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MULTIPLICATION OF NEUTRONS IN SMALL SPHERES OF ACTIVE MATERIAL

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ABSTRACT

A calculation has been made of the result of the following experiment: a source of neutrons is surrounded by a sphere of active material and a fission counter is placed nearby. The count is compared when the sphere is in place and when the sphere is removed; the ratio (M) of the two counting rates is a measure of the reactive properties of the sphere. The results of our calculations are compared with the results of the experiments of Group R-3:

Material	Sphere Diameter	Counter	M_{calc}	M_{obs}
73% 25 + 27% 28	2.5"	63% 25 + 37% 28	1.381	1.35 ± .02
		28	1.082	1.13 ± .02
	2.0"	63% 25 + 37% 28	1.262	1.28 ± .06
		28	1.032	1.07 ± .05
Normal Uranium	2.5"	63% 25 + 37% 28	1.044	1.031 ± .011
		28	0.707	0.760 ± .012
Plutonium	.9"	63% 25 + 37% 28	1.199	1.202 ± .012
		28	1.176	1.175 ± .015

In the case of the 2.5" 25 sphere the outcoming spectrum has also been calculated. The agreement with the spectrum as measured by Richards is satisfactory.

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MULTIPLICATION OF NEUTRONS IN SMALL SPHERES OF ACTIVE MATERIAL

1. INTRODUCTION

Recently a number of experiments have been performed which measure the multiplication of neutrons by small spheres of active material. (Compare also LAMS-227, 230.) These experiments are essentially differential and serve to check our data on the neutron processes in active material.

We shall calculate theoretical quantities to be compared with the result of the experiment performed by R. L. Walker, J. H. Manley[1)] and co-workers of Group R-3. The experiment is performed by surrounding a source of mock fission neutrons with a sphere of active material. A suitable distance away a fission counter records the counts when the sphere is in place and when the sphere is taken away. The ratio of the counting rates is a weighted measure of the production of neutrons due to fissions taking place in the sphere. We will call this ratio the multiplication[2)] and will denote it by M.

The experiment has been made using spheres of 73% 25 + 27% 28, normal uranium and plutonium. Furthermore with each sphere the measurement has been made using both a 25 foil of 63% 25 + 37% 28 and a 28 foil as detector. The 25 foil responds to neutrons of all energies; the 28 foil on the other hand has a threshold at around 1.1 Mev. The 28 counter, therefore, will be much more sensitive to inelastic collisions taking place in the sphere.

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1) LAMS-207, p. 16; LAMS-222, p. 9; LA-191.

2) Strictly speaking the term multiplication applies only to the ratio of the counting rates when the counter has a flat response. In this case the multiplication represents exactly the number of neutrons produced per neutron from the source and not merely a rather complicated weighted measure of this quantity.

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2. EXPANSION IN NUMBER OF COLLISIONS

The spheres of active material used in the experiment had radii, a, which were less than a mean free path. This means that most of the neutrons that make collisions will make only one collision in the sphere. A few will make two and a very few three. This suggests a study of the problem in the form of an expansion in the number of collisions that the neutrons make. The natural parameter of such an expansion will be $a\sigma$, where σ is the reciprocal of the mean free path. For our spheres then $\alpha \equiv a\sigma < 1$. In the calculations we shall take the unit of length to be the radius of the sphere.

The effects of surrounding the source by the sphere of active material are twofold: first, the flux of neutrons that came directly from the source in the no-sphere experiment has been attenuated by the collisions performed in the sphere; second, the collisions occurring in the sphere multiply the number of neutrons and present to the counter a distributed source in which the intensity is a function of the distance of the point of the sphere from its center. The flux of neutrons that comes directly from the center is attenuated by the factor $e^{-\alpha}/4\pi R^2$. Let us understand this from the point of view of an "expansion in collisions". We need only study $e^{-\alpha}$ since $1/4\pi R^2$ is a purely geometric attenuation. Consider then a slab of active material of thickness equal to b. Suppose one neutron enters the slab n normally how many neutrons emerge which have not suffered any collisions at all?

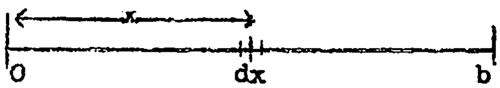

Fig. 1

Transmission through a Slab

Let σ be the probability of a collision per cm. If we now consider only single collisions, we say that the number of neutrons lost in dx is σdx and therefore

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throughout the slab the loss is σb. The number of neutrons that emerge is therefore $1-\sigma b$. If we now include the consideration of double collisions we would say that we have subtracted out too many neutrons because we have just seen that due to single collisions σx fewer get to x. Therefore at x, there will be $\sigma x \cdot \sigma dx$ fewer neutrons taken out of the beam. And across the whole slab $\sigma^2 b^2/2$. This argument in terms of collisions is continued and we see that we obtain the development of $e^{-\sigma b}$. This argument in terms of collisions is continued and we see that we obtain the development of $e^{-\sigma b}$. The term in $(\sigma b)^n$ represents the effect of n collisions contributing positively or negatively to the outcoming beam.

