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# MULTIPLICATION OF FISSION NEUTRONS IN A SPHERE OF 25

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## ABSTRACT

The multiplication in a sphere of 25 of fission neutrons from a shell source on the surface of the sphere was observed by small 25 and 28 detectors at its center. The values for the multiplication registered by the 25 counter are in agreement with calculations by Rarita for both a 1-1/2"- and a 2"- diameter sphere. The experimental values for the 28 counter, however, disagree with similar calculations by four times the probable error. Multiplication of a normal alloy sphere was measured with a 28 counter. In addition, the standard indium-foil source-comparison technique was used as another measurement of the multiplication in a 1-1/2" sphere. These results are also in agreement with theory. From these measurements we conclude that the effective value of $\nu-1-\alpha$ assumed in the calculations is substantially correct, but the assumed value of the cross section for inelastic scattering from above to below the 28 fission threshold is 30 per cent too high.

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## MULTIPLICATION OF FISSION NEUTRONS IN A SPHERE OF 25

The critical mass of 25 has been calculated in terms of its differential constants and the fission neutron spectrum. In these calculations, it has been necessary to assume a value for the radiative capture cross section and to use values for the transport and inelastic scattering cross sections which are not yet as precise as might be desired, because of the limited amounts of material available to date for measurements. Even the fission spectrum is not very well known. Under these conditions the true value of the critical mass is clearly in considerable doubt. While a critical mass of 25 is not yet available, nevertheless it is possible to devise integral experiments in which, to evaluate the results, the same differential constants of 25 are used in a manner comparable to the critical-mass calculation. These experiments are designed to test as well as possible the present critical-mass estimates, while using only the amounts of 25 now available. One such experiment is to measure in a 25 sphere the multiplication of fission neutrons from a source localized on its surface. Indeed, Nelson has shown that if one assumes the existing measurements of the transport cross section to be correct, a plot of the reciprocal multiplication versus the sphere radius may be extrapolated to give the critical mass itself.

To this end, the multiplication in 1-1/2" and 2" $\beta$-stage spheres was measured in the thermal column in building X. In such a measurement, the almost purely thermal neutrons in the 3' x 3' x 3' cavity in this column are used to produce fissions in a rather thin spherical shell of $\beta$-stage material. These fissions are the source of the fast neutrons on

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which measurements are made. The shell is thick enough that ideally the slow neutrons do not penetrate through it, although it is thin for fast neutrons. The flux of fast neutrons is measured at the center of the sphere both with and without a practically solid sphere of $\beta$-stage material filling the space between the shell and the detector at the center. The ratio of these two measurements indicates the multiplication of the fission neutrons by fission in the solid sphere. The detector at the center is either a 25 or a 28 fission chamber. Although the measurement with the 25 counter is more directly connected with the critical mass, the 28 counter measurement was taken as an additional check on the differential constants, primarily the inelastic scattering to which it is quite sensitive.

## Experimental

During the measurement of multiplication the 25 assemblies were mounted in the center of the three-foot cavity with the counter in the center of the 25 shell or shell-plus-core (see block arrangement, diagram Fig. 1). The core, which was made in hemispheres for easy assembly, had a small cavity and radial hole machined out to contain the counter. Although the geometry and thickness of active material in these chambers deprived them of plateaus (for detail of fission chambers, see Fig. 2), their sensitivity was maintained constant by using them in conjunction with the very stable Model 100" amplifier at a set discriminator bias. Repeated α count on the 25 counters taken at a low bias showed that in the course of a run the gain drift was less than one or two per cent. However, in this time the shell and the shell-plus-core had been interchanged so often that the effect of this drift on the observed multiplication was vastly reduced.

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The small-fission-chamber counting rates were always taken relative to a monitor of the thermal neutron flux in the block. This monitor was checked continually against two other monitors and found to perform reliably throughout the entire course of the measurements.

