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# ANALYTIC OPTIMIZATIONS IN CRISIS STABILITY

by

Gregory H. Canavan

## ABSTRACT

Second strikes are dominated by submarine launched missiles in the absence of defenses, but shift to aircraft at modest levels of defense. Defenses protect some retaliatory missiles, but not enough to retaliate strongly. With defenses, missiles should be vestigial and could be eliminated without penalty. Then aircraft could also be significantly reduced without impacting stability. The combination of parameters that maximizes cost effectiveness also maximizes midcourse effectiveness and crisis stability.

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## I. INTRODUCTION

Crisis stability models must include a large number of seemingly complicated interactions, which can make it hard to understand the sources of important results.<sup>1</sup> This note gives a simple, approximate version thought to contain most of the features needed to understand the behavior of stability indices and their optimization by proper mixtures of defenses.

## II. ELEMENTS

The main elements of crisis stability models are inter-continental and submarine-launched ballistic missiles (ICBMs and SLBMs) and bombers, cruise missiles, and carriers--or aircraft for short. The number of reentry vehicles (RVs) that penetrate given boost-phase space-based interceptor (SBI) defenses can be evaluated exactly, but the results are awkward to manipulate.<sup>2</sup> A simple analytic function approximating the number of ICBM reentry vehicles (RVs) penetrating a constellation of K SBIs is<sup>3</sup>

$$R \approx mM \cdot e^{-fK/M}, \quad (1)$$

where  $m \approx 10$  is the number of RVs per missile and  $M \approx 270$  is the total number of Soviet heavy missiles. About half of them are fixed and half mobile, but given their long burn times, it is not necessary to distinguish between them for START near-term force levels and basings. The fraction of the SBIs within range of launch is  $f$ . It is about 20% for current basing, but would drop to about 13% for fixed heavy missiles and 10% for mobile heavy missiles if under START, all were located in the current heavy missile launch area.<sup>4</sup>

Fast mobile singlets such as SS-25s have  $f \approx 2-3\%$ . Thus, optimal allocations essentially give them a free ride through the SBIs, although the  $\approx 350$  singlets contemplated under START do not carry enough RVs to impact stability significantly.<sup>5</sup> Their inclusion has been studied,<sup>6</sup> but they are ignored below. The fraction of RVs penetrating the boost phase from Eq. (1) is exact for both small and large  $K$  but is overestimated by 10%-20% for intermediate  $K$ .<sup>7</sup> Thus, the calculations below underestimate the effectiveness of boost-phase defenses.

It is useful to write the number of penetrating RVs as  $R = mMx$ , where  $x = e^{-fK/M}$  is the fraction of the heavy ICBM RVs that penetrate boost. For 270 START-constrained heavy missiles, 2,000 SBIs give  $x \approx \exp(-0.13 \cdot 2000/270) \approx 0.4$ . The general relationship of  $x$  to  $K$  is shown by the top curve of Fig. 1. According to Eq. (1) increasing the number of missiles or replacing current missiles with faster ones would only rescale

the number of SBIs required. Thus, it is not necessary to vary them to study boost-phase requirements and effectiveness.

The Soviet Union has about 20 submarines with a total of  $H \approx 400$  SLBMs with an average of  $n \approx 6$  RVs each. The fraction that penetrate the SBIs can be written as approximately  $e^{-fK/\phi H}$ , where the factor  $\phi \approx 1/4$  is included to incorporate clustering before launch, without which few SLBMs would penetrate.<sup>8</sup> For START-constrained forces,  $fK/\phi H \approx (fK/M)(4M/H) \approx (fK/M)(4 \cdot 300/400) \approx 3(fK/M)$ , so that  $e^{-fK/\phi H} \approx x^3$ , as shown by the second curve on Fig. 1. It shows that when ICBMs were suppressed by a factor of 0.4, SLBMs would be suppressed by a factor of  $\approx 0.05$ , i.e. about an order of magnitude more.

Since  $nH \approx mM$ , when the SLBMs' contribution is added to that of the ICBMs, the total number of RVs penetrating the defender's SBIs is

$$R \approx mM(x + x^3), \quad (2)$$

so that the total number of penetrating RVs at  $x \approx 1$  is about twice that from ICBMs alone, but the SLBM contribution decays rapidly. For that reason the SLBM RVs, and with them the submarines' contribution to deterrence, can be safely ignored above 1,000-2,000 SBIs. Phase 1 contemplates the deployment of 2-4 times that number.

The variation of performance and indices with offense and defense parameters is studied elsewhere;<sup>9</sup> the purpose of this note is to point out a few important relationships. Accordingly, it is assumed below that the same offensive forces apply to both sides and that both sides deploy identical defenses.

### III. MIDCOURSE DEFENSES

Midcourse defenses can be preferential or non-preferential. The latter would give essentially another factor of  $x$  attrition; the former could be more effective. Midcourse interceptors are assumed to act preferentially and to have long enough ranges to cover the whole target set. Taking them to act adaptively would improve performance slightly at low levels, but would not be



consistent with the sensors likely to be available in the near and midterms.<sup>10</sup>

If  $R$  penetrating RVs attacked  $M$  targets, there should be  $\approx R/M$  RVs per target. Thus,  $R/M$  interceptors could defend any given target, and  $I$  interceptors could defend  $I/(R/M)$  targets. If there were  $D$  decoys per RV, there would be a total of  $(1+D)R$  threatening objects, so that  $I$  interceptors could defend

$$S = I/[(1+D)R/M] = M[I/(1+D)R] = Ie^{fK/M}/(1+D)m \quad (3)$$

targets, or a fraction  $\Gamma \equiv S/M = I/(1+D)R$  of them up to  $I = (1+D)R$ , for which  $S = M$ . The choice of  $\Gamma$  is somewhat judgmental. For cost-effectiveness studies it is usually sufficient to observe that  $S \approx 300$  surviving missiles out of  $M \approx 1,000$  deployed, which would represent a robust deterrent, would correspond to a fixed  $\Gamma = 0.3$ . That would determine the  $I$  needed for any  $R$ . For example,  $R = 1,000$  RVs, each with 10 credible decoys, gives  $I \approx 0.3 \cdot 11 \cdot 1,000 \approx 3,300$  interceptors. Their cost, together with those of the SBIs, would determine the variable defense costs, whose ratio to the RV costs would determine the defense's cost effectiveness.

For crisis stability analyses,  $S$  and the restrike vary.  $S/M$  scales on  $I/(1+D)R$ , so it is useful to absorb  $(1+D)$  into  $I$  to make the midcourse interceptors "ideal," i.e. net of decoys, which is done below. Since Eq. (3) only holds for  $I < R$ , it is convenient to replace  $\min(I/R, 1)$  by  $1 - e^{-I/R}$ . The substitution is not essential, but the error is modest.

The RVs penetrating the boost-phase defenses can be used on ICBMs, aircraft, or value targets, i.e. projection forces. It has, however, been shown that a targeting strategy that allocates about a third of the penetrating RVs to each of the three induces defenses that allocate their interceptors similarly. That joint allocation is not overly sensitive to changes by either side.<sup>11</sup>

Figure 2 shows the portion of the first strike reaching the defender's value. At the left border, from the bottom the curves are for  $I = 2,000, 1,000, 500,$  and  $10$  midcourse interceptors. The top curve is the total number of penetrating RVs from Eq. (2), about one third of which would be directed towards value.

Both sides have  $V \approx 2000$  value targets of their own to protect and a like number of the other's to hold at risk. The number of RVs is less than 20%-30% of the strike on value above  $\approx 1,000$  boost-phase defenders. The number of RVs striking value is not large with defenses; it is taken to be a third of the difference between the penetrating RVs and interceptors. Aircraft weapons are added to determine the total strike on value, which falls to about 1,400 weapons by 1,000 boost-phase defenders.

If each of the three target sets has about the same ratio of interceptors to RVs, missiles and bombers have the same survival function

$$\epsilon = 1 - e^{-I/R} = 1 - e^{-I/mMx} = 1 - \exp(-Ie^{fK/M}/mM), \quad (4)$$

which is shown in Fig. 3. For zero SBIs,  $\epsilon$  starts at  $I/R$  and increases monotonically with  $I$  for each number of SBIs and with SBIs for each  $I$ . Note that for  $I \rightarrow 0$ , weak or no preferential layers,  $\epsilon \rightarrow 0$  for all SBIs. Eq. (4) only holds for  $R > M$ , i.e. more than enough penetrating weapons to cover all targets. For first strikes that condition becomes  $R/3 = mMx/3 > M$ , or  $x > 3/m \approx 0.3$ , which from Fig. 1 is often met.

Knowing  $\epsilon$  also makes it possible to evaluate the effectiveness of the offensive missiles. The possibility of an ICBM RV penetrating the boost-phase defenses is  $x$ ; the probability of its penetrating all the way to a missile is roughly  $1 - \epsilon$ . Thus, its overall probability of destroying a target is  $\approx x(1 - \epsilon)$ , which is shown in Fig. 4. The compound probability falls strongly, but for  $I \approx 0$  it is still approximately 15% at 4,000 SBIs. The reduction is larger for large  $I$ . For 2,000 SBIs and midcourse interceptors the probability of destroying a target is about 7%; for 3,000 SBIs, less than 1%.

The dependence of  $\epsilon$  on  $I$  is the core of the exchange. It is also the basis for confusion about crisis stability models.  $\epsilon$  is sensitive to modest defenses. For a near-term  $I/mM \approx 1/3$ , START-constrained offenses, and  $K \approx 2,000$  SBIs,  $\epsilon \approx x \approx 0.4$ , both of which are appreciable, so midcourse defenses are pivotal. To get the most out of them, the defender combines  $I$  and  $K$  to maximize

$Ie^{fK/M}$ , thereby maximizing the survivability of his retaliatory assets.

By Eq. (3) this is the same combination of parameters that is maximized in cost-effectiveness analyses.<sup>12</sup> That means that the combination of parameters that maximizes cost effectiveness also maximizes midcourse effectiveness and crisis stability, which implies that maximizing cost effectiveness could simultaneously maximize crisis stability.

If midcourse was non-preferential, Eq. (3) would not apply. An RV's penetration probability would then be  $x$ . The number of RVs targeted on each missile about  $mM/3M \approx 4$ . Thus, for  $x = 0.4$  a missile's survival probability would be about  $(1-x)^4 \approx 13\%$ . For  $\approx 20$  airbases the number of RVs targeted on each might be  $\approx 20-50$ . If so, the survival probability of a non-alert aircraft would be about  $(1-x)^{20} \approx 4 \cdot 10^{-5}$ , which is well below the  $\approx \epsilon$  bases expected with preferential midcourse defenses.

Without a preferential layer the defender's ICBMs would be drawn down strongly, SLBMs more strongly, and non-alert aircraft eliminated altogether. All of the major components of his retaliatory strike would be reduced without compensation, which would degrade crisis stability. With preferentiality, at least some missiles and aircraft would remain.

Aircraft arrive long after the RVs. All of their weapons would be deposited on value, since there would be little else left to strike. If the attacker had  $B$  aircraft-borne weapons that struck from an alert rate  $\alpha$ , and the rest of his aircraft were destroyed, that would give  $\alpha B$  weapons. Most should arrive because they should be immune to missile defenses once in flight. Ideally, the attacker would strive to make  $\alpha \rightarrow 1$  to maximize his first strike, but that could be detected. If so, the defender could disperse his aircraft and increase his alert rate, which could negate the benefit of striking first. Thus, the attacker's alert rate could be little more than a nominal  $\alpha \approx 30\%$ .