In the case of the sphere of active material we will use exactly this type of reasoning with the addition that neutrons can be created by fission. The calculation will be made in the following way. We will first assume that all the neutrons have the same energy, and that the foil is activated by neutrons of this energy. We will then calculate the multiplication on this basis. From this we will get the correct multiplication by introducing the proper averages in each collision.

3. SINGLE COLLISIONS

Let the source emit 4π neutrons per sec, and let the counter be at a distance R from the source. Then the flux at the counter when the sphere is absent is R^{-2} sec^{-1}. In calculating the multiplication we always require the number of counts with the sphere relative to a single count without the sphere. This conversion may be obtained by multiplying the flux at the counter with the sphere in place by R^2.

In the spherical case we also use the method of collisions which was illustrated above in the case of the slab. In this method, the multiplication is given by the sum

$$1 + M_1(\text{single collisions}) + M_2(\text{double collisions}) + \ldots$$

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We will now calculate M_1, the contribution due to single collisions. Let (r,μ) represent any point in the sphere, and let ρ be the distance of (r,μ) to the counter.

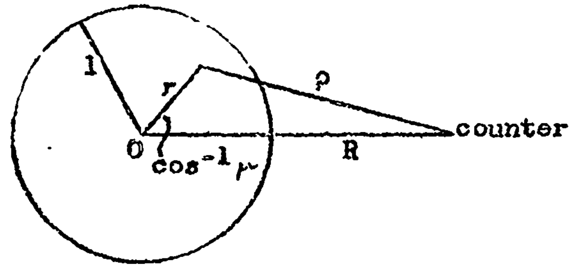

Fig. 2

Geometry for Single Collisions

From the figure we have $\rho^2 = R^2 + r^2 - 2rR\mu$. The flux of neutrons from the source at any point r is $1/r^2$. The element of spherical volume is $2\pi r^2 dr d\mu$. Therefore the number of collisions per sec taking place in $dr d\mu$ is $\alpha \cdot 2\pi dr d\mu$. Let $1+f$ neutrons come off after each collision. The contribution of these neutrons is

$$\frac{\alpha(1+f)}{2}\int_0^1 dr \int_{-1}^1 \frac{d\mu}{\rho^2} = \frac{\alpha(1+f)}{2}\int_0^1 dr\,\frac{R}{r}\ln\frac{1+r/R}{1-r/R} = \alpha(1+f)\left(1+\frac{1}{9R^2}+\frac{1}{25R^4}+\ldots\right)$$

The net contribution due to single collisions is this plus the term, $-\alpha$, resulting from single-collision attenuation of the direct beam from the source to the counter

$$M_1 = \alpha(1+f)(1+1/9R^2+1/25R^4+\ldots) - \alpha \qquad (1)$$

If $R \rightarrow \infty$ we obtain just αf. This is the number of neutrons produced by a neutron traversing the radius of the sphere. If in Eq. (1) we let $f = 0$ we have $M_1 = \alpha(1/9R^2 + 1/25R^4 + \ldots)$ which means that even if only scattering occurs, there is still multiplication. This clearly comes about from the fact that the sphere of material effectively brings the source closer to the counter. In our experiments $R \sim 2$ and the geometric multiplication is $\sim 3\%$.

4. DOUBLE COLLISIONS

When only single collisions are considered, we say that a ray of neutron starts from the source, makes a collision at some point in the sphere, and then goes

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to the counter. The consideration of double collisions is a refinement of this picture. Some neutrons will be scattered out of the ray before they can produce this collision. We say that these neutrons undergo a double collision process. Suppose that the collision has taken place and a ray of neutrons is on its way to the counter. This ray must pass through some matter in the sphere and a certain number of neutrons will be scattered out of this beam. This is a second double collision process. A third double collision process takes place when the neutrons perform a second collision after their first collision and then go to the counter. Finally there will be the contribution of the direct beam from the source in double collisions.

The contribution to the multiplication of the direct beam is $\alpha^2/2$. The contribution of the two fission collisions can be calculated in the following way. The first collision produces a flux of neutrons at (r',μ) of $2\pi\alpha(1+f)dr'd\mu$. Let the second collision take place at r. Then the flux of neutrons at r where $\widehat{rr'} = \cos^{-1}\mu$ is

$$\frac{\alpha^2(1+f)}{2}\int_0^1 dr'\int_{-1}^{1}\frac{d\mu}{r^2+r'^2-rr'\mu} = \frac{\alpha^2(1+f)^2}{2r}\int_0^1\frac{dr'}{r'}\ln\frac{r'+r}{|r'-r|}$$