The counting rates had to be corrected for small backgrounds of two types. One was caused by epi-cadmium neutrons in the graphite column even when no fissionable material was present. This background was measured by the counting rate per monitor count when, in the geometry used in the multiplication measurement, the 25 shell was covered by a cadmium shell. As to be expected, when the core was present within the shell, this background was somewhat smaller, because the core filters out some of the epi-cadmium neutrons.

The second background to the measurements was caused by fission neutrons reflected and moderated by the graphite walls of the cavity. This arises because many of the reflected neutrons are of high enough energy that they are not stopped by the shell source and can penetrate inside either to count directly in the counter or to produce fissions in the core. Surrounding the 25 counter with boron, or the 28 counter with cadmium, when making measurements on the shell, reduced the former of these effects. Meanwhile, it was found that both effects could be measured in a very simple way. First, a source of fission neutrons was placed at the center of the cavity. Then one of the small counters was used to plot counting rate as a function of position in the cavity. After subtracting off the counts caused by neutrons coming directly from the source by means of a $1/r^2$ fit, the counting rate caused by reflected neutrons was found to be constant over the cavity to the accuracy required

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for a background correction. The second measurement was to place the counter half way between the central source and the walls, where for practical purposes the counting rate caused by direct neutrons was already negligible; then the source and counter were repeatedly interchanged in position. It was found that the counting rate was the same in both arrangements. These two results meant that we could measure the background of graphite scattered neutrons as follows:

The shell (or shell-plus-core) is covered by the cadmium shell and mounted at the center of the three-foot cavity as for measurement of the first type of background. Then a fission source of known strength relative to that of the shell is placed in the cavity half way between the center and a wall. The counting rate at the center of the shell less the background of type one gives the desired back scattering background altered by the ratio of the strength of the auxiliary source to that of the shell.

The procedure of a measurement, then, was as follows: a bias curve was run on the counter and the bias set so that the alpha count was zero; then the backgrounds were measured as described; finally, a series of alternate measurements of central fluxes in sphere and sphere-plus-shell were run. Frequent α counts at another fixed bias checked the gain during the 25-counter measurements. Such a "run" is self-sufficient for a multiplication determination. Nevertheless, when possible, several such runs were made to check each other and give better statistics.

## Results on the 2" Sphere

The 2-inch sphere was of $\beta$-stage material containing about 70 per cent of 25 and was of density 18.7. The source was a close-fitting shell of

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the same material, 90 mils thick. About 99.5 per cent of the thermal neutrons were stopped on their way into the shell. When measurements were made on the shell alone, the counters were surrounded with a close-fitting shield of cadmium and then a boron carbide layer ~ 2 gms/cm. In this way, measurements were made with both the 25 and 28 detector.

Two separate runs with the 25 detector gave for the multiplication of the 2" sphere

1.43 ± .03

and 1.46 ± .03,

or an average value of 1.445 ± .02

One run with the 28 detector gave the value

1.21 ± .02

Here and in the following results, the probable errors are obtained from the self-consistency of the data, not from counting statistics.

Results on the 1-1/2" Sphere

The 1-1/2" sphere was also of $\beta$-stage material of about 70 per cent 25 content and of density 18.4. However, the close-fitting shell source in this case was not as satisfactory as in the case of the 2" shell. It was 65 mils thick and made of 44 per cent 25 alloy. This thickness corresponds to about two mean free paths of thermal neutrons; consequently, an appreciable number of thermal neutrons leak through. This lack of opacity of the shell causes several effects which must be corrected for. These corrections are dealt with in some detail in appendix 1; here we shall only indicate how they come about. It is shown in the appendix that 8.5 per cent of the thermal neutrons leak through into the interior of the shell and that 1.2 per cent

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leak all the way in and out again and do not produce any fissions at all. Since, when the solid sphere is in place, all the thermals are stopped, this means that we have effectively a 1.2 per cent stronger source. However, this is not quite the full story. When we make measurements on the shell alone, we surround the counter with either boron or cadmium as explained above. The presence of these absorbers inside the shell effectively stops a fraction of the (8.5 - 1.2) per cent of thermals which would be stopped on their way out, and hence further reduces the source strength of the shell as measured by the counter. Finally, there is a third effect of the partial transparency of the shell. The 8.5 per cent of thermals which leak through the shell are stopped in the first millimeter of the core and produce a neutron source which is somewhat closer to the counter and therefore counted more efficiently. It is shown in the appendix that this amounts to about .7 per cent.

