>no limit</td>
</tr>
<tr>
<td>planar</td>
<td>24 in. (61.0 cm)</td>
<td>no limit</td>
</tr>
<tr>
<td></td>
<td>21 in. (53.3 cm)</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>16 in. (40.6 cm)</td>
<td>32</td>
</tr>
<tr>
<td>volume</td>
<td>36 in. (91.4 cm)</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>30 in. (76.2 cm)</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>24 in. (61.0 cm)</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>20 in. (50.8 cm)</td>
<td>12</td>
</tr>
</tbody>
</table>

Rather arbitrarily, arrays such as those listed were considered isolated from one another if more than 13 ft apart or separated by concrete at least 1 ft thick.

Additional subcritical measurements were made on large capsules arranged in more practical storage configurations, such as that of two facing planar arrays separated by various distances (Fig. 7). Each plane contained 12 capsules spaced 30 in.
center-to-center. As the reciprocal cross-multiplication curve shows, interaction between planes is small when they are separated by more than 10 ft. At a separation of 6 ft, the multiplication was not affected by the persons shown in Fig. 7, standing as they might in the corridor of a storage vault.

Similar measurements on larger numbers of the same capsules in an actual storage vault are described by Fig. 8. If cross multiplication were not allowed to exceed two, the permissible number of these units would be about 25 at 19-in. center-to-center spacing and 50 at 21-in. spacing. When four people stood in the corridor, neutron counting rate actually was reduced somewhat. There was no measurable effect from the illustrated 25 units in a neighboring enclosure separated by a 12-in.-thick concrete wall.

The last two series of tests led to the concept of "associated arrays" consisting of two facing planar arrays separated by less than 13 ft. This concept was used in special rules applicable to capsule storage in cramped quarters, such as on shipboard. Modified rules for associated arrays are as follows.

<table>
<thead>
<tr>
<th>Minimum Center-to-Center Spacing of Units</th>
<th>Allowable Number of Units in Arrays</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Between Arrays</td>
</tr>
<tr>
<td>30 in. (76.2 cm)</td>
<td>90</td>
</tr>
<tr>
<td>24 in. (61.0 cm)</td>
<td>68</td>
</tr>
<tr>
<td>21 in. (53.3 cm)</td>
<td>45</td>
</tr>
<tr>
<td>19 in. (48.3 cm)</td>
<td>30</td>
</tr>
</tbody>
</table>

These limits also could be refined, but have served well enough as they stand.

Flooding Test. Although cadmium plating would protect against criticality if only one of the large capsules should be flooded, there was some question about the effect of rising water on an array of such capsules in their containers. Again, shipboard storage compartments were of principal concern. Because the container consists of a relatively small cylinder within a tubular framework, or "birdcage," units in a flooded array would be isolated by large water thicknesses. But effects of partial flooding were not well known, so an experiment with the setup shown in Fig. 9 was undertaken at LASL. A tank containing eight units in contact was surrounded by concrete walls and closed on top by boxes of paraffin. As the water level rose in the tank, neutron response differed little from that expected for a single unit.

**WEAPON STORAGE**

One of the biggest hazards in the storage of complete implosion-type fission weapons is the conventional explosive. In most such weapons the explosive is effective in preventing criticality, but, of course, introduces risks of its own. In our experience, a 4-in. thickness of explosive between nuclear components of fission weapons (2 in. per weapon) provides a neutron shield that will prevent criticality for unlimited storage in any arrangement.

For a few weapons of the fission-fusion type, there are restrictions on numbers that can be stored in volume (three-dimensional) arrays. For some of these weapons, special subcritical measurements were made to establish or confirm permissible storage arrangements. Incidentally, in no case was there a restriction on linear or planar arrays of such weapons in their usual containers.

One set of tests explored the safety of the two-high arrangement (Fig. 10) which would conserve deck space on a ship. The results, given in Fig. 11, show that the length of this array could be extended indefinitely without exceeding a cross-multiplication value of two. They further demonstrate the negligible effect of another unit that might be above the array while being moved into its proper location.