The contribution to the multiplication is therefore

$$\frac{\alpha^2(1+f)^2}{4}\int_0^1 dr\,r\int_0^1\frac{dr'}{r'}\ln\frac{r'+r}{|r'-r|}\int_{-1}^{1}\frac{d\mu}{\rho^2/R^2}$$

$$=\frac{\alpha^2(1+f)^2}{2}\int_0^1\frac{dr'}{r'}\int_0^1 dr\,r\ln\frac{r'+r}{|r'-r|}\left(1+\frac{1}{3}\left(\frac{r}{R}\right)^2+\frac{1}{5}\left(\frac{r}{R}\right)^4+\dots\right)$$

Performing these integrations we find

$$M_2(\text{two real coll.}) = \alpha^2(1+f)^2\left(\frac{1}{4}\left(\frac{\pi^2}{4}\ \ 1\right)+\frac{1}{24R^2}\left(\frac{\pi^2}{4}+\frac{2}{3}\right)+\dots\right) \qquad (2)$$

The contribution due to attenuation and then fission is obtained as follows. The number of neutrons taken out of the beam is $(1/r^2)\cdot\alpha r = \alpha/r$. Therefore the loss in multiplication because these neutrons did not produce fission, is

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$$-M_2(\text{att.}) = \frac{\alpha^2(1+f)}{2}\int_0^1 r\,dr\int_{-1}^{1}\frac{d\mu}{\rho^2/R^2} = +\alpha^2(1+f)\left(\frac{1}{2}+\frac{1}{3\cdot 4R^2}+\frac{1}{5\cdot 6R^4}+\dots\right) \qquad (3)$$

Finally we come to the loss in multiplication due to the neutrons lost on the way to the counter after fission took place. If the distance traversed in the active material starting at (r,μ) is Z, then the loss in multiplication is

$$-M_2(\text{att.}) = \frac{\alpha^2(1+f)}{2}\int_0^1 dr\int_{-1}^{1}\frac{d\mu\, Z(r,\mu)}{\rho^2/R^2} \qquad (4)$$

Let us find what this distance $Z(r,\mu)$ is. From Fig. 3 we have

$$(r+Z\mu')^2+Z^2(1-\mu'^2) = 1$$

$$Z = r\left(\sqrt{\mu'^2+(1-r^2)/r^2}-\mu'\right)$$

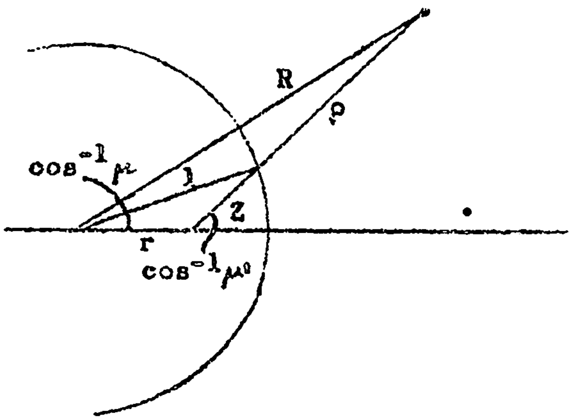

Fig. 3
The Attenuation Distance Z

We will now change variable of integration from μ to μ' in Eq. (4). Let $r/R = x$.

$$\mu' = \frac{R\mu - r}{\rho} = (\mu - x)\frac{R}{\rho}$$

$$\rho/R = \sqrt{1+x^2-2\mu x} = 1-\mu x+\tfrac{1}{2}x^2(1-\mu^2)$$

and

$$\mu' = \mu+(\mu^2-1)\,x-\frac{3}{2}(1-\mu^2)\,\mu x^2$$

neglecting higher powers of x than the second. To solve for μ in terms of μ' let $\mu = \mu'+x\mu_1+x^2\mu_2$. If we substitute this in the previous equation and then set the separate coefficients of different powers of x equal to zero we find $\mu_1 = 1-\mu'^2$, $\mu_2 = -\mu'(1-\mu'^2)/2$

$$\mu = \mu'+(1-\mu'^2)\,x-\tfrac{1}{2}\mu'(1-\mu^2)\,x^2$$

$$\frac{d\mu}{d\mu'} = 1-2\mu' x-(1-3\mu'^2)\frac{x^2}{2}$$

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[redacted]

$$\int_{-1}^{1} \frac{Z\,d\mu}{\rho^2/R^2} = \int_{-1}^{1} \frac{Z\left(1 - 2\mu' x + \frac{1}{2} x^2 (1-3\mu'^2)\right)}{1 - 2\mu x + x^2}\, d\mu'$$

$$= \int_{-1}^{1} r\left(\sqrt{\mu'^2 + (1-r^2)/r^2} - \mu'\right)\left(1 + \tfrac{1}{2}\,(1-\mu'^2)\, x^2\right) d\mu'$$

On carrying out the integrations we find

$$- M_2(\text{att.}) = \frac{\alpha^2(1+f)}{2} \int_0^1 dr \int_{-1}^{1} \frac{Z\,d\mu}{\rho^2/R^2} = \alpha^2(1+f)\left(\tfrac{1}{4}\left(\frac{\pi^2}{4}+1\right) + \frac{\pi^2}{128R^2} + \dots\right)$$