Stability indices are sensitive to  $\alpha$ , since the contribution from aircraft is larger than that from RVs for moderate and strong defenses. For that reason results are sensitive to

measures such as attacks by close-in SLBMs, particularly those on depressed trajectories, which could greatly reduce the warning time and hence effective alert rates.

#### IV. RESTRIKES

The defender's restrike can be calculated by reversing the procedure. If before the attack he has the same number of missiles and RVs as the attacker, the number he can launch is reduced by the attacker's first strike to  $\epsilon \cdot M$  missiles. The fraction of them penetrating the attacker's boost phase defense is shown in Fig. 5. The curves fall strongly for small numbers of SBIs, and then hold up at a reduced number for large numbers. Three factors contribute to the latter behavior. The first is  $\epsilon$ , the probability of surviving the strike, discussed above.

The second is the restrike missiles penetrating the boost phase defenses. SBI performance improves for smaller strikes, so the restrike's penetration would be  $y = e^{-fK/\epsilon M} = (e^{-fK/M})^{1/\epsilon} = x^{1/\epsilon}$ .  $y$  is small compared to  $x$  for  $\epsilon$  small, but for large defenses,  $\epsilon \rightarrow 1$  and  $y \rightarrow x$ . Figure 6 shows restrike penetration as a function of the number of SBIs and midcourse interceptors. For  $I > 0$  it has a long tail that leads to the similar behavior of Fig. 5.

The third factor influencing the effectiveness of RV restrikes is their probability of penetrating the attacker's midcourse defense, which is essentially the fraction of the RVs in excess of the total number of attacker midcourse interceptors. That is shown in Fig. 7. Only for  $I \approx 0$  is there any significant penetration for large numbers of SBIs. For 500 midcourse interceptors, penetration falls to 0 at 1,500 SBIs. For 1,000 it falls to 0 at 1,000; for 2,000 at 500 SBIs. For modest defenses the RV restrike is small.

Those three factors are combined in Fig. 8, which shows the overall probability of an ICBM RV penetrating to target. For  $I$  small the probability is small because few survive the first strike. For  $I = 500$  the probability is non-zero out to about 1,500 SBIs, but never greater than about 7%. For  $I = 1,000$  it

has a greater maximum, but only extends out to 1,000 SBIs. At 2,000 it reaches over 13% but does not extend beyond 500 SBIs. Significant missile defenses protect some retaliatory missiles, but not enough to retaliate decisively.

Thus, the bulk of the restrike must be carried by aircraft. The defender's midcourse defenses should be able to defend his air bases against threats that were attrited by modest boost-phase defenses. All aircraft on alert are assumed to survive. The fraction of those not on alert that survive because they are defended should be about that given by Eq. (4). Once airborne, aircraft are assumed immune to missile defenses, although there are some caveats.<sup>13</sup> The total number of aircraft weapons in the restrike would thus be

$$W = [\alpha + (1-\alpha)\epsilon]B \equiv \beta B, \quad (5)$$

where the first term is the  $\alpha \approx 30\%$  of the aircraft normally on alert and the second is the fraction not on alert that was defended. The overall form of  $\beta$  as a function of  $K$  is shown in Fig. 9. It resembles  $\epsilon$  of Fig. 3, although shifted up by  $\alpha$  and compressed by  $\alpha/(1-\alpha)$ .

The total second strike is the sum of the missile and aircraft strikes, which is shown in Fig. 10. It is clear that the dominant contribution below  $\approx 500$  SBIs is the large number of SLBM RVs from Fig. 5, but beyond that the dominant contribution is the aircraft weapons from Fig. 9. The shift from RV to aircraft retaliation occurs at quite modest numbers of SBIs. It is interesting that for large  $K$  the restrikes converge to two trajectories. The one for  $I \approx 0$  falls throughout. Those for large  $I$  converge to about 4,500 weapons because as more SBIs are added, eventually all of the aircraft are protected and can restrike. Adding midcourse interceptors simply makes it possible to approach that condition with fewer SBIs.

## V. COSTS

The costs for the first striker are those for imperfect limiting of damage to his value and for his imperfect strike on the other's value.<sup>14</sup> The two costs can be expressed in terms of

the first,  $R_1$ , and second,  $R_2$ , strikes on value discussed above. The total costs for the two sides are weighted averages of them. For exponential damage functions, the costs for striking first and second are<sup>15</sup>

$$C_1 = 1 - e^{-R_2/V} + Le^{-R_1/V} \text{ and} \quad (6)$$

$$C_2 = 1 - e^{-R_1/V} + Le^{-R_2/V}, \quad (7)$$

where  $V$  is the number of targets held at risk.  $L$  is the relative weighting of value strikes and damage limiting.  $L = 0.3$  is used below; the results are not sensitive to the precise value.

Figure 11 shows the costs of first strikes for the  $R_1$  from Fig. 2 and the  $R_2$  from Fig. 10. They start at  $\approx 0.95$  for no SBIs and broaden out to 0.7-1. Once again, however, the curves converge to two trajectories. That for  $I \approx 0$  falls to  $\approx 0.7$ ; the rest converge to  $\approx 1.1$ . Apart from a dip at  $\approx 1,000$  SBIs, for all large  $I$  the costs to the attacker for striking first increase to levels greater than those in the absence of defenses.

Figure 12 shows the costs of second strikes. They start at about 0.8 and then all fall. The cost for  $I \approx 0$  falls to  $\approx 0.7$ ; the rest approach  $\approx 0.5$ . The costs for waiting rather than striking first fall monotonically with increasing defenses.

These two costs can be combined into a stability index. While a heuristic derivation is possible, here it is simply observed that increasing the cost for striking first or decreasing the cost for striking second appear to be stabilizing, so that an useful index of stability is  $C_1/C_2$ . Figure 13 shows  $C_1$ ,  $C_2$ , and  $C_1/C_2$  as functions of the number of SBIs for a mid-term  $I = 1,000$ . The trends in the costs have been noted above; their ratio leads to the index of the top curve. It is relatively flat to  $\approx 1,000$  SBIs. Then it climbs, asymptoting to about 2, which is set by the penetration of restrike aircraft.

Figure 14 shows the indices for several values of  $I$ . Again, that for small  $I$  falls monotonically, indicating that pure boost-phase defenses would be destabilizing in this metric. Those for large  $I$  increase, apart from a small dip at small defenses, converging to about 2. The index is a monotonically increasing function of  $I$  for every number of SBIs. The increase is small

for few SBIs, which indicates that midcourse defenses of these sizes would not by themselves increase stability significantly. For 1,000 or more SBIs the increase with  $I$  is strong, which indicates that defenses with comparable numbers of boost-phase and midcourse defenses could be stabilizing.

## VI. PROTECTION OF FIRST-STRIKE AIRCRAFT

The indices of Figs. 13 and 14 rise to above unity because for large defenses the first and second strikes, largely from aircraft, tend to  $\alpha B$  and  $B$  weapons, respectively. The model above assumes that the attacker strikes from some alert rate  $\alpha$ , so that  $1 - \alpha$  of his aircraft are lost, but the defender can asymptotically defend all of his aircraft and deliver a larger strike in retaliation. Under some conditions that assumption is appropriate, e.g. if the defender used close-in SLBMs to underfly missile defenses and destroy all non-alert aircraft immediately after the first attack.

It is, however, useful to study the completely symmetric case in which the attacker can also defend his aircraft. Then if the defender did not retaliate against those aircraft, all would survive to deliver their full inventory. If, however, he diverted all of his penetrating RVs to them, he would be unable to destroy any value. Intermediate allocations seem appropriate.

Fig. 15 shows the survivability of the attacker's non-alert aircraft under the assumption that half of the restriking RVs are allocated to the aircraft and the rest to value. In contrast to Fig. 3, except for small  $I$  the attack aircraft survivability function increases rapidly with  $K$  and  $I$  to  $\approx$  unity, reflecting in part Fig. 8's result that except at  $K \approx 0$  there are few restriking RVs left to allocate to aircraft suppression. Overall, most attack aircraft survive.

Figure 16 shows the resulting first strikes. The curve for  $I \approx 0$  drops down to  $\approx 2,000$ , but the rest remain about constant at about 4,500. The top curve is the number of attacking RVs penetrating boost, which is dominant for few SBIs, but falls away quickly. The differences from Fig. 2 are due to the greater

number of aircraft in the attack. All of the additional weapons are delivered late and on value. Thus, the restrike is not impacted, other than by the need to partition RVs between value and aircraft.

Figure 17 shows  $C_1$ . While superficially different from Fig. 11, its underlying structure is essentially the same. The three curves for  $I > 0$  again converge. The limit is lower than in Fig. 11, but still higher than that for first strikes in the absence of defenses.

Figure 18 shows  $C_2$ , which does differ in essential ways from Fig. 12. The curve for  $I$  small has shifted from the top to the bottom, reflecting the defense's preference for the attacker's non-alert aircraft to be undefended. The other three curves again converge, but to a value only slightly lower than that without defenses. The curve for 2,000 SBIs falls monotonically, but only  $\approx 5\%$ . That for 1,000 increases a few percent before falling. The curve for  $I = 500$  increases about 7% by 1,000 SBIs and then falls. None of the increases appear significant.

As noted above, Figs. 15-18 were calculated under the assumption that half of the restriking RVs were allocated to aircraft and half to value. Figure 19 shows the effect of varying that allocation at 1000 SBIs, where there are still some RVs to allocate. The top curve is  $C_2$ ; the bottom  $C_1$ . Both fall as more restrike RVs are allocated to aircraft. Neither change is large, essentially because there are too few RVs to greatly impact aircraft contributions. The defender would like to minimize  $C_2$  by favoring aircraft in his allocation. That would, however, decrease  $C_1$  more. Their ratio is not, however, greatly different across the range. Thus, the earlier figures should not be sensitive to this allocation.

Figure 20 shows the stability indices. The variations are essentially the inverses of those in Fig. 18. The index for  $I \approx 0$  falls throughout; that for 2,000 increases throughout. Those for intermediate values first fall slightly and then rise and converge to unity. For 0 SBIs the index for  $I \approx 0$  is about 0.92, as before. Then for  $K = 0$  the index falls monotonically with



increasing I, which indicates that midcourse defenses alone would reduce stability under the stability metric used. By 1,000 SBIs and 2,000 midcourse interceptors, 2,000 SBIs and 1,000 midcourse interceptors, or 3,000 SBIs and 500 midcourse interceptors, however, the indices would have returned to about the no-defense level. They would then continue to climb above it as more SBIs or midcourse interceptors were added.

A trajectory that followed the curve for  $I = 0$  out to  $K = 3,000$  and then added 500 midcourse interceptors would appear to have the smallest transient degradation. Thus, the overall interpretation is much the same as that of Fig. 14. With or without defense of non-alert aircraft, modest defenses increase stability indices. Without their defense, however, it would appear best to deploy midcourse interceptors first. With their defense it would appear best to deploy some number of SBIs first.

## VII. ASYMPTOTIC CRISIS STABILITY INDICES

It is useful and plausible that the stability indices in Figs. 14 rise for large K and that those in Fig. 20 approach unity there. It is possible to see why they approach that asymptote. The investigation also sheds some light on the behavior of crisis stability in the long term. The discussion treats ICBM and aircraft strikes; SLBM penetration is higher order in  $x$ , so it can be ignored for large defenses.