Most of the experiments described above would be unnecessary today. To a large extent, powerful computational techniques, such as Monte Carlo, with large banks of experimental data for validation, would take
the place of subcritical tests. Of course, there remains the possibility of measurements for the purpose of confirming questionable computed margins below criticality.

EVOLUTION OF DENSITY-ANALOG TECHNIQUES

Initial Density-Analog Formulation.

In the course of weapon and component tests a rather specific model for critical arrays was hypothesized and became known as the density-analog method. Although published elsewhere, a review of this scheme, with its subsequent improvements, and its shortcomings, fits into this discussion.

The hypothesis was that the simple extrapolation formula of relation (1) could be turned into a more specific expression patterned after (2). Namely, it was assumed that the critical mass of a cubic array could be expressed as

\[ M_c = m_{co}(\bar{\rho}/\rho_o)^{-s}, \]

where \( m_{co} \) is the critical mass of a cube of the fissile material at full density \( \rho_o \), reflected like the array. Conservative interpretations of subcritical measurements, including those discussed above, suggested the following correlations between the exponent \( s \) and the "fraction critical" of a unit \( f \):

\[ s = 2(1-f) \quad \text{for unreflected arrays,} \quad (4a) \]

\[ s = 1.4(1-f) \quad \text{for heavily reflected arrays.} \quad (4b) \]

The quantity \( f \) is the same as that in Fig. 5. This model gives the following simple expression for a reflected array for which \( f \approx 0.3 \):

\[ M_c(\text{refl}) = [m_{co}(\text{bare})/R](\rho_o/\bar{\rho})^{2(1-f)}. \]

These results, obtained from near-equilateral critical clusters of solution containers, appear as the deviant points in Fig. 12.

Modified Formulations. D. R. Smith recognized that the value of \( s \) for extremely large reflected arrays must approach that for unreflected arrays of similar units, as is the extreme case for density exponents of isolated bare and reflected spheres. The implication, of course, is that relation (4b), implying a constant value of \( s \) for reflected arrays, is not a good model.

To improve matters, Smith continued to use expressions (3) and (4a) for unreflected arrays, but applied a computed "reflection factor" \( R \) to obtain the critical mass of the corresponding reflected array. Thus the resulting critical mass was expressed by

\[ M_c(\text{refl}) = [m_{co}(\text{bare})/R](\rho_o/\bar{\rho})^{2(1-f)}. \]

Smith also computed the following values of \( R \) for various homogeneous systems at very low densities.

LIMITING RATIOS OF CRITICAL MASSES OF BARE AND WATER-REFLECTED SPHERES AT LOW DENSITY

<table>
<thead>
<tr>
<th>Core Composition</th>
<th>( R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U(93) ) metal</td>
<td>13</td>
</tr>
<tr>
<td>( U(93)O_2 )</td>
<td>8.0</td>
</tr>
<tr>
<td>( U(93)F_6 )</td>
<td>6.0</td>
</tr>
<tr>
<td>( U(93)C_8O )</td>
<td>2.7</td>
</tr>
<tr>
<td>( U(93) ) solution, ( H/^{235}U = 60 )</td>
<td>5.4</td>
</tr>
<tr>
<td>( U(93) ) solution, ( H/^{235}U = 400 )</td>
<td>2.7</td>
</tr>
<tr>
<td>Pu metal</td>
<td>19</td>
</tr>
</tbody>
</table>

Somewhat later, when more became known about critical cubic lattices, J. T. Thomas pointed out that for a still better approximation the constant value \( s = 1.8 \) could be adopted for all large cubic arrays of practical-size units. He suggested that the density-analog concept be used only for extrapolating to smaller \( \bar{\rho} \) from an experimental
(or computed) critical array of reasonable size, instead of from a full-density critical mass. It follows that the simple extrapolation formula (1) becomes

\[ N_c = A(p_o/p)^{1.8}, \]

where the constant A is evaluated in terms of the critical values of N and \( p_o/p \) for a known reference array of the same units.