We have therefore the following result

$$M_2 = \frac{\alpha^2}{2} + \alpha^2(1+f)^2\left[\tfrac{1}{4}\left(\frac{\pi^2}{4}+1\right) + \frac{1}{24R^2}\left(\frac{\pi^2}{4}+\frac{2}{3}\right) + \dots\right] - \alpha^2(1+f)\left[\tfrac{1}{2} + \frac{1}{12R^2} + \dots\right]$$
$$- \alpha^2(1+f)\left[\tfrac{1}{4}\left(\frac{\pi^2}{4}+1\right) + \frac{\pi^2}{128R^2} + \dots\right] \qquad (5)$$

5. REMAINDER

With the calculation of the contribution to the multiplication due to neutrons that have made two collisions, we bring to a close the calculation in terms of collisions, and we will now obtain the remainder of the multiplication by an approximate method.

We will adopt a method first applied by Feynman and Ashkin to problems of this nature. Consider the integral equation describing our system. In the one velocity approximation we can write it in the following form

$$n(r) = S(r) + (1+f)\int K(r,r')\, n(r')\, dr' \qquad (6)$$

where n/r is the flux, S/r is the source function and K the kernel. Suppose now that we have an approximate solution to n. This we denote by $n^{(i)}$. Let the remainder be $\overline{n^{(i)}}$. Then $n = n^{(i)} + \overline{n^{(i)}}$. We will now find an approximate expression for $\overline{n^{(i)}}$. Substituting in Eq. (6) we have [redacted]

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$$\overline{n^{(i)}} = S - n^{(i)} + (1+f)\int K\, n^{(i)}\, dr' + (1+f)\int K\, \overline{n^{(i)}}\, dr'$$

$$\overline{n^{(i)}} = S_1 + (1+f)\int K\, \overline{n^{(i)}}\, dr' \tag{7}$$

where $S_1 = S - n^{(i)} + (1+f)\int K(r,r')\, n^{(i)}\, dr'$. Thus $n^{(i)}$ satisfies the same integral equation as n except for a different source function.

We can write down the formal solution to this equation immediately

$$n^{(i)} = \sum_{n=1}^{\infty} \frac{1+f_n}{f_n-f}\left(\phi_n(r)\, S_1(r)\, dr\right)\phi_n(r) \tag{8}$$

where

$$\phi_n = (1+f_n)\int K(r,r')\,\phi_n(r')\, dr' \tag{9}$$

The proof of Eq. (8) is simple. The symmetric kernel $K(r,r')$ generates a complete orthonormal set of eigenfunctions ϕ_n. The associated eigenvalues are $1+f_n$, as shown in Eq. (9). Let us then expand $\overline{n(i)}$ and S_1 in terms of these eigenfunctions

$$\overline{n^{(i)}} = \sum_{n=1}^{\infty} C_n\, \phi_n \tag{8'}$$

$$S_1 = \sum_{n=1}^{\infty} S_{1n}\,\phi_n \qquad S_{1n} = \int S_1\,\phi_n\, dr$$

Substituting in Eq. (7) and using Eq. (9) we find

$$C_n = \frac{1+f_n}{f_n-f}\, S_{1n}$$

If this is substituted in Eq. (8') we obtain Eq. (8).

The equation for $\overline{n^{(i)}}$ is an exact expression. We now made the approximation $f_n = f_o$ for all n. This means that we put all the neutrons from the source in the normal mode.

$$\overline{n^{(i)}} = \frac{1+f_o}{f_o-f}\sum_n (\phi_n\, S_1)\,\phi_n = \frac{1+f_o}{f_o-f}\, S_1$$

$$\overline{n^{(i)}} = \frac{1+f_o}{f_o-f}\left\{S - n^{(i)} + (1+f)\int K(r,r')\, n^{(i)}\, dr'\right\}$$

The contribution to the multiplication corresponding to $n^{(i)}$ we write $M^{(i)}$. If we say that the counter is at infinity we have

$$\overline{M^{(i)}} = \alpha f \int_0^1 \left(\frac{\overline{n^{(i)}}}{r}\right) r^2\, dr$$

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6. CORRECTIONS

The spheres of active material used in the experiments each had a small spherical hole at the center which served to hold the source of neutrons. The radius of this inner sphere was 0.396 cm. The source of neutrons was approximately spherical in shape of radius 0.336 cm. Our calculations so far have assumed a point source and no hole, and it is now necessary to find the effects of these perturbations. They are by no means negligible.

The material in the source has approximately the same scattering properties as the active material. We make a negligible error if we assume the scattering properties to be exactly the same, and furthermore if we assume for simplicity that the space between the source and the active material is filled with inactive material but scattering like the active material.