The results on 1-1/2" sphere corrected for effects caused by the partial transparency of the shell as well as for the two types of background described on pages 5 - 6 are given in Table I.*

Table I. Multiplication by 1-1/2" Sphere.

| 28 Detector | 25 Detector |
|---|---|
| 1.13 ± .03 | 1.26 ± .01 |
| 1.14 ± .02 | 1.26 ± .02 |
| 1.14 ± .016 | 1.28 ± .02 |
| 1.15 ± .014 | 1.24 ± .01 |
| 1.15 ± .016 | |
| 1.08 ± .02 | |
| Averages<br>1.13 ± .01 | 1.26 ± .01 |

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*In addition to the measurements in the 3' cubical cavity some measurements were performed on the 11" cavity. These were corrected for back scattering using the results of Feynman and Reines based on the "age theory". Because these corrections were fairly large, especially in the case of the 25 counter and because their predictions on several details did not agree well with experiment these values are considered to be somewhat in doubt and were not averaged in with the large cavity measurements to obtain our final result.

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## Measurements on Tuballoy Sphere

A measurement was carried out with the 28 counter on a 1-1/2" normal tuballoy sphere. This result had to be corrected in a manner similar to those on the 1-1/2" 25 sphere since the same shell source was used. The corrected result for the multiplication is .89 ± .03.

## Discussion

Rarita has calculated the multiplication in 25 spheres to be expected under the conditions of these observations. Using in his calculation .068 as the average cross section for radiative capture of 25, the value of $\nu(25) = 2.44$, assuming the inelastic scattering of 25 and 28 are the same and that all inelastic scattering sends the neutrons below the 28 threshold, using the Staub-Richards fission spectrum (LA Handbook, Edition 2), and assuming $\sigma_f$ drops off at high energies as given by Bretscher, then one obtains values listed in columns two and three of Table II.

Table II. Calculated Multiplication.

| Detector \ Sphere Size | 1.5" | 2" | 2" |
|---|---|---|---|
| 25 | 1.23 | 1.42 | 1.47 |
| 28 | 1.06 | 1.125 | 1.12 |

Using Manley's 25 inelastic cross section measurements, the new Richards' spectrum, and assuming $\sigma_f$ becomes flat above 600 kv, the 2" sphere values were recalculated to give column four in Table II. Rarita also calculated the value for the 1-1/2" normal alloy sphere. He obtained .83, which is below the experimental value by twice the probable error.

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It is seen that the first set of calculations on the measurements with a 25 detector are a percent or so lower than the experimental values, the recalculation on the large sphere is as much higher. The agreement of either calculation for the 25 detector with the measurements lies almost within the experimental probable error. In contrast to this agreement, the 28-detector calculations lie four probable errors away from the experimental numbers. This discrepancy is not caused by the form of the fission spectrum assumed since the two calculations on the 2" sphere give almost the same result even though the spectra used are quite different. The effective inelastic cross sections in the two calculations were almost the same, Manley's number being the higher. In fact, we show in appendix 2 that if one demands a given result for the 25 calculations the result of the 28 calculations may only be altered appreciably by altering the inelastic cross section. On this basis, a reduction by 30 per cent in the inelastic scattering of the fission neutrons from above to below the 28 theshold would bring the calculations on both the small and large spheres into agreement with experiment. Hence we conclude that the effective cross section for this process should be 1.2 barns instead of the 1.7 used in the calculation. Such a reduction in the inelastic cross section tends to lower the calculated 25 numbers very slightly.