### A. Preferential Two-Layer Defenses

The non-preferential layer is assumed to be followed by I ideal preferential interceptors. For large K few RVs penetrate. It is assumed that all B first strike aircraft weapons penetrate, so the first strike is  $\approx B$  weapons. The survival probability of targets defended by both layers is from Eq. (4)  $\epsilon \approx 1 - e^{-I/R}$ . The number of second-strike RVs is higher order in  $x$  and can be neglected. The number of aircraft restrike weapons is from Eq. (5)  $W = [\alpha + (1-\alpha)\epsilon]B \equiv \beta B$ . For  $x$  small, the cost of striking first is

$$C_1 = 1 - e^{-k\beta B} + Le^{-kB} \approx 1 - z^\beta + Lz, \quad (8)$$

where  $z \equiv e^{-kB}$ , and  $k$  is determined by the number of targets held at risk. The cost of restrike is then

$$C_2 = 1 - z + Lz^\beta. \quad (9)$$

Their difference is

$$\delta C_\beta \equiv C_1 - C_2 = (1 + L)(z - z^\beta). \quad (10)$$

Because  $0 < z < 1$  and  $0 < \beta < 1$ , then  $z < z^\beta$ ,  $C_1 - C_2 < 0$ , and  $C_1/C_2 < 1$ . Thus, defenses reduce crisis stability indices to some extent. However, for  $\epsilon \rightarrow 1$ , i.e. good defenses,  $\beta \rightarrow 1$ , so  $z^\beta \rightarrow z$ ,  $C_1 - C_2 \rightarrow 0$ , and  $C_1/C_2 \rightarrow 1$ , which is crisis stable. For  $\beta \rightarrow 1$   $C_1/C_2 \rightarrow 1$ .

#### B. Non-Preferential Defenses

If  $I \rightarrow 0$ , the only remaining defenses are the  $K$  SBIs, which are non-preferential. Then by Eq. (4)  $\epsilon \rightarrow 0$ ,  $\beta \rightarrow \alpha$ ,

$$C_1 = 1 - z^\alpha + Lz, \text{ and} \quad (11)$$

$$C_2 = 1 - z + Lz^\alpha. \quad (12)$$

The difference between the the strike costs is then

$$\delta C_\alpha \equiv C_1 - C_2 = (1 + L)(z - z^\alpha). \quad (13)$$

For  $\epsilon \rightarrow 0$ ,  $z - z^\alpha = \text{constant} < 0$ , so  $C_1 - C_2 < 0$ , and  $C_1/C_2 < 1$ , which indicates loss of stability.

The difference between Eqs. (13) and (10) is

$$\delta C_\alpha - \delta C_\beta = (1 + L)(z^\beta - z^\alpha). \quad (14)$$

Since  $z^\beta - z^\alpha < 0$ , the difference between first and second strike costs is always more negative for  $I = 0$ . For large defenses  $\beta \rightarrow 1$ . For example, for START's  $B \approx 4,500$  and  $V \approx 2,000$  targets

$$\delta C_\alpha - \delta C_\beta \rightarrow 1.3 \cdot (0.1 - 0.1^{0.3}) \approx -0.52, \quad (15)$$

so the difference can be large as seen in Figs. 14 and 20.

The presence or absence of a preferential layer has a much larger effect than the imperfect defense of restrike aircraft. The stability metric enters only through the monotonicity of the damage functions and  $C_1$  and  $C_2$  and the multiplier  $1 + L$ . Large variations in the weights given to damage limiting and damage would not alter the conclusions.

The discussion above concerns the limit of the transition from deterrence with missiles to deterrence with aircraft. If, after the latter was achieved, the missiles were eliminated, then

according to Eq. (10) for  $\beta \rightarrow 1$  the aircraft could also be reduced without loss of stability, because

$$\delta C_{\beta} = (1 + L)(z - z^{\beta}) \rightarrow (1 + L)(z - z) \approx 0 \quad (16)$$

for any  $z$  or  $B$ . The drop in their stability indices would be slight even for an order of magnitude decrease in the number of aircraft. Thus, the protection of aircraft and elimination of missiles followed by the elimination of the aircraft, too, would not appear to have any adverse asymptotic stability issues.

#### VIII. SUMMARY AND CONCLUSIONS

This note has derived a simple, approximate model for crisis stability that appears to contain most of the features needed to understand the variation of stability indices with defenses. ICBMs are attenuated exponentially; SLBMs more strongly; singlets are not attenuated but do not matter at START levels. For modest, symmetrical defenses, most of the first and second strikes are carried by aircraft. If attack aircraft cannot be defended, restrikes can be larger than first strikes, and stability indices can be larger than one. If they can be defended, first strikes can be as large as restrikes, and stability indices tend toward unity.

Under either assumption, first and second strikes are dominated by SLBMs in the absence of defenses, but they transition over to aircraft at modest levels of defense. Significant defenses protect some retaliatory missiles, but not enough to retaliate strongly. That makes detailed discussions of factors that might reduce aircraft pre-launch survivability interesting. This note does not address the ultimate goals of strategic defenses; only the transition from deterrence with missiles to deterrence with aircraft. It indicates, however, that in the latter, missiles would be vestigial and could be eliminated without penalty, and aircraft could be significantly reduced without loss of stability.

The combination of parameters that maximizes cost effectiveness also maximizes midcourse effectiveness and crisis

stability, which implies that maximizing cost effectiveness could simultaneously maximize crisis stability.

#### ACKNOWLEDGMENT

The author would like to acknowledge stimulating discussions with Lt. Gen. Glenn Kent (USAF Ret.), useful comments from Dr. Albert Petschek, and useful comments on the interpretation of asymptotic crisis index scaling from Dr. David Lee.

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Fig. 1. Penetration probabilities