It is easy to show that if we have a spherical source of radius b and source strength q per unit volume, then the flux, F, at a point distance r from the center of the sphere is given by

$$F(r) = \frac{q}{2r}\left(\frac{b^2-r^2}{2}\ln\frac{r+b}{|r-b|}+rb\right) \tag{10}$$

It is convenient to have $F(r)$ as a series in r. We normalize the total source strength to 4π neutrons per sec, $\emptyset \equiv \frac{4\pi}{3} b^3 \cdot q = 4\pi$. We have then

$$r \geq b \qquad F(r) = \frac{1}{r^2}\left(1+\frac{3}{3.5}\left(\frac{b}{r}\right)^2+\frac{3}{5.7}\left(\frac{b}{r}\right)^4+\ldots\right) \tag{10'}$$

$$r \leq b \qquad F(r) = \frac{3}{b^2}\left(1-\frac{1}{1.3}\left(\frac{r}{b}\right)^2-\frac{1}{3.5}\left(\frac{r}{b}\right)^4+\ldots\right) \tag{10''}$$

It is now necessary to go through our previous calculations taking into account this more complicated flux distribution and also the fact that in the region which we designate $0 \leq r \leq C$ no fissions take place corresponding to the hole in the sphere of active material. The calculation is straightforward and we now give the new expressions for the contributions to the multiplication of the different collision processes, here $b/C = d$.

[redacted]

$$M_1 = \alpha(1+f)\left[1 - C + \frac{3}{1.3.5}\, db(1-C) + \frac{3}{3.5.7}\, d^3 b(1-C^3) + \dots + \frac{1}{3R^2}\left(\frac{1}{3}\,(1-C^3) + \frac{3}{3.5}\, b^2(1-C) + \frac{3}{1.5.7}\, db^2(1-C) + \dots\right) + \dots\dots\right]$$

$$+ \alpha\left[C(1-d) + \frac{3}{1.3.5}\, b(1-d) + \frac{3}{3.5.7}\, b(1-d^3) + \dots + \frac{1}{3R^2}\left(\frac{C^3}{3}\,(1-d^3) + \frac{3}{3.5}\, b^2 C(1-d) + \frac{3}{1.5.7}\, b^3(1-d) + \dots\right) + \dots\dots\right]$$

$$+ \alpha b\left[\frac{3}{4} + \frac{b^2}{R^2}\left(\frac{1}{5} - \frac{1}{1.3.7} - \frac{1}{3.5.9} - \dots\right) + \frac{1}{5}\left(\frac{b}{R}\right)^4\left(\frac{1}{7} - \frac{1}{1.3.9} - \frac{1}{3.5.11} - \dots\right)\right]$$

$$- \alpha\left[\left(1 - \frac{b^2}{5} - \dots\right) + \frac{1}{5}\left(\frac{b}{R}\right)^2\left(1 - \frac{2b^2}{7} - \dots\right)\right] + \dots\dots$$

$$M_2(\text{two real coll.}) = \alpha^2 J_1 + \alpha(1+f)\,\alpha J_2 + \alpha\cdot\alpha(1+f)\, J_3 + \alpha^2(1+f)^2\, J_4$$

$$J_1 = 3bC\left[\frac{1}{4} - \frac{1}{1.3}\, d^2\left(\frac{1}{5} - \frac{1}{1.3.7} - \frac{1}{3.5.9} - \dots\right) - \frac{1}{3.5}\, d^4\left(\frac{1}{7} - \frac{1}{1.3.9} - \frac{1}{3.5.11} - \dots\right)\right]$$

$$+ C\left[C(1-d) + \frac{3}{1.3.5}\, b(1-d) + \frac{3}{3.5.7}\, b(1-d^3) + \dots\right.$$

$$- \frac{1}{1.3}\left(\frac{1}{3}\, C(1-d^3) + \frac{3}{1.3.5}\, d^2 C(1-d) + \frac{3}{1.5.7}\, d^2 b(1-d) + \dots\right)$$

$$\left. - \frac{1}{3.5}\left(\frac{1}{5}\, C(1-d^5) + \frac{3}{3.3.5}\, db(1-d^3) + \frac{3}{5.7}\, d^4 C(1-d) + \dots\right) + \dots\dots\right]$$

$$J_2 = C\left[C\left(\frac{1}{1.3}\,(1-C) + \frac{1}{3.3.5}\,(1-C^3) + \frac{1}{5.5.7}\,(1-C^5) + \dots\right)\right.$$

$$\left. + \frac{3}{3.5}\, db\left(\frac{1}{1.3.3}\,(1-C^3) + \frac{1}{3.5.5}\,(1-C^5) + \dots\right) + \frac{3}{5.7}\, bd^3\left(\frac{1}{1.3.5}\,(1-C^5) + \dots\right)\right] + \dots\dots$$

[redacted]