Since the radiative capture assumed has just the effect equal to the difference between the two calculations, 1.42 and 1.47, we can say only that we are in essential agreement with the assumed value until a calculation based on more accurate values of the other constants is possible.

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Indium Foil Measurements

The multiplication in a 1-1/2" $\beta$-stage sphere of fission neutrons from a shell source on its surface was also measured by indium foils. For this measurement the shell and shell-plus-core were alternately suspended at the center of an 11" spherical cavity in the graphite block. The source strength of each was measured using standard, cadmium-covered indium foils placed along a radius of the cavity and on the side away from the cyclotron at distances ranging from 13.9 to 89.3 cm from the center of the cavity. The saturation activity, A, acquired by these foils was measured on a standard Chicago-type, thin-walled aluminum counter. When these activities are reduced to unit monitor of thermal flux in the block and corrected for a 5 per cent background of epi-cadmium neutrons from the cyclotron, the net activity obtained is proportional to q(r), the slowing-down density of 1.4-e.v. neutrons at the foil's location, caused by the source of fission neutrons. The total source strength of fission neutrons is proportional to

$$\int_0^\infty A(r)\,r^2 dr$$

where r is the distance from the center of the cavity. The ratio of this quantity for the shell-plus-core to that for the empty shell gives the multiplication.

Since the function A(r) had the same shape for both shell and solid sphere the multiplication was actually determined by

$$\sum_j (A_j r_j^2\, g_j)_{\text{solid sphere}} \Big/ \sum_j (A_j r_j^2\, g_j)_{\text{shell}}$$

where $g_j$ is the statistical weight of the measurement on the j'th foil.

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These measurements were repeated three times for the shell and four for the shell-plus-core. The average ratio is 1.152 ± .003. This number must be corrected for the following effects: the shell transparency to thermal neutrons was 1.2 per cent; the presence of boron or cadmium shields at the center of the shell for simultaneous fission counter measurements was 3.8 per cent. There is an additional multiplication in the solid core caused by neutrons reflected by the graphite. According to Feynman and Reines this number is .7 per cent of the source strength. It must be multiplied by $\nu/(1 + a)$ and by 1.03 for the greater effectiveness of fission neutron sources distributed throughout the volume of the core. This correction then amounts to 1.7 per cent. These corrections all act in the same direction to reduce the measured number to $M = 1.076 \pm .008$ in the ideal case of an opaque shell and no graphite reflector. Rarita has also calculated the result of this experiment and obtains 1.070.

A two-collision approximation to the multiplication expected is given by

$$M = 1 + \sigma_f(\nu-1-a)\, a \left\{ \frac{1}{2} + \sigma_T a \left[ .348(1 + f) - \frac{1}{3} \right] \right\}$$

where $\sigma_f$ is the fission cross section, $\nu$ is the number of neutrons per fission, $a\sigma_f$ is the radiative capture cross section and $\sigma_T$ is the transport cross section, these quantities being averages over the fission spectrum. $1 + f$ is this quantity averaged over a first-collision spectrum and a is the sphere radius. From this expression, it is clear that agreement between calculation and measurement means that the assumed value of $\nu-1-a$ is correct to a probable error of the measurement, 10 per cent.

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## Appendix

### Calculation of Thermal-Neutron Transmission of 1-1/2" Shell.

For an isotropic thermal-neutron flux incident on a spherical absorbing shell whose thickness is small compared to its radius, the fraction $F_1$ of neutrons which reach the interior is given by averaging the transmission over all velocities, $v$ and angles of incidence, $\theta$. The result is

$$F_1 = (8/\pi^{1/2}) \int_0^{\infty} x^2 e^{-x^2} dx \int_0^{\pi/2} \sin\theta \cos\theta \, e^{-t_0/x \cos\theta} d\theta$$

where $x = v/\sqrt{2KT/m}$ and $t_0$ is the shell thickness in mean free paths for neutrons of velocity $v = \sqrt{2KT/m}$ and direction of incidence normal to the shell surface. If we let $Z = x \cos\theta$ this becomes

$$F_1 = 4 \int_0^{\infty} (1 - \mathrm{Erf}\, Z) Z e^{-t_0/Z} dZ$$

We may observe that if the flux incident on the shell is not isotropic but comes from the walls of a large cavity as in our measurements, this result still gives the average transmission since rotating the sphere about its center cannot affect this value. The value obtained for the 1-1/2" shell is $F_1 = .085$.