$\alpha=0.3, m=10, M=270, B=4500, V=2000$

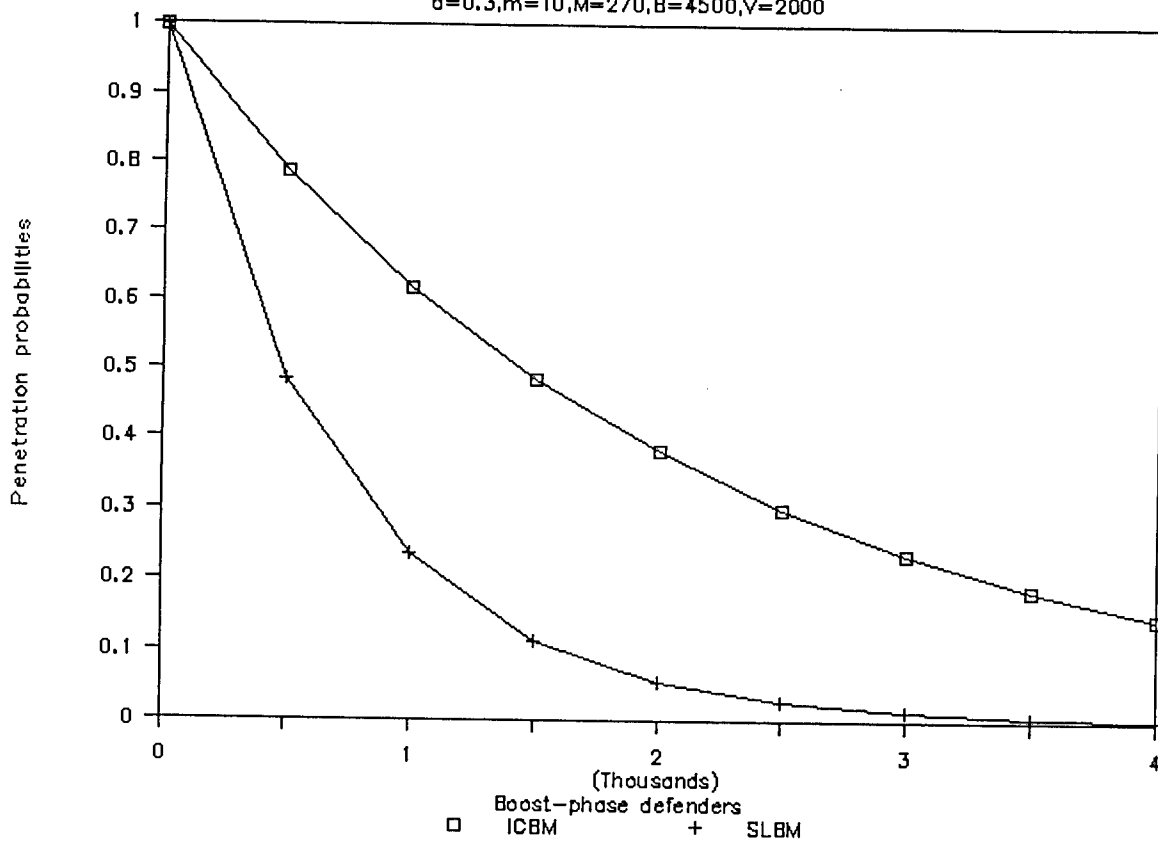


Fig. 2. First strike on value

$\alpha=0.3, m=10, M=270, B=400, V=2000$

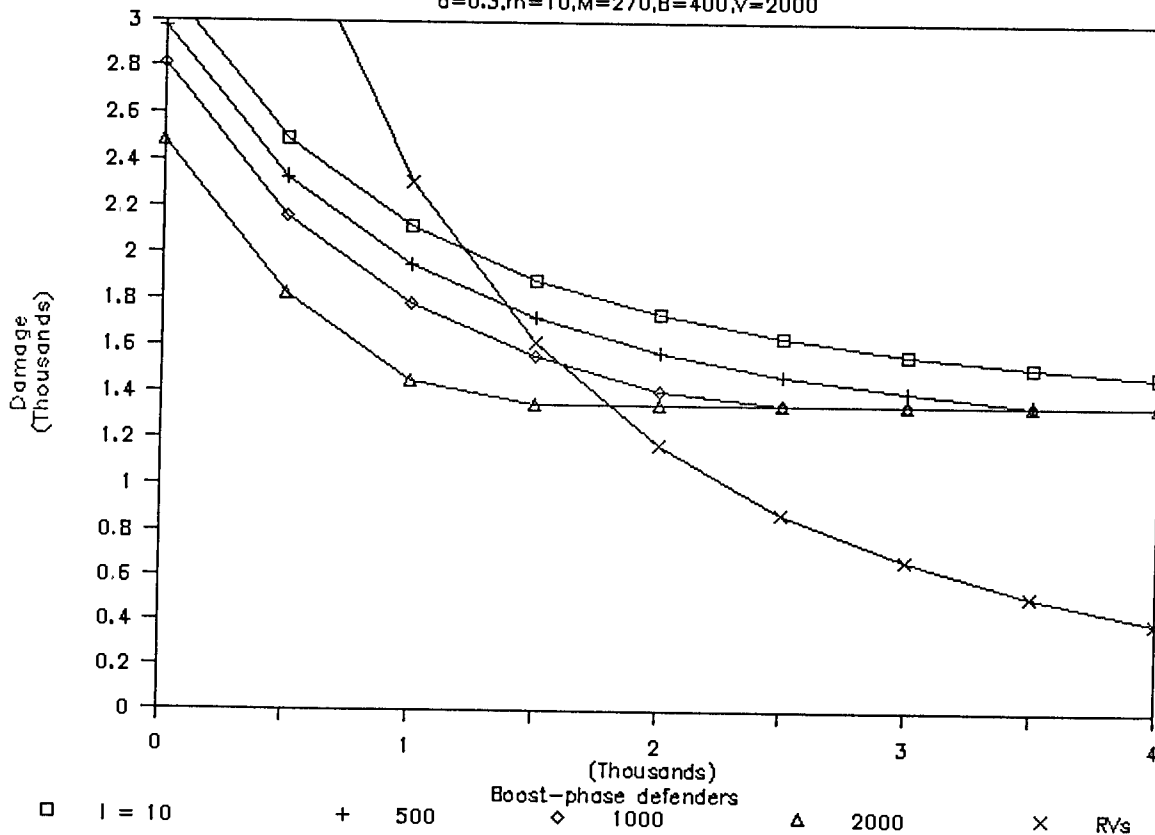


Fig. 3. Survival probability

$\sigma=0.3, m=10, M=270, B=400, Y=2000$

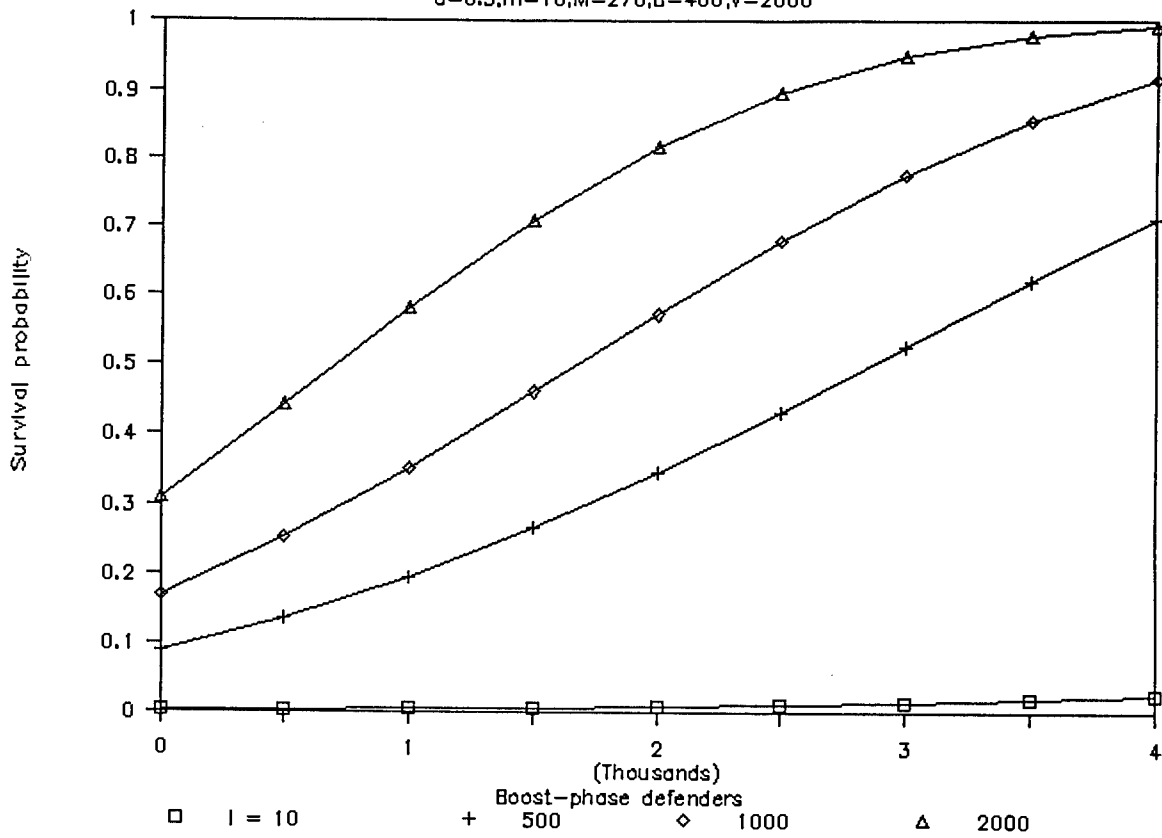


Fig. 4. First strike RV penetration prob

$\sigma=0.3, m=10, M=270, B=400, Y=2000$

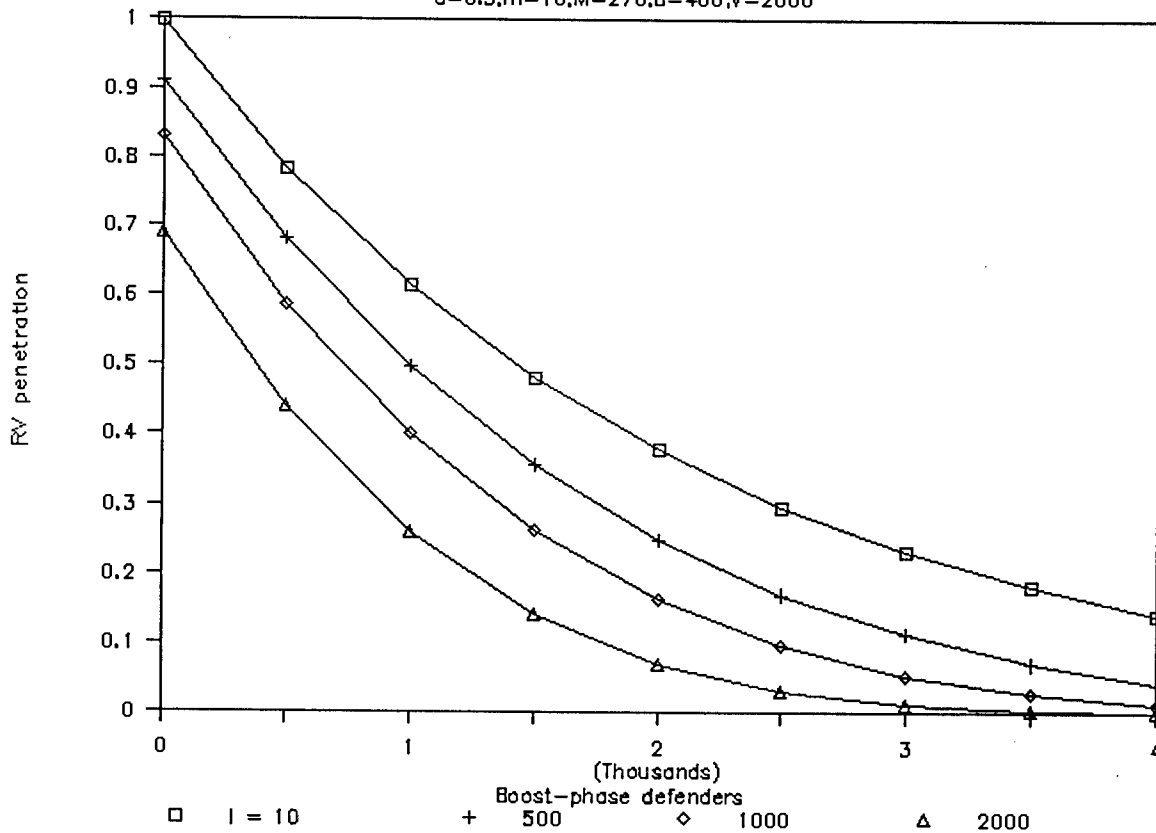