$J_3 = H_3 - J_1$

$$H_3 = 3b\left[\frac{1}{4} - \frac{b^2}{1\cdot3}\left(\frac{1}{5} - \frac{1}{1\cdot3\cdot7} - \frac{1}{3\cdot5\cdot9} - \dots\right) - \frac{b^4}{3\cdot5}\left(\frac{1}{7} - \frac{1}{1\cdot3\cdot9} - \dots\right)\right] + C(1-d) + \frac{3}{3\cdot5}\,b(1-d)$$

$$+ \frac{3}{3\cdot5\cdot7}\,b(1-d^3) + \frac{3}{5\cdot7\cdot9}\,b(1-d^5) + \dots - \frac{1}{1\cdot3}\left(\frac{1}{3}\,C^3(1-d^3) + \frac{3}{3\cdot5}\,b^2C(1-d) + \frac{3}{5\cdot7}\,b^3(1-d) + \dots\right)$$

$$- \frac{1}{3\cdot5\cdot}\left(\frac{1}{6}\,C^6\left(1 - \frac{b^6}{C^6}\right) + \dots\right) + \frac{1}{3R^2}\left(C - \frac{2}{3}\,b\right) + \dots\dots$$

$J_4 = H_4 - J_2$

$$H_4 = 1 - C + \frac{3}{1\cdot3\cdot5}\,db(1-C) + \frac{3}{3\cdot5\cdot7}\,d^3b(1-C^3) + \dots - \frac{1}{1\cdot3}\left(\frac{1}{3}\,(1-C^3) + \frac{3}{3\cdot5}\,b^2(1-C)\right.$$

$$\left. + \frac{3}{1\cdot5\cdot7}\,db^3(1-C) + \dots\right) - \frac{1}{3\cdot5}\left(\frac{1}{5}\,(1-C^5) + \frac{3}{3\cdot3\cdot5}\,b^2(1-C^3) + \dots\right) + \frac{1}{24R^2}\left(\frac{\pi^2}{4} + \frac{2}{3} - \frac{8}{3}\,C\right) + \dots\dots$$

$$-\ M_2(\text{att. then coll.}) = -\alpha^2(1+f)\left[\frac{1}{2}\,(1-C^2) + \frac{1}{12R^2}\,(1-C^4) + \dots\right]$$

$$-\alpha^2\left[\frac{1}{2}\,C^2(1-d^2) + \frac{C^4}{12R^2}\,(1-d^4) + \dots\right] - \alpha^2 \cdot \frac{2}{5}\,b^2 + \dots\dots$$

$$-\ M_2(\text{coll. then att.}) = \alpha(1+f)\alpha\left[1 - C - \frac{1}{1\cdot3^2}\,(1-C^3) - \frac{1}{3\cdot5^2}\,(1-C^5)\ \dots\right.$$

$$+ \frac{1}{R^2}\left(\frac{1}{9}\,(1-C^3) - \frac{2}{75}\,(1-C^7) - \dots\right) + \frac{3}{3\cdot5}\,b^2\left(\frac{1-C}{C} - \frac{1}{3}\,(1-C) - \dots\right)$$

$$\left. + \frac{3}{3\cdot5}\left(bd(1-C) - \frac{b^2}{3}\,(1-C) - \dots\right) + \frac{3}{5\cdot7}\left(\frac{1}{3}\,bd^3(1-C^3)\dots\right) + \dots\dots\right]$$

$$+ \alpha^2\left[C - b - \frac{1}{9}\,C^3(1-d^3)\dots + \frac{3}{3\cdot5}\,b\left(1 - d - \frac{b^2C}{3}\,(1-d) + \dots\right) + \dots\dots\right]$$

$$M(\text{direct beam}) = \frac{\alpha^2}{2}\left[1 - \frac{b^2}{5} + \frac{1}{R^2}\left(\frac{b^2}{5} - \frac{3b^4}{5\cdot7}\right) + \dots\right]$$

[redacted]

7. DATA AND RESULTS

The mock source of fission neutrons was obtained by the reactions $Po + NaBF_4 + 2\%\ BeF_4$. Natural alphas on boron do not produce a spectrum of sufficiently high average energy; so a small amount of beryllium was added. The resulting spectrum as measured by Richards[3] is given in Fig. 4 together with the latest fission spectrum.[4]

The method of calculating the averages to be put in each collision has been given by Kurath and Rarita[5]. In the case of 25 and 28 we possess rather extensive data and here there is no difficulty in obtaining the proper energy averages[5]. On the other hand for 49 only the fission cross section is known as a function of energy. We have therefore used what we believe to be reasonable values and in the case of the experiment with the 28 counter the calculation has been made for three values of the inelastic cross section: .75, 1.0, and 1.25 barns. The other data used were: $\sigma_f = 1.80$ b, $\nu-1-\alpha = 2.02$, or more significantly, $\sigma_f(\nu-1-\alpha) = 3.64$, and $\sigma_{total} = 4.77$ b.

The tables below give the calculated values of M the ratio of the counting rates when the sphere is in place and when the sphere is removed. For comparison we have added the experimentally observed values of M.

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3) H. T. Richards, LA-201.

4) H. T. Richards, LA-200.

5) D. Kurath and W. Rarita, LA-197. To the data used here must be added a 3% reduction in σ_f for pure 73% 25, indicated by the multiplication experiments with the flat counters.