The fraction, $F_2$, of neutrons incident on the shell which pass into it and out again is given by the above expression using twice the shell thickness for $t_0$. This follows since any ray through the shell enters and emerges with the sample angle to its surface. For the 1-1/2" sphere, $F_2 = .012$.

### Calculation of Absorption of Thermal Neutrons by Opaque Sphere at Center of 1-1/2" Shell

Let the shell radius be a, the sphere radius b. Then the desired

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result is obtained by averaging the flux over angles 0 to $\sin^{-1} b/a$ instead of 0 to $\pi/2$ as in $F_1$. The result for our shell and the boron absorber of average diameter .7" gives $F_{1\ b/a} = .082$. A similar integration for the fraction of neutrons captured by the boron which would otherwise have passed out of the sphere gives $F_{2\ b/a} = .012$. Hence the net chance of fission source strength caused by the boron is $F_{1\ b/a} - F_{2\ b/a} = .070$. For the cadmium we obtain similarly .025.

Because of the possibility of slight inhomogeneities (of the order of .005" in a total wall thickness of 0.65" which might put the above calculation in error) it was felt desirable to measure this correction to the shell-source strength. To do this a cadmium-covered fission chamber was placed at one side of the 11" cavity with the shell supported in the center. Counts in this chamber were then proportional to the source strength of fission neutrons from the shell. The counting rates with and without the boron or cadmium shield inside of the shell were compared and gave fractional differences of .07 and .023, respectively, in agreement with the above calculations. This agreement gives confidence in the calculation of the quantity which we did not measure experimentally, the 1.2 per cent of neutrons which pass entirely through the empty shell.

Effect of Source Distribution in 1-1/2" Sphere.

The central counters measure source strength per unit area. Hence we must average the thermal flux per $4\pi r^2$ for all radii to obtain the effective source strength. In the case of the 1-1/2" sphere this average for the shell and core will be different from that for the empty shell because of its partial transparency. The correction to be applied is to increase the measured flux at the center of the empty shell. It is very

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closely

$$(F_2 - F_1)\left[\frac{a^2}{(a-\ell_1)^2} - \frac{a^2}{(a-\ell_2)^2}\right]$$

where a is the radius of the sphere and $\ell_1$ and $\ell_2$ are respectively $\sqrt{2}$ (mean free path) for the sphere and shell. For the 1-1/2" sphere this correction is .7 per cent.

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## Appendix 2

## Calculations on Multiplication in a Small Sphere

Assume a shell source of fission neutrons on the surface of the sphere, a point detector at its center. In the one-collision approximation, the multiplication M observed by the detector is given by

$$\frac{M-1}{a} = \frac{n^2}{8}\int_0^\infty \nu(E)\chi(E)\sigma_f(E)\,dE + \frac{(n^2/3-1)\int_0^\infty \chi(E)\sigma_{e\ell}(E)\sigma_D(E)\,dE}{\int_0^\infty \chi(E)\sigma_D(E)\,dE}$$

$$- \frac{\int_0^\infty \left[1+\alpha(E)\right]\chi(E)\sigma_f(E)\sigma_D(E)\,dE}{\int_0^\infty \chi(E)\sigma_D(E)\,dE} + \frac{\int_0^\infty \chi(E)\sigma_{in}(E)\left\{\frac{n^2}{8}\int_0^E \psi(E,E')\sigma_D(E')\,dE' - \sigma_D(E)\right\}dE}{\int_0^\infty \chi(E)\sigma_D(E)\,dE}$$