Fig. 5. Restrike through boost defenses

$\sigma=0.3, m=10, M=270, B=400, V=2000$

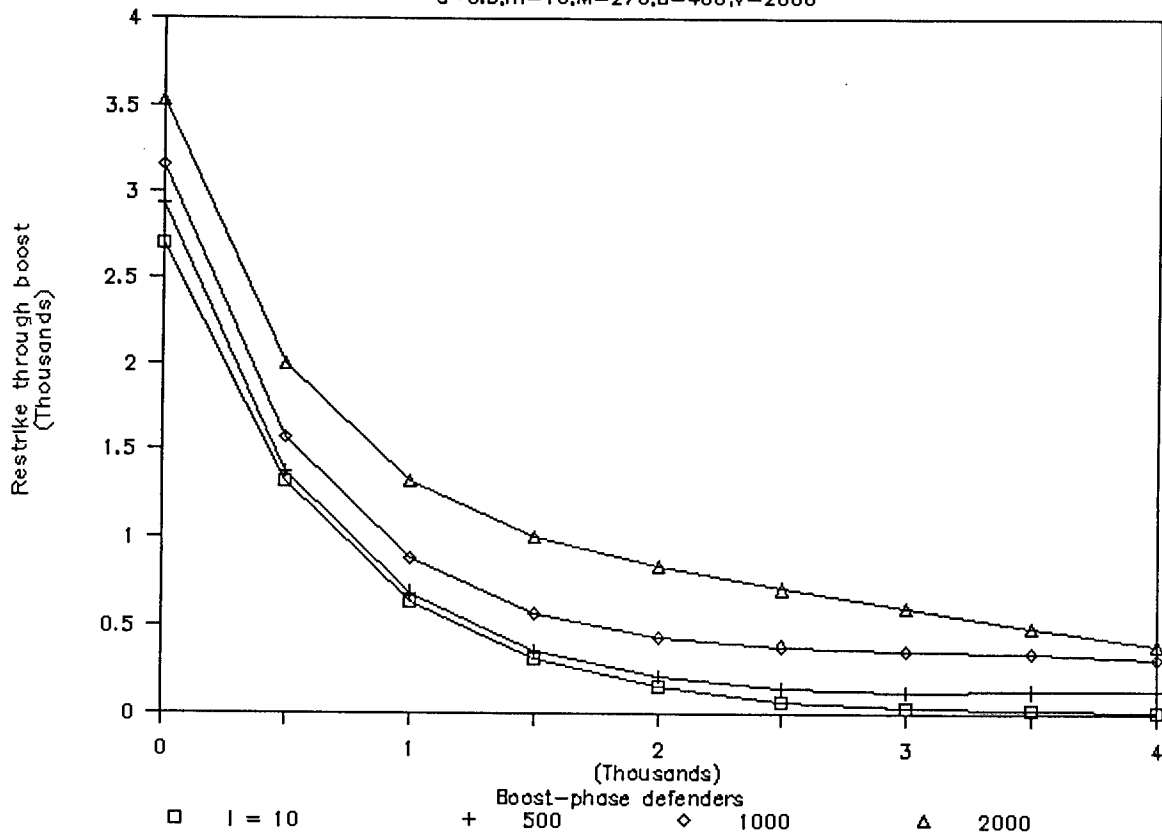


Fig. 6. Restrike penetration of boost

$\sigma=0.3, m=10, M=270, B=400, V=2000$

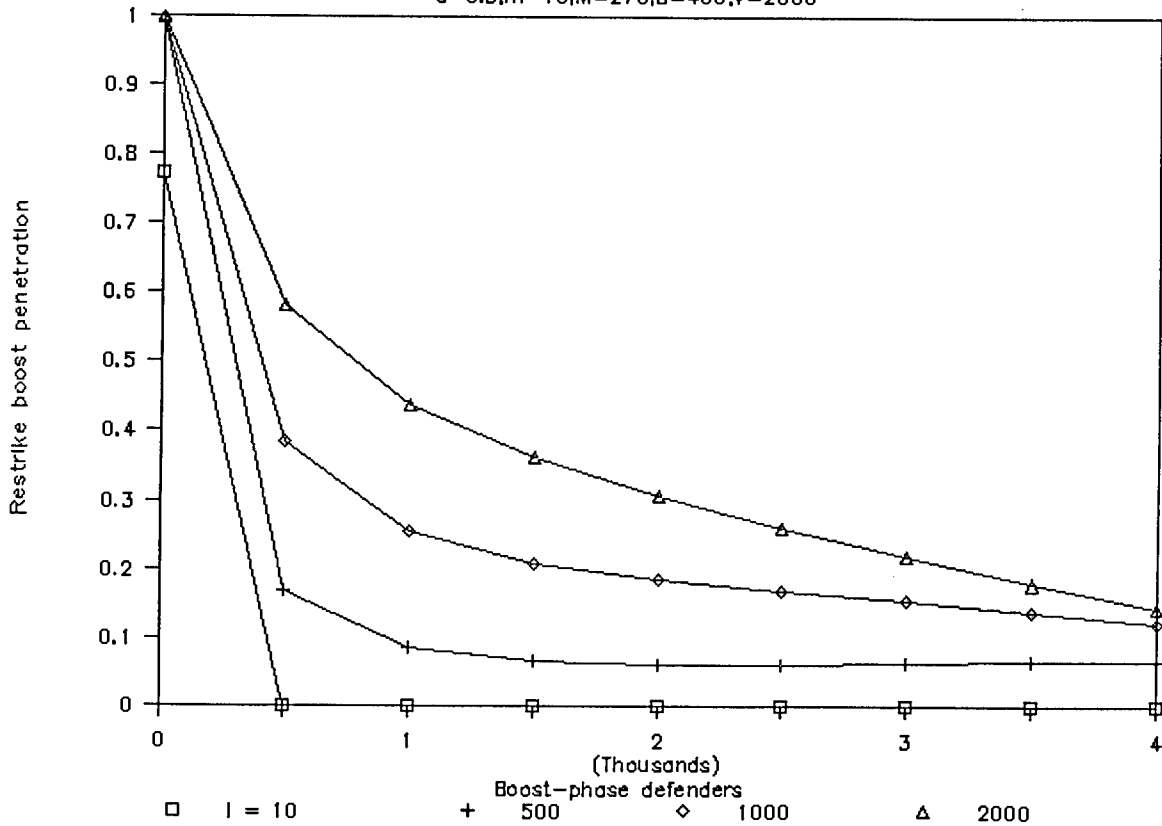


Fig. 7. Restrike penetration of terminal

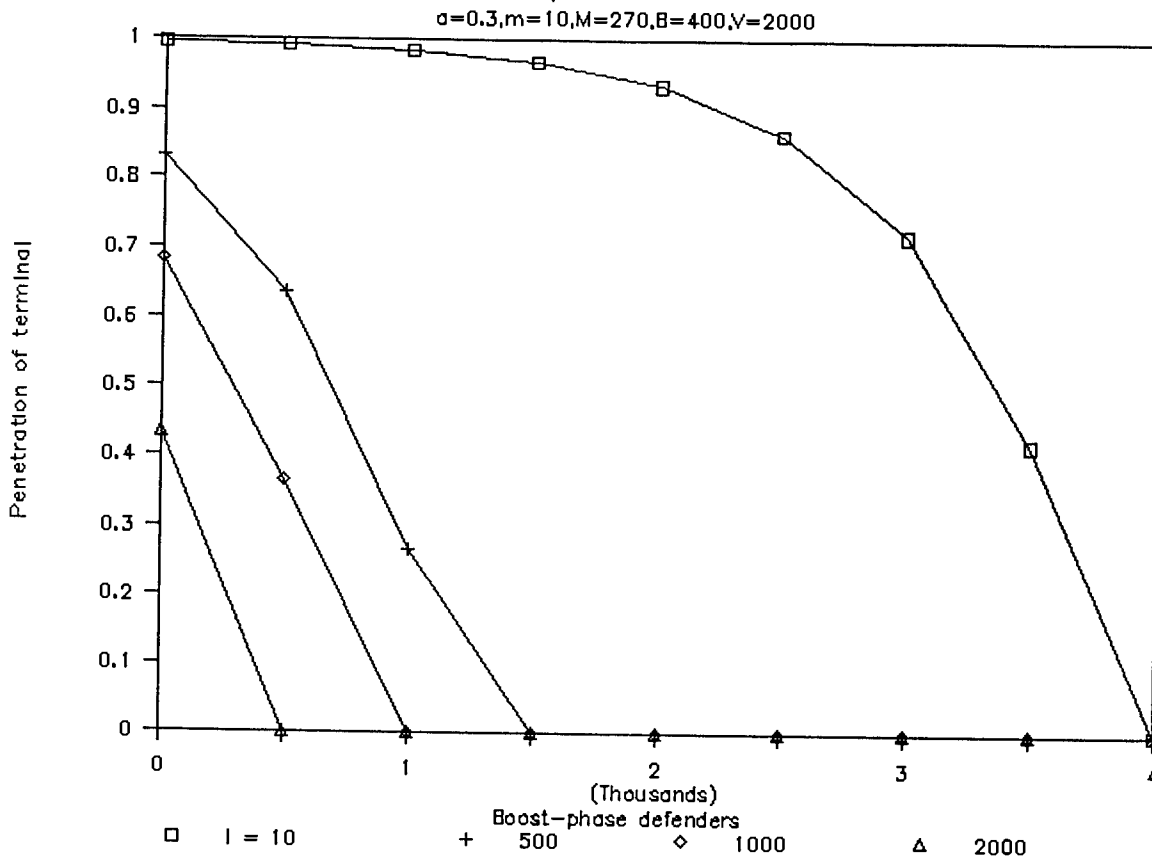


Fig.8. Overall restrike penetration prob

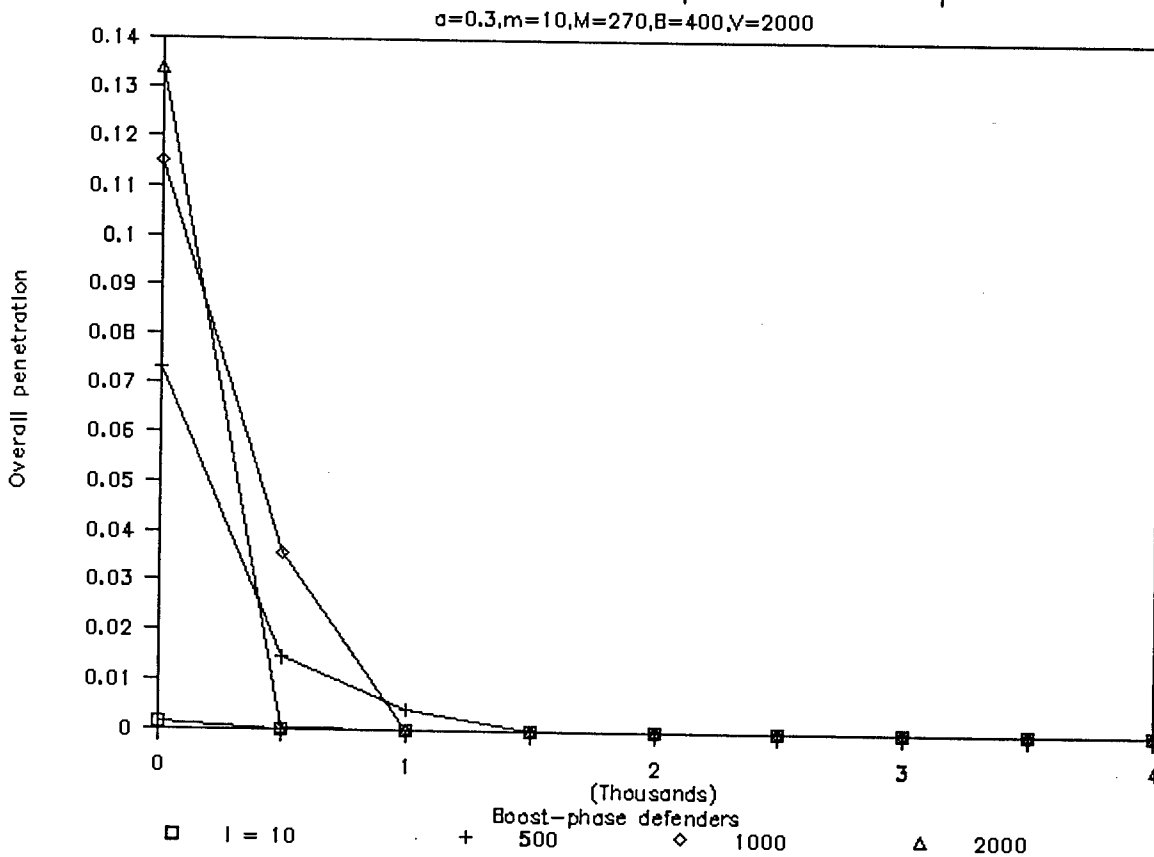