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TABLE I: Experiments with 25 (73% 25, 27% 28)

Diameter	Counter	M_{calc}	M_{obs}
2.5"	25	1.383	1.35 ± .02
	28	1.082	1.13 ± .02
2.0"	25	1.262	1.28 ± .06
	28	1.032	1.07 ± .05

TABLE II: Experiments with normal uranium

Diameter	Counter	M_{calc}	M_{obs}
2.5"	25	1.044	1.031 ± .011
	28	0.707	0.760 ± .012

TABLE III: Experiments with 49

Diameter	Counter	M_{calc}	M_{obs}
.9"	25	1.199	1.202 ± .013
	28_1	1.187	
	28_2	1.176	1.175 ± .015
	28_3	1.165	

As an example of the contribution of the various terms to M let us consider the 2.5" sphere of 25 used in conjunction with the 25 counter. The zero collisions give 1. The first collision gives 0.2935 the second 0.0915 the remainder 0.0388. Total 1.4238. Correcting for the fact that the 25 foil is 63% 25 and 37% 28 we find that the result of the experiment should be 1.383. The slight difference between the experimental values here listed and those in Ref. 3 arises from the fact that the latter have been corrected to correspond to a pure 25 detector.

Richards[6] has measured the spectrum of neutrons emerging from the 2.5" sphere of 25. Fig. 5 shows the calculated spectrum together with Richards[3] results.

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6) Private communication.

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On the whole the agreement between theory and experiment is satisfactory except for the multiplication of the 25 spheres as measured by a 28 counter. Such a discrepancy has also been reported by Kurath and Rarita[7] when they calculated the result of similar experiments performed by Snyder, Wilson, and Woodward[8]. In our case a reduction of ~35% in the inelastic scattering is required to bring the theoretical results in agreement with experiment. It is not possible to learn anything from the comparison of the measurements of the spectrum that comes out of the $2\frac{1}{8}$" sphere with the theory on this point because the difference in the spectrum (Fig. 5) sought is far inside the probable error.

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7) Loc. Cit.

8) LA-189.

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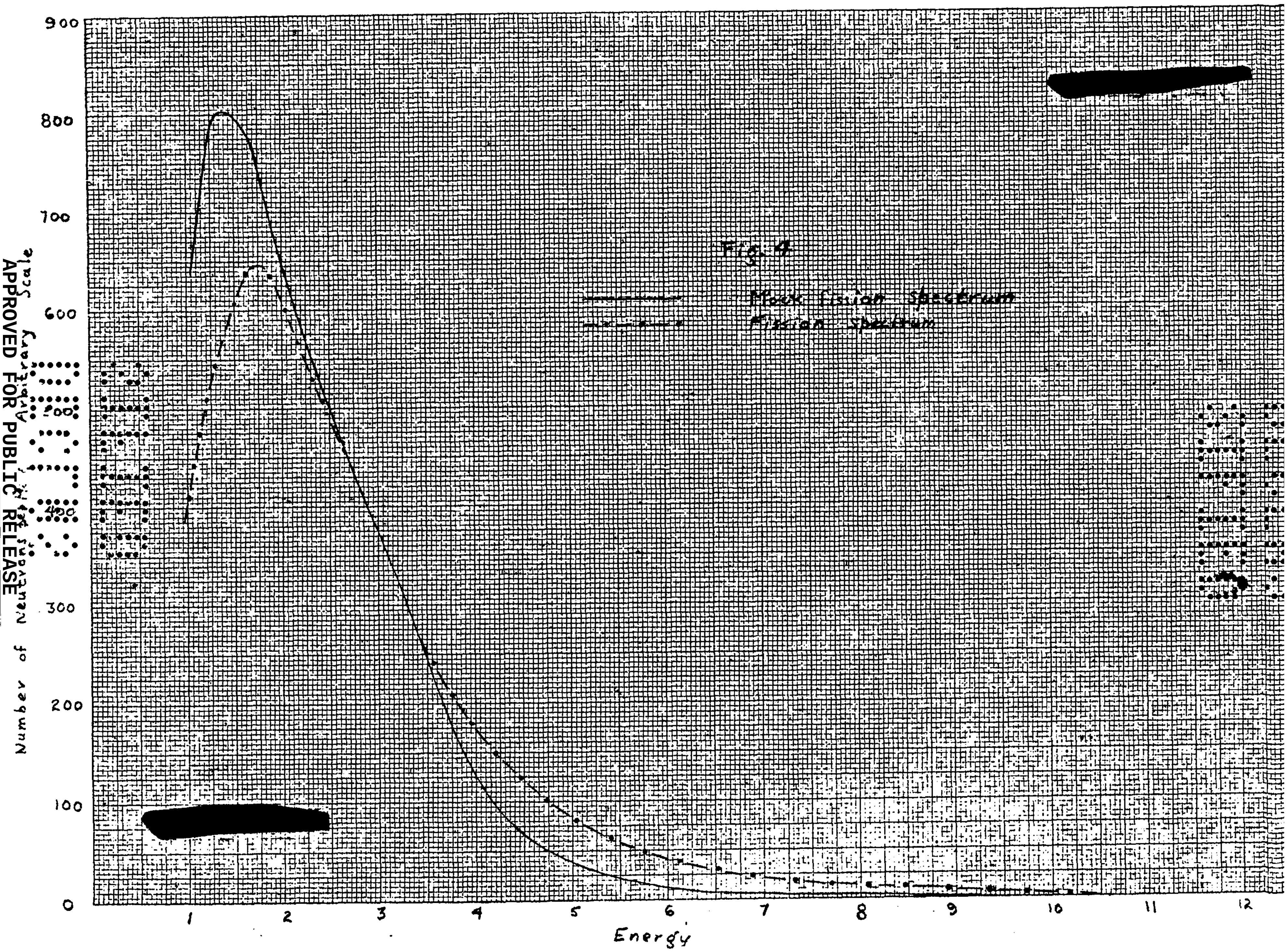