where a is the sphere radius, $\chi$ is the fission neutron spectrum, $\sigma_f$, $\sigma_{e\ell}$, $\sigma_D$, $\alpha\sigma_f$, $\sigma_{in}$ are respectively the fission, elastic, detector, radiative capture and inelastic cross sections, and $\psi(E, E')$ is the inelastic spectrum for neutrons of primary energy E. $\psi$ is normalized so that

$$\int_0^E \psi(E,E')\,dE' = 1$$

Since the major part of the multiplication in the small spheres used in our measurements arises from the first collision of neutrons from the source, we may use this expression for multiplication to notice by what effects the 25 and 28 detector results are mainly caused.

We will begin by making simplifications of the expression for M-1. First we assert that for 25 or 28 detectors $\sigma_{e\ell}(E)$, $\sigma_{in}(E)$ and $1+\alpha(E)$ are slowly varying functions of the energy compared to $\chi(E)$ and $\sigma_f$. Hence they may be given their values at the maximum of $\chi(E)\ \sigma_D(E)$ and $\chi(E)\sigma_D(E)\ \sigma_f(E)$ respectively and removed from the integration. Next we

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break the region of integration into two parts for 28: above and below the threshold, $E_{28}$, for 28 fission. $\sigma(28)$ may then be removed from the integration. The W group results show that $\sigma_f$ becomes essentially constant from .5 to 3 Mev. The $\nu\sigma_f(25)$ experiments in X show that Bretscher's low $\sigma_f(25)$ at high energies is improbable and indicate that it remains constant to higher energies. Therefore it may be removed from the integration in the high-energy region. We further remark that $\sigma_{e\ell}$ at the maximum of $\chi\sigma_f(25)$ is almost the same as at the maximum of $\chi\sigma_f(28)$. Hence the contribution of the elastic term to both detectors is the same, i.e.

$$(\pi^2/8 - 1)\ \sigma_{e\ell}(E_{28})$$

Now consider in detail the $-(1 + a)$ term which is the attenuation caused by absorption. It has become for the 28 detector

$$-\left[1 + a(E_{28})\right] \int_{E_{28}}^{\infty} \chi\sigma_f\, dE \Big/ \int_{E_{28}}^{\infty} \chi\, dE$$

$\sigma_f$ is essentially constant above $E_{28}$, hence this reduces further to

$$-\left[1 + a(E_{28})\right]\ \sigma_f(E_{28})$$

For the 25 detector we have to take $a(E)$ at the maximum of $\chi\sigma^2_f(25)$. Since $a$ varies slowly, we take this to be the same as $a(E_{28})$ and obtain

$$-\left[1 + a(E_{28})\right] \int_0^{\infty} \chi\sigma_f^2\, dE \Big/ \int \chi\sigma_f\, dE$$

$\sigma_f$ is constant over most of the fission spectrum. Hence this quantity is just slightly less than unity times $\sigma_f(25)(E_{28})$. Again the 25 and 28 detector terms are the same. This means that any appreciable difference between the calculated results for 25 and 28 detectors must be found in the inelastic term.

This difference can indeed be large. The term, if all the neutrons

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are scattered below the 28 threshold as assumed in Rarita's calculations, has a contribution to the 28 detector $-\sigma_{in}(E_{28})$ and if none go below the threshold,

$$+(\pi^2/8 - 1)\sigma_{in}(E_{28})$$

By contrast consider the 25 detector. If none of the neutrons go below the constant region of $\sigma_f(25)$ the value is again about $(\pi^2/8 - 1)\sigma_{in}(E_{28})$. If they do not go below $E_{28}$, then the multiplications observed by the 28 and 25 detectors are nearly identical. If all go below the constant region of $\sigma_f(25)$ then the effective $\sigma_f$ may become extremely large, and an enormous multiplication in the 25 detector results.