**Tests of Density-Analog Schemes.** The role of density-analog models is influenced profoundly by the reliable parameters that exist now for a great variety of critical cubic lattices. Most significantly, reliance upon such models has all but disappeared, although the simpler forms still have their place as convenient formulas for extrapolation or for gross sorting of safety features. Because of these residual uses, we repeat here evaluations of the various density-analog schemes.

Results of these schemes are compared with reliable parameters for families of critical cubic lattices of U(93) metal, U(93) solution, plutonium metal, and UO\(_2\) units. Figure 13 illustrates serious deficiencies of the initial specific density-analog model. It shows that the version of this model represented by Eq. (5), for \( f \approx 0.3 \), falls far short of critical numbers in large reflected arrays, while it is actually nonconservative for small arrays. The considerable improvement from Smith's modification, Eq. (6), is shown by Fig. 14. Even this form is highly conservative at small lattice densities.

To illustrate the still further advantage of the form suggested by Thomas, constant A has been chosen such that Eq. (7) represents a somewhat conservative envelope for the families of arrays we are considering. For this purpose, the value of A corresponding to \( f = 0.4 \) is adjusted by the factor \( 0.4/f \) to reduce the penalty for arrays of less reactive units. Equation (7) then becomes

\[ N = (0.012/f)(p_o/p)^{1.8} \text{ for } f \leq 0.4, \]

which is represented in Fig. 15, together with the reference families of lattices. Although this is the best of the specific density-analog forms, it is not simple enough to be an easily remembered substitute for the extensive tabulations that Thomas has provided.

**Density-Analog Schemes at Rest.** Density-analog methods have served acceptably during our period of ignorance about parameters of large critical lattices, but the comprehensive tabulations that appear in Ref. 4 are more reliable and as easy to use. The density-analog concept persists in the simple extrapolation formula [Eq. (7)]

\[ N_c = \text{const} (p_o/p)^{1.8}, \]

which provides a handy means of extending known criticality data. Although we now realize that it is extremely crude, another very simple expression [Eq. (5)]

\[ M_c > m_{co} (p_o/p), \text{ for } f < 0.3 \text{ and small } p, \]

can be useful for immediately distinguishing between highly subcritical lattices and those that require better evaluation. These density-analog fragments are all that we recommend retaining.

A final observation is that density-analog pictures of cubic lattices do not extend readily to the more haphazard layouts that are customary in processing plants. Here, some version of the critical surface density concept introduced in Fig. 5 is probably more applicable.

**REFERENCES**


**Fig. 1.** Setup for vault tests of 1947.

**Fig. 2.** Reciprocal-multiplication and cross-multiplication values for arrays of fissile units in concrete vaults.
Fig. 3. Extrapolated critical data for arrays of weapon components in concrete vaults.

Fig. 4. Planar array of large capsules; neutron counters are overhead. The reciprocal cross-multiplication curves are for various values of center-to-center spacing.

Fig. 5. Extrapolated critical surface densities for infinite planar arrays of large capsules and massive low-density units. The abscissa is the ratio of the mass of a unit to that of a similar critical unit.
Fig. 6. Reciprocal cross-multiplication curves for linear, planar, and cubic arrays. A composite of data for the large capsules is reduced to 16-in. spacing.

Fig. 7. Effect of separation between two planar arrays on overall cross multiplication. Each plane consisted of 12 large capsules at 30-in. spacing; neutron counters were overhead. Persons between planes did not change results.

Fig. 8. Change of reciprocal cross multiplication as large capsules were positioned in a storage vault at spacings of 19 and 21 in. One side was filled before the other was started. The lone point, for 19-in. spacing, shows the apparent effect of four persons in the corridor.
Fig. 9. Setup for water-flooding test on eight large capsules in their carrying cases.

Fig. 10. Fission-fusion weapons in a simulated shipboard storage array for subcritical tests.
Fig. 11. Reciprocal cross-multiplication data obtained while building up the array of fission-fusion weapons shown in Fig. 10.

Fig. 12. Lattice-density exponent vs "fraction critical" of an isolated unit for various degrees of external reflection.

Fig. 13. Evaluation of the original density-analog formulation.
Fig. 14. Evaluation of Smith's modification.

Fig. 15. Evaluation of an adaptation of Thomas' version.