Fig. 9. Restrike

$\sigma=0.3, m=10, M=270, B=400, Y=2000$

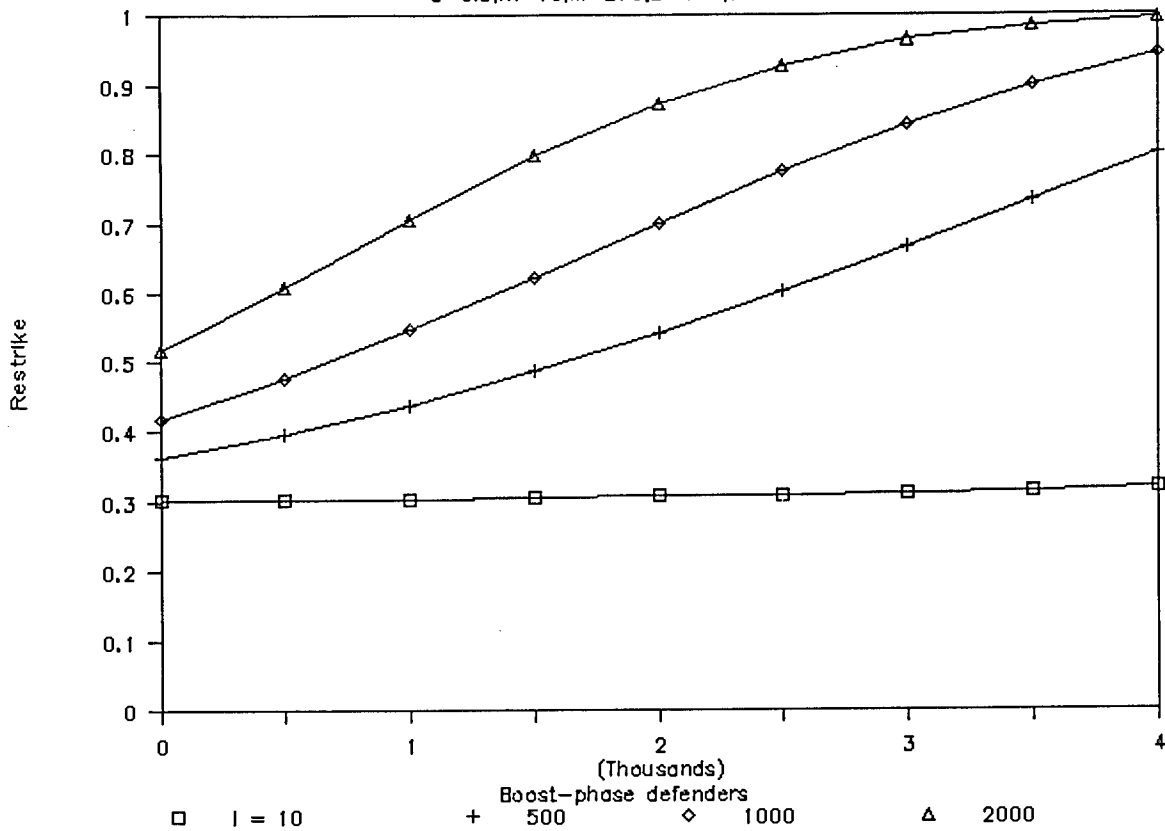


Fig. 10. Restrike

$\sigma=0.3, m=10, M=270, B=400, Y=2000$

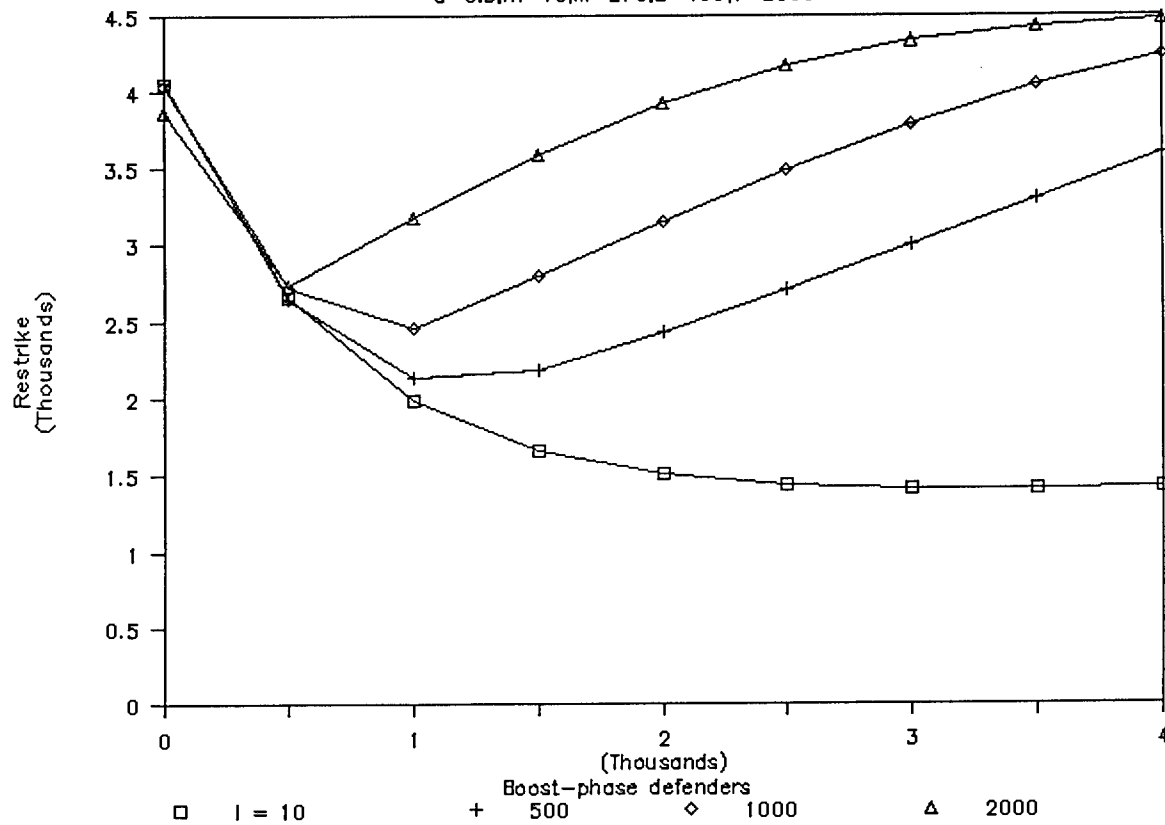


Fig. 11. Cost of first strike

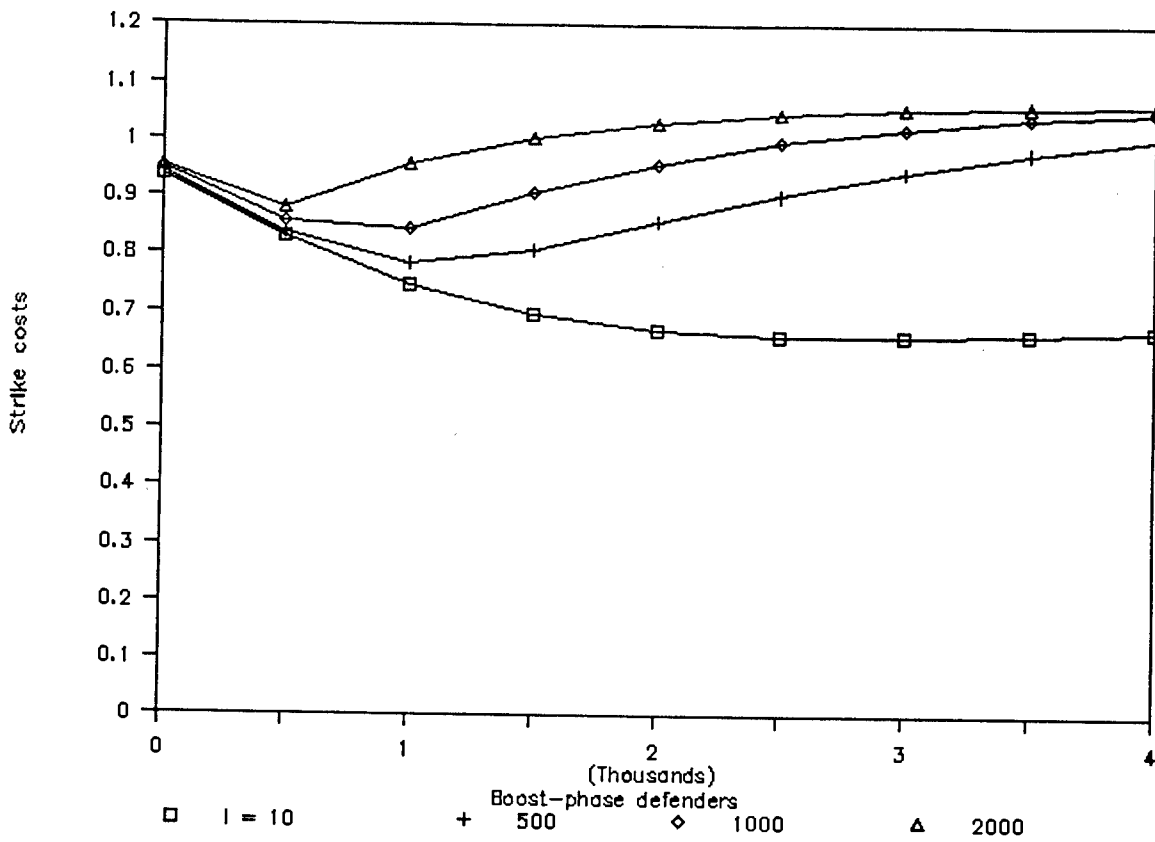


Fig. 12. Restrike costs

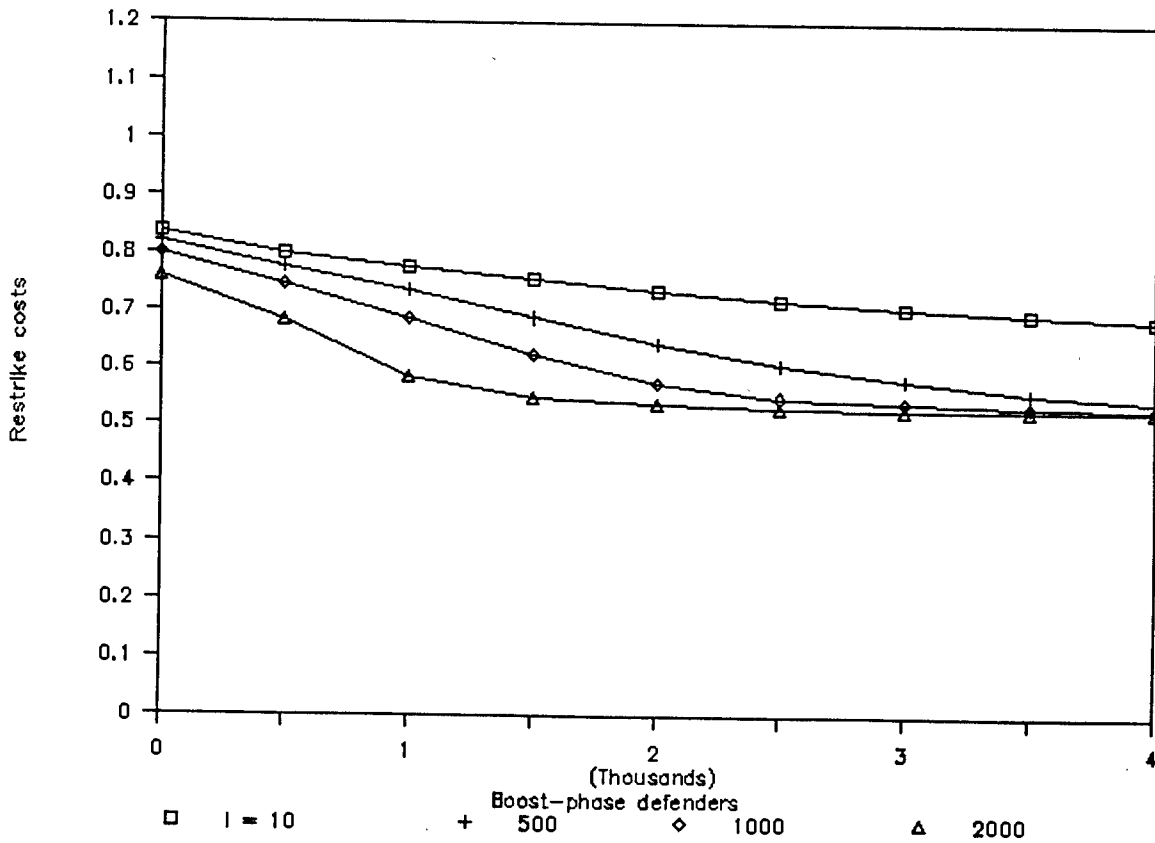


Fig. 13. Costs and stability index

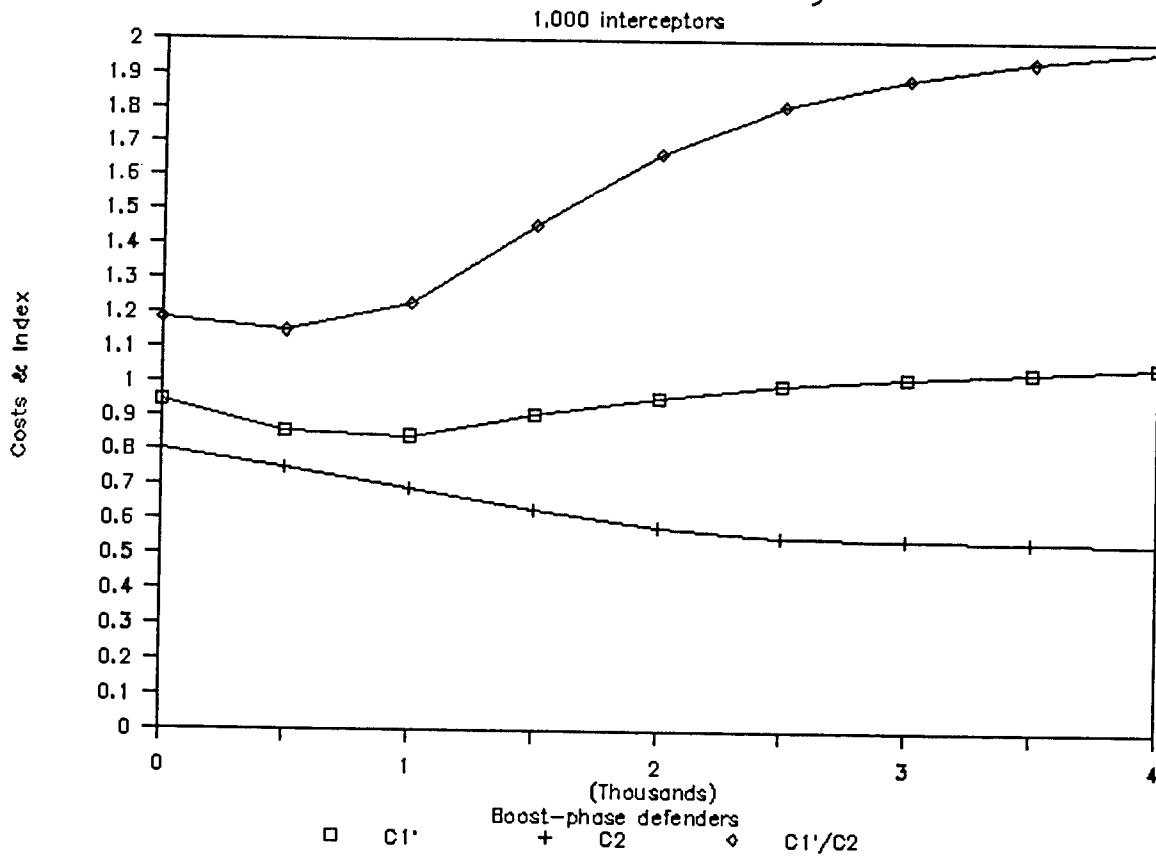