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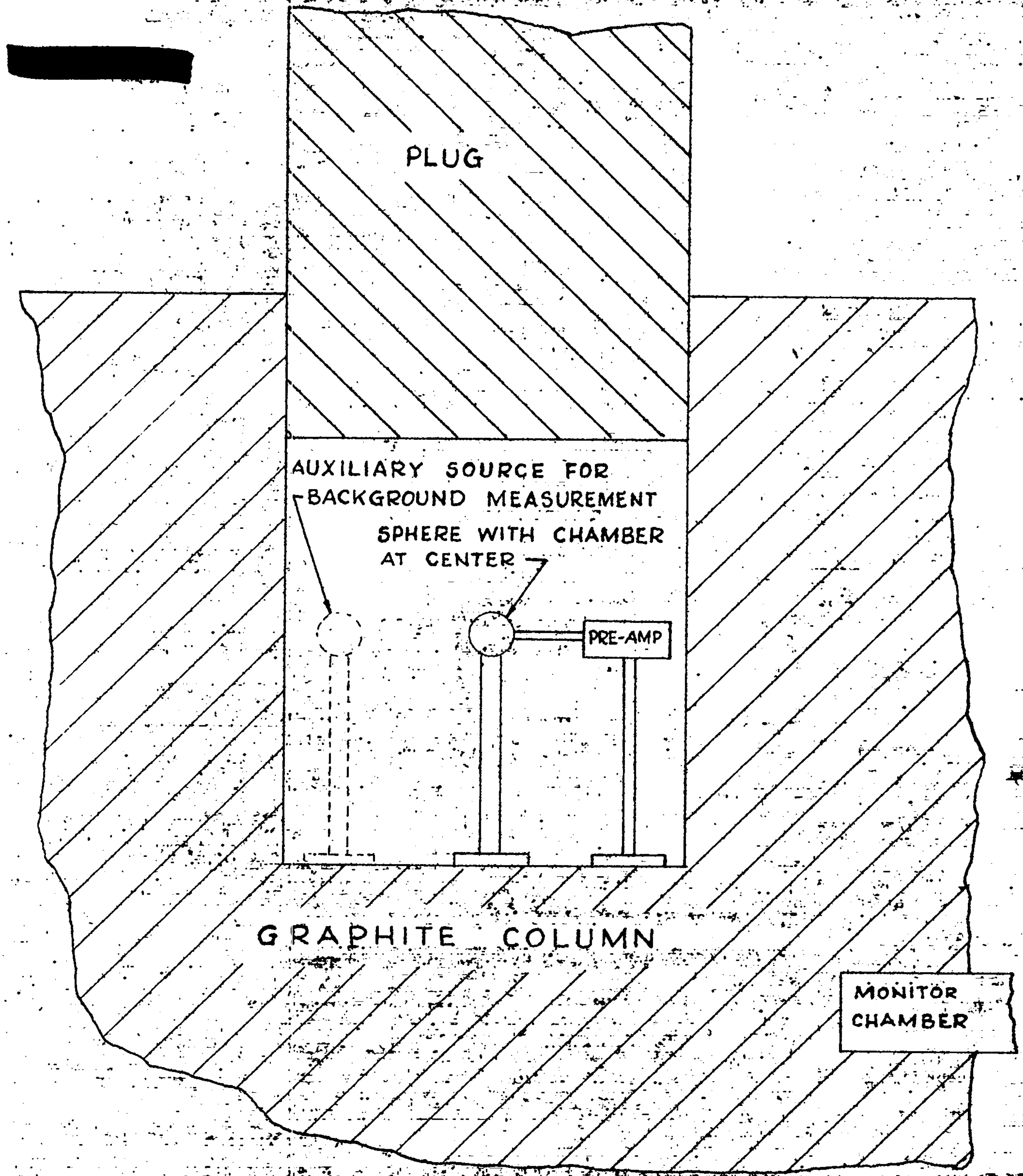