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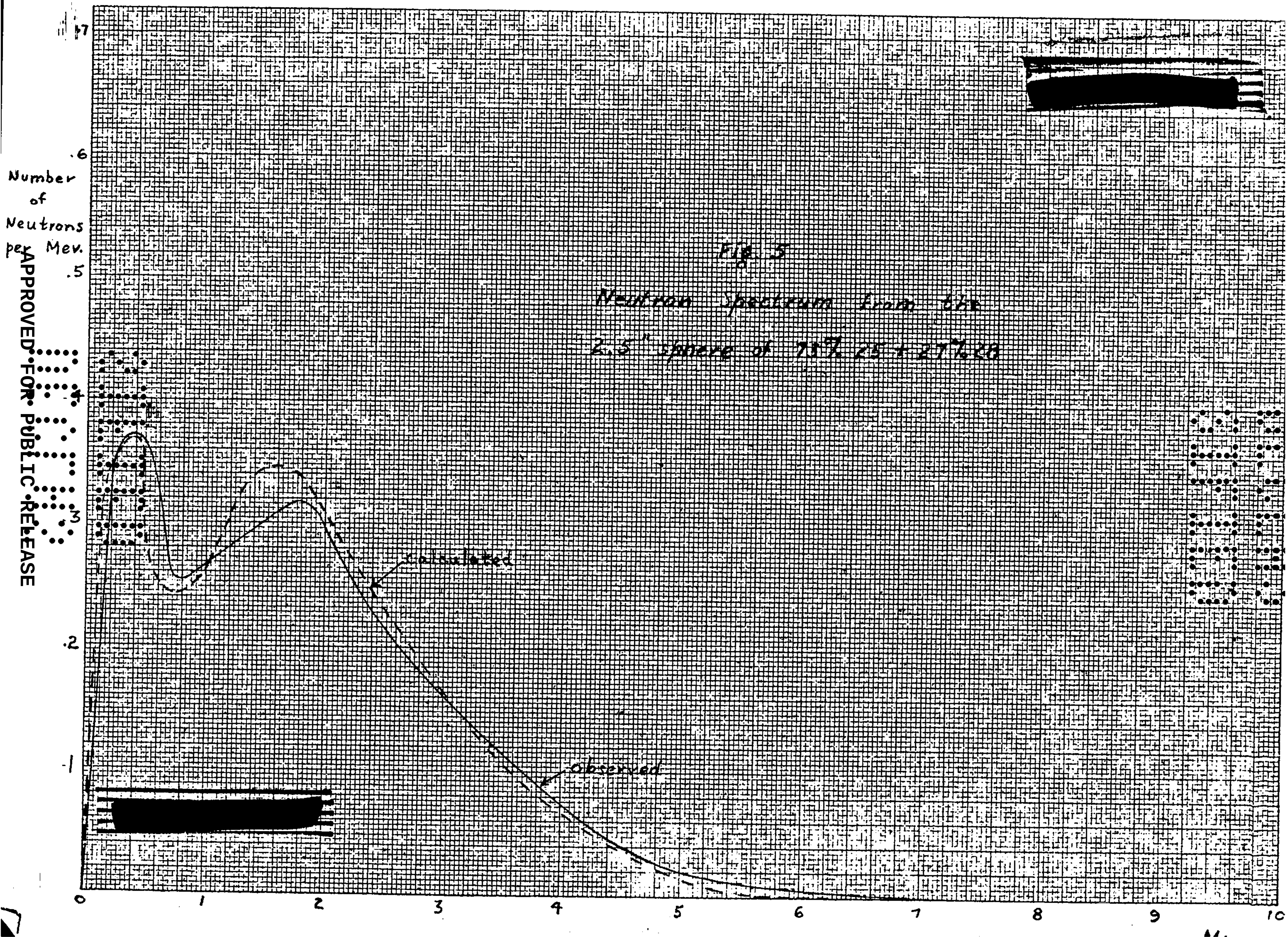

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