Fig. 14. Stability index

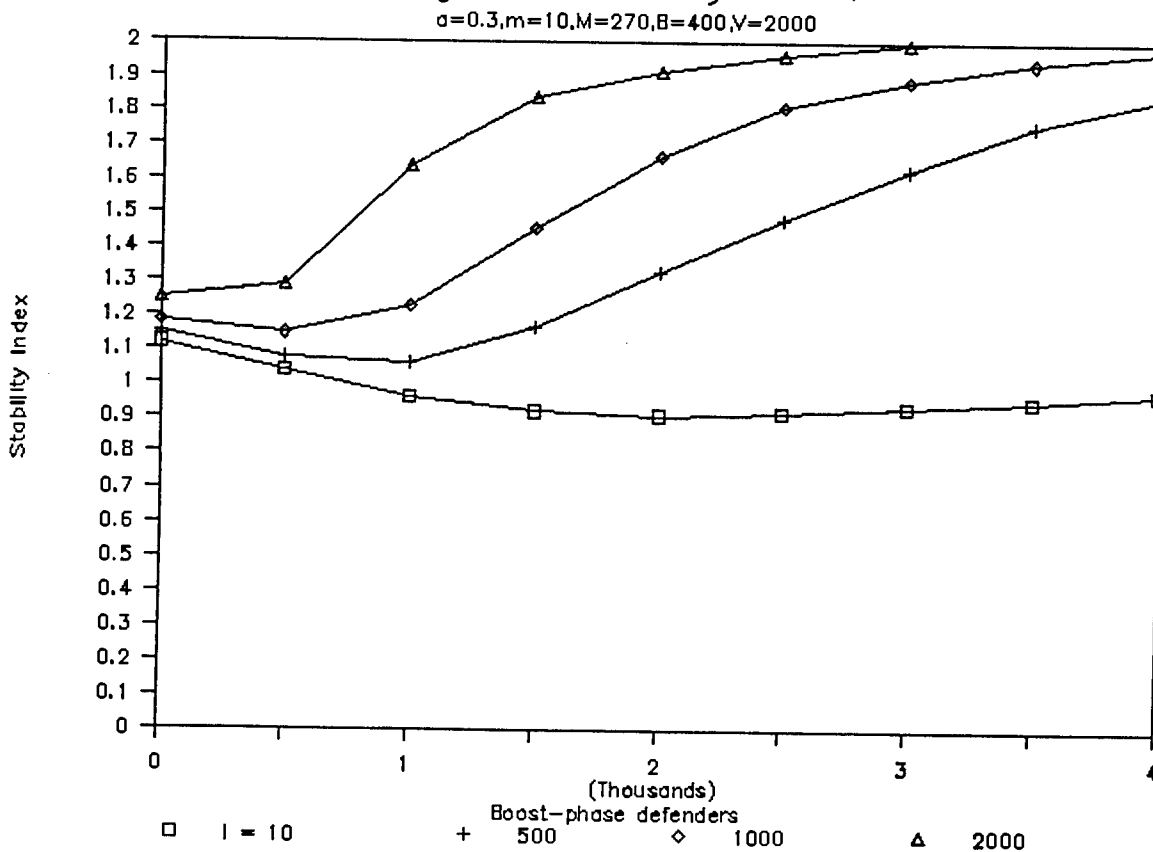


Fig. 15. Non-alert survival probability

$\alpha=0.3, m=10, M=270, B=400, V=2000$

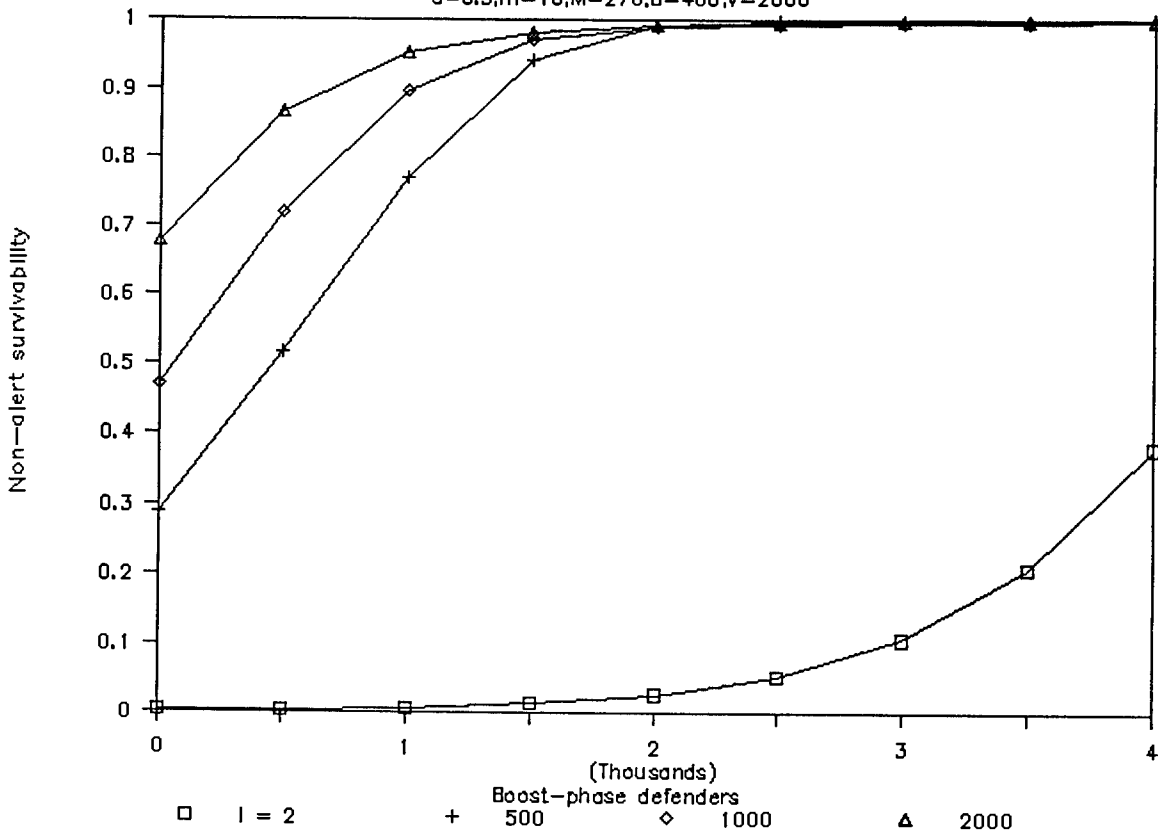


Fig. 16. Strike with aircraft defense

$\alpha=0.3, m=10, M=270, B=400, V=2000$

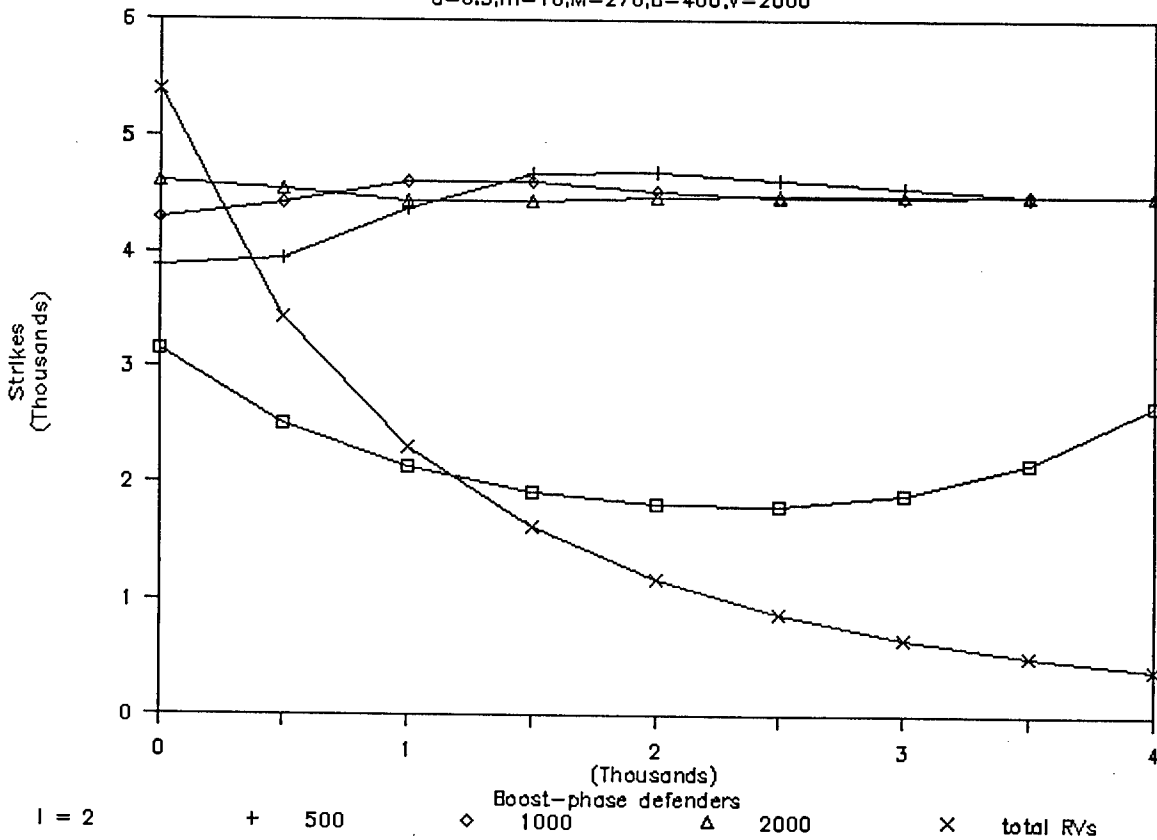


Fig. 17. First strike costs

$\sigma=0.3, m=10, M=270, B=400, V=2000$

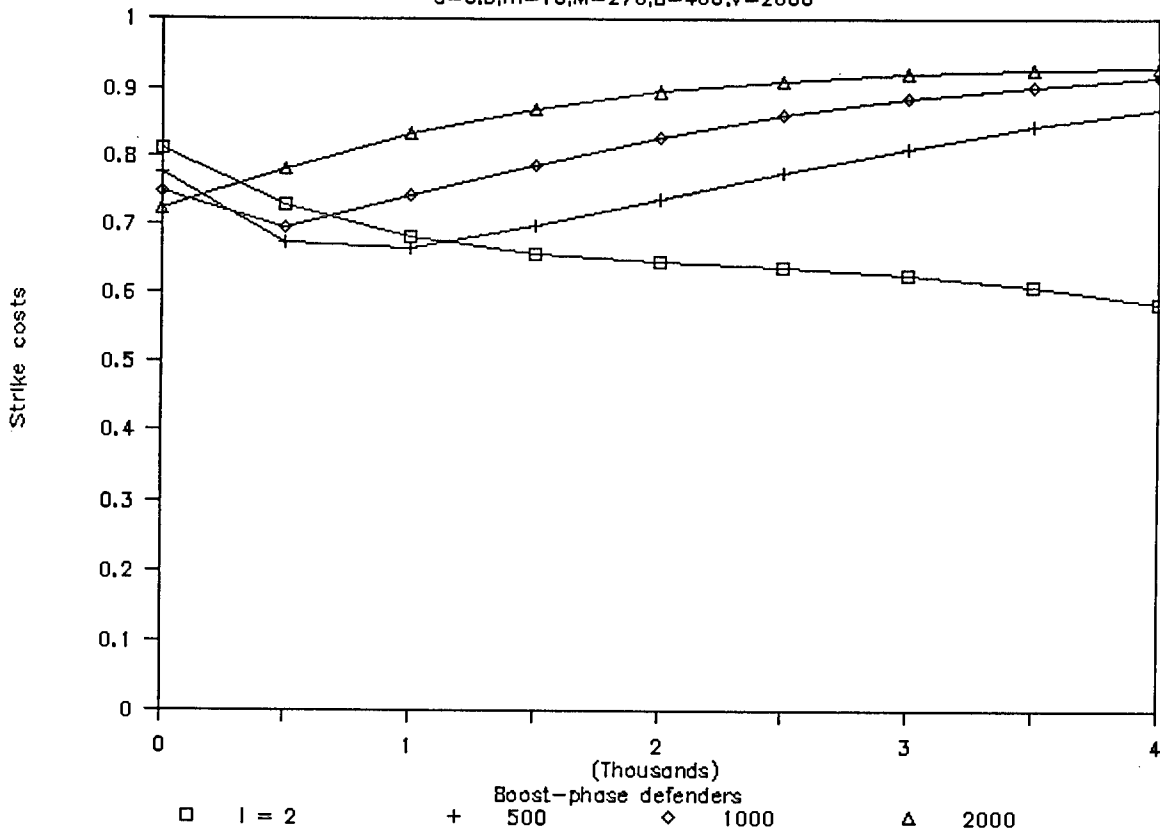