FIGURE 1

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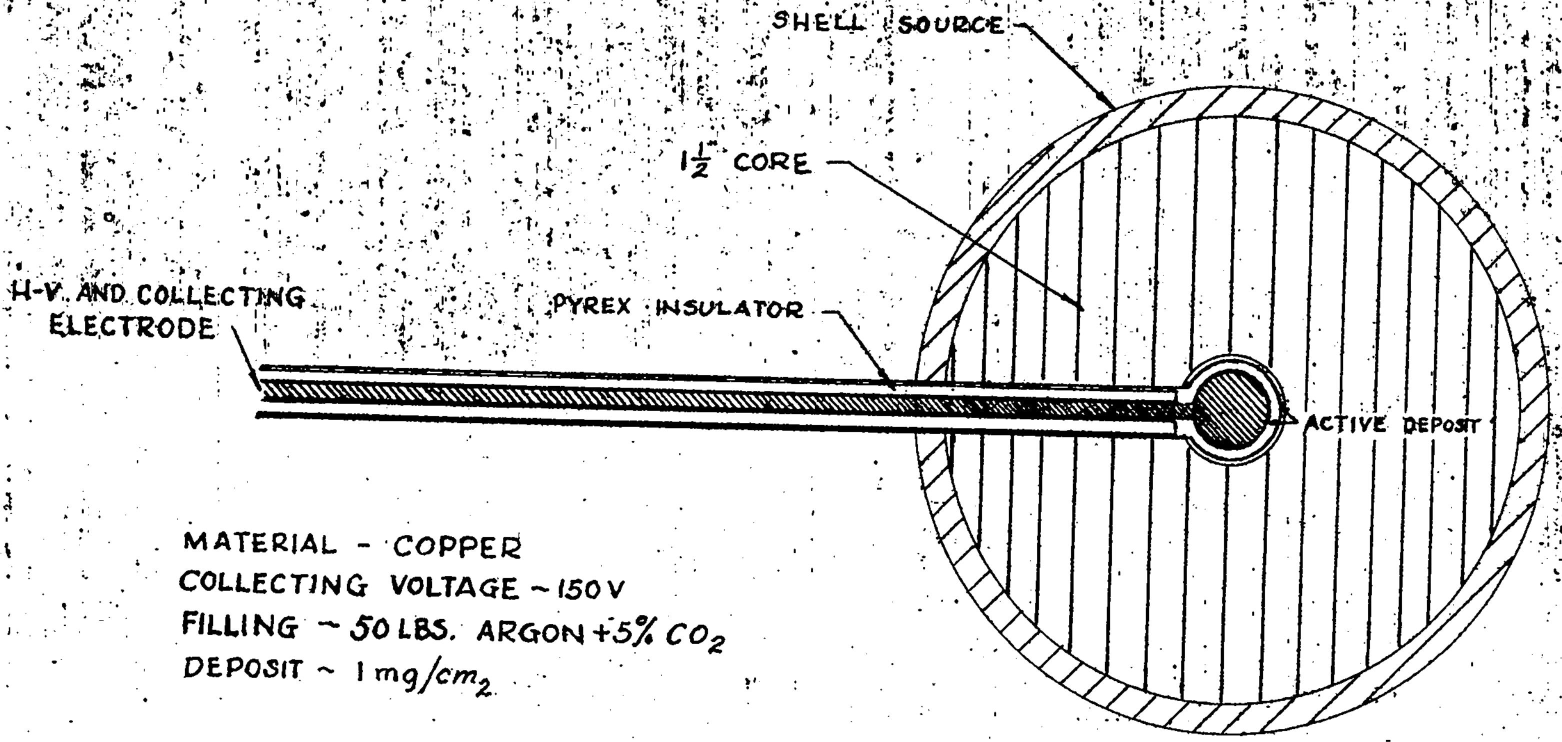

MATERIAL - COPPER
COLLECTING VOLTAGE ~ 150 V
FILLING ~ 50 LBS. ARGON + 5% $CO_2$
DEPOSIT ~ 1 $mg/cm_2$

FIGURE 2
SCALE 2 - 1

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