Fig. 18. Restrike costs

$\sigma=0.3, m=10, M=270, B=400, V=2000$

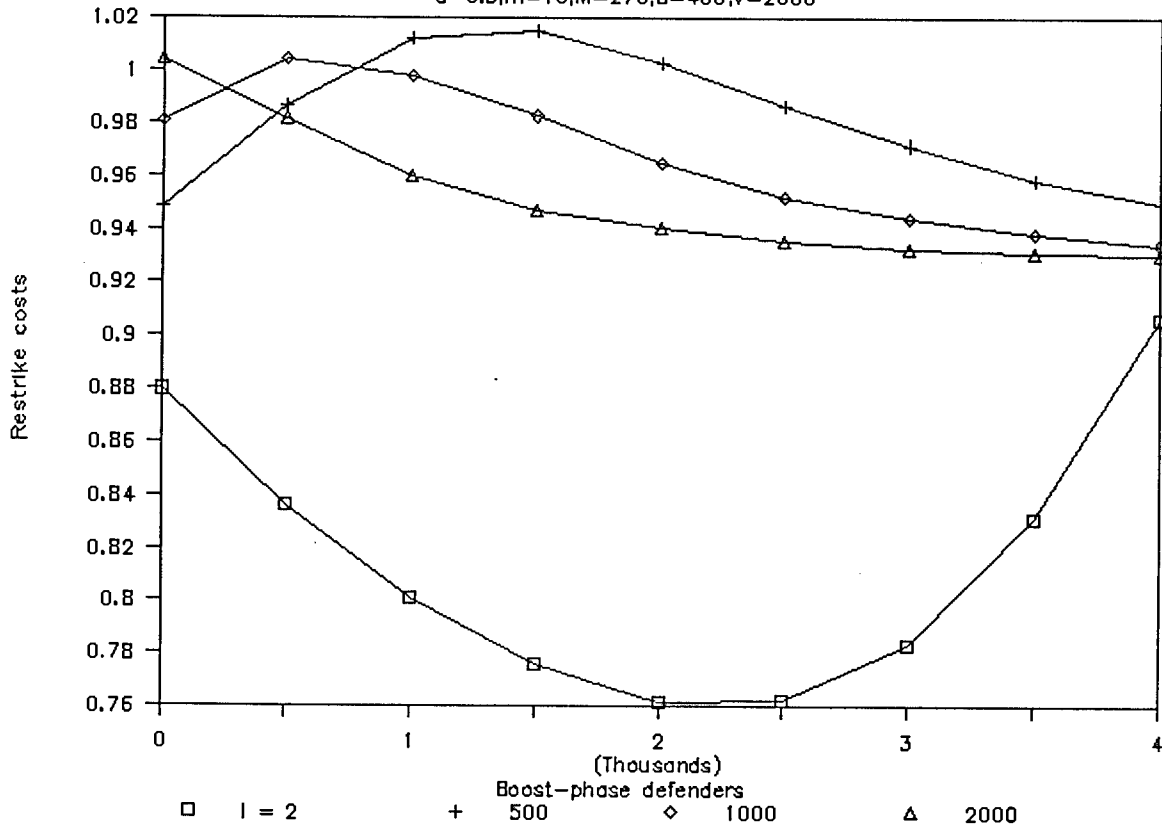


Fig. 19. Strike costs vs RV restrike %

$\alpha=0.3, m=10, M=270, B=400, V=2000$

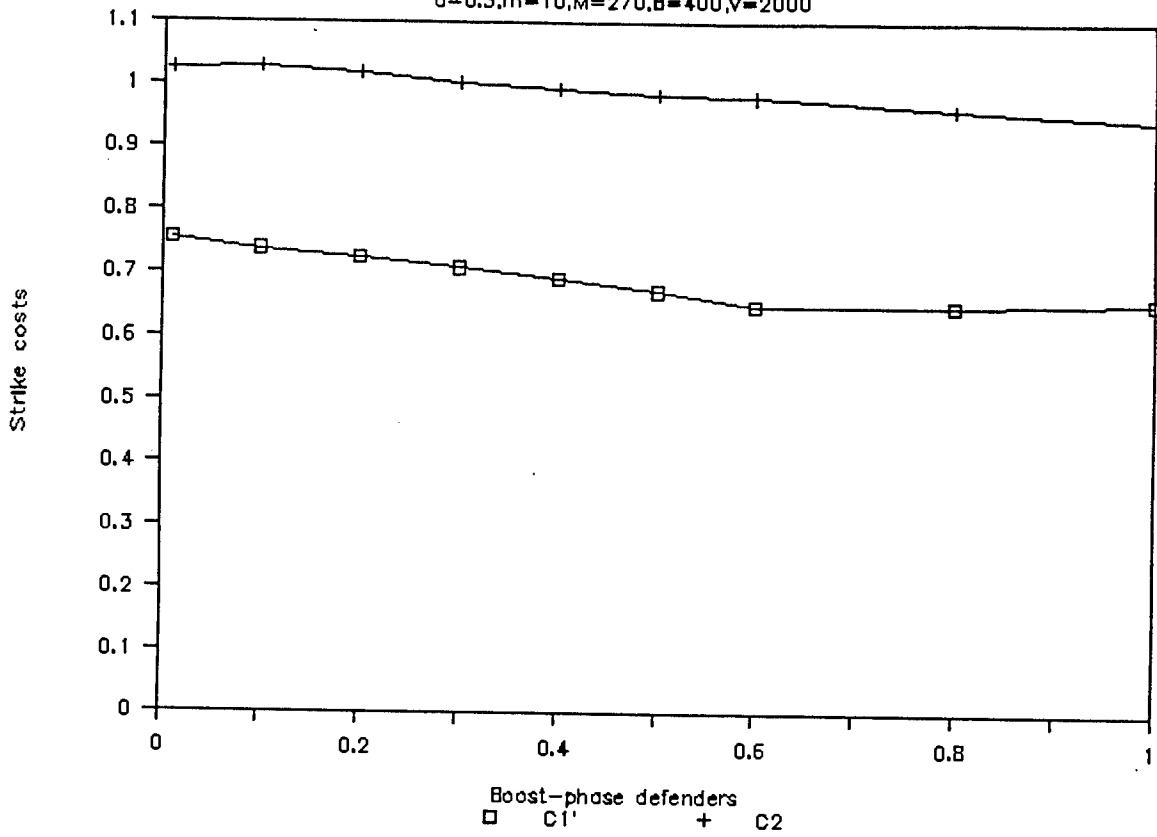
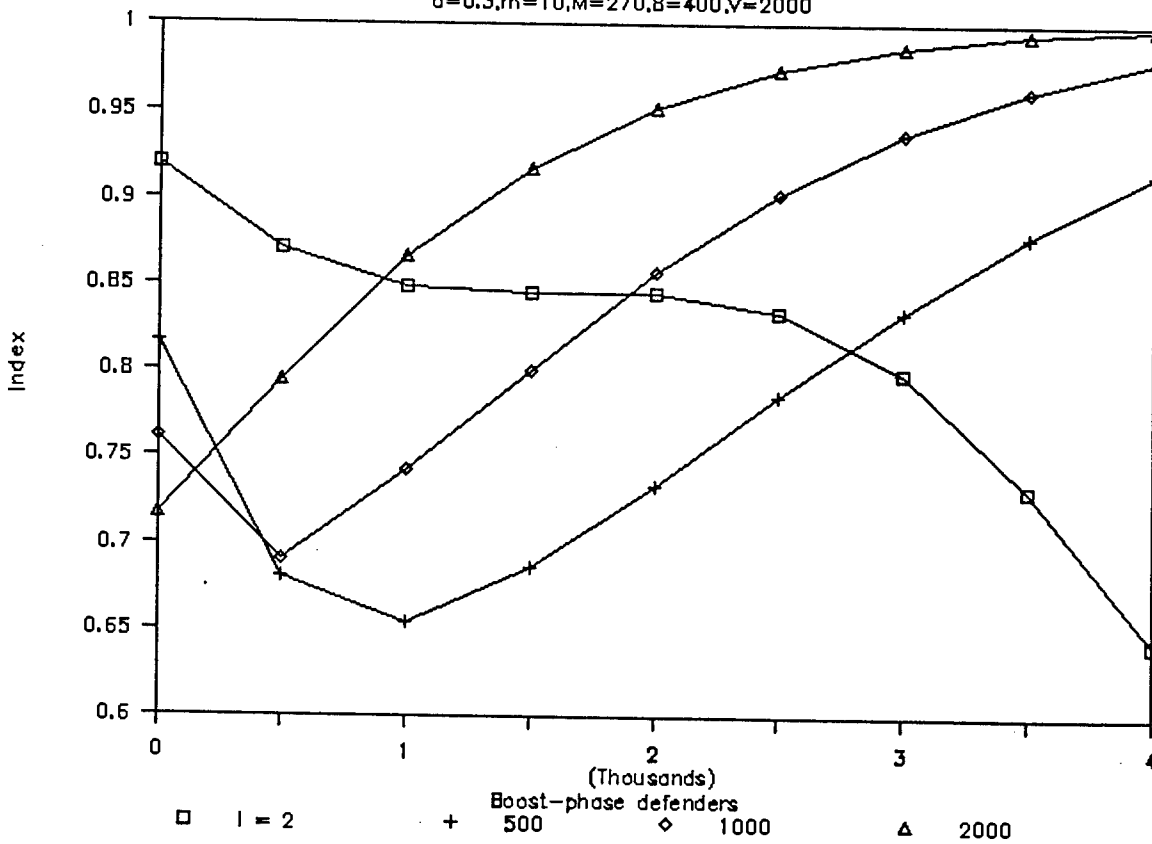


Fig. 20. Crisis stability index

$\alpha=0.3, m=10, M=270, B=400, V=2000